Open Geoboard – a Platform for Art, Math and Inspiration

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Abstract

The Open Geoboard is an upgrade to the classical geometric board. The enhancements we have introduced are: high-precision machined wooden removable pegs and a plywood board with high resolution grid of holes, ergonomic pegs of different height (transforming it into the first 3D geoboard) and various paper templates that change the board's application when placed on top of the board. We propose a workshop which reveals the extended capacity of the "Open Geoboard" as a tool for creating exciting 2D and 3D interdisciplinary projects. The workshop combines hands-on activities with different teaching techniques like demonstration, guided insight and free improvisation. It is suitable for students above 8 years of age, parents and teachers in primary and high school.

Introduction

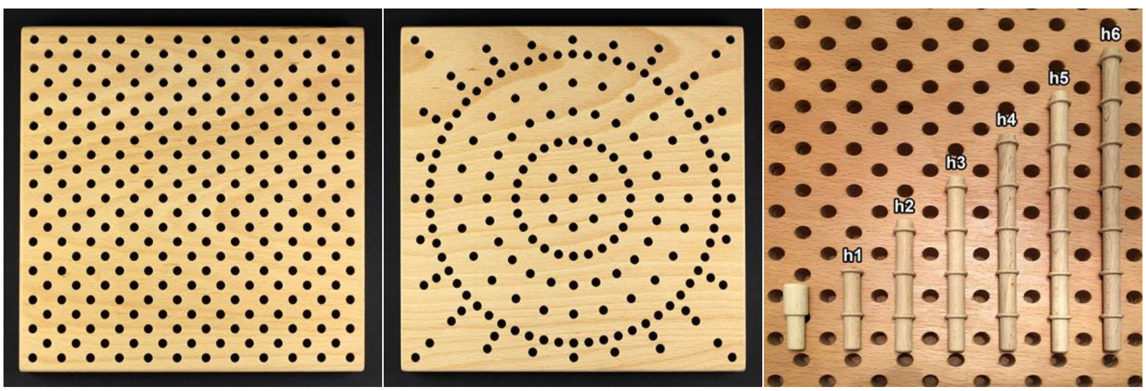
The Geoboard or the Geometric Board was invented and popularized in the middle of the 20th century by the Egyptian mathematician Caleb Gattegno [1, 2]. Basically the Geoboard consists of a square wooden base with fixed pegs or nails arranged in columns and rows. Rubber bands can be wrapped around the pegs and thus form a variety of shapes. Initially, Gattegno proposed the Geoboard as a tool for energy efficient learning of math concepts such as *counting*, *adding*, *subtraction*, *area*, *perimeter*, *position*, *pattern*, *scaling*, *plane shapes, angles, rotation*, *reflection*, etc. [7]. But it turned out that the Geoboard can also be widely used as a platform for art work, design and game play. Applied at an early age the Geoboard develops fine motor skills and enhances learning. It is also suitable as a teaching tool for students with special needs.

The "Open Geoboard" – an upgrade. Our backgrounds as parents, makers and teachers, combined with daily observations of our children playing and learning in different environments (including Montessori and Waldorf), triggered our imaginations how to combine our passions for making and teaching. Inspired by the simplicity of Caleb Gattengo's invention, the Open Geoboard was born in the SmartFabLab [5] in Sofia, Bulgaria. Our contribution to his idea evolved further into an open design platform, through which students can explore and play with mathematical concepts like figures, forms, symmetry, 2D and 3D space and much more. It can be used as a learning material at home and in the classroom by students and their parents, teachers and educators. The geometric board and different rubber bands, wooden pegs, templates and other elements are the tools for making diverse forms, figures, objects, art and much more in two- and three dimensional space. Changing the template on the board makes it a base for board games like Ludo, Chess, Maze, etc.

Innovations in the "Open Geoboard". With respect to the initial design and use we introduce a completely new set of features and ideas to the Geoboard. Those are:

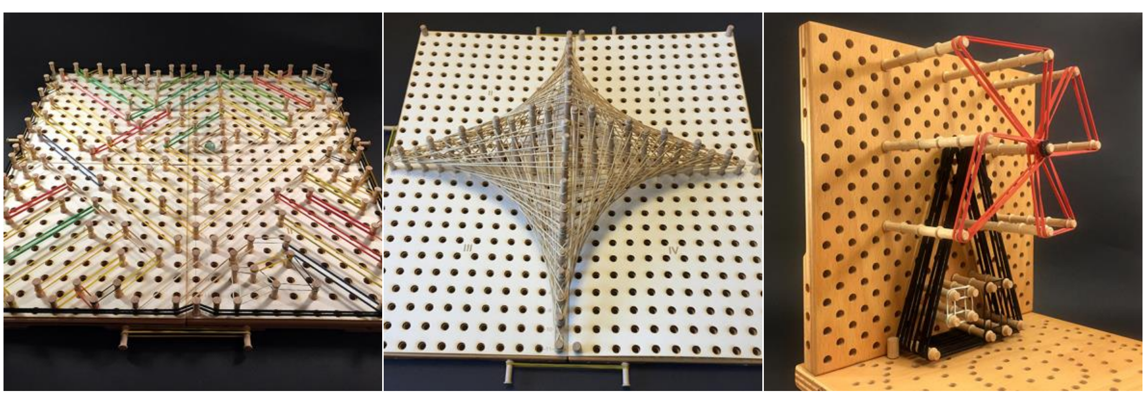
- *Wooden boards* – Each Open Geoboard is a solid wooden board made out of beech plywood. It is a square 24 by 24 cm, 1,8 cm thick and weighs around 800 gr. It feels very natural and heavy in one's hands. It provides a strong structural integrity and is a reliable fundament for building patterns. The holes are ¼ inch (6,35 mm) in diameter and are placed 2 cm apart. The plans of the wooden parts and templates of the Open Geoboard will be available to download at www.geoboard.co under a Creative Commons license. The size and shape of the board can vary if needed. It is also possible to be made of different materials.

- *Enhanced coordinate systems* – Unlike the small number of pins in the traditional geoboards, the Open Geoboard set consists of 2 boards with high resolution Cartesian (Figure. 1a) and Polar (Figure 1b) coordinate systems with 265 and >170 holes respectively. Therefore, along with classical geometry shapes, a whole new set of patterns can be built – more complex and more detailed.
- *Movable pegs in different lengths and modularity* – Pegs are ¼ inch (6,35 mm) in diameter, made of beech wood, which are high-precision machined on lathe so that they fit snugly into the holes. The pegs have ergonomic design which prevents the rubber bands from slipping away, come in 6 different heights, plus there is an anchor peg for the templates (Figure. 1c).
  Figure 2. illustrates that all pegs are movable which enables the teacher to guide the building process where students go through planned sequences of steps. For example, they can start with a rubber band and two pegs when they are asked to build a line, three pegs for a triangle and four for a square. This is particularly useful when a new topic is introduced and the teacher would like to draw the student's attention to some crucial points. The boards can also be paired in a plane (Figure. 2a and 2b) creating a bigger workspace, or at a 90 degree angle to create a 3D space (Figure. 2c.).
- *3D upgrade* – Figures 2. a, b and c demonstrate how by only varying the length of the pegs we can plunge ourselves right into the third dimension.



(a) (b) (c)

Figure 1: *The Open Geoboard. (a) Plywood board with Cartesian Coordinate system, (b) Plywood board with Polar Coordinate system, (c) Wooden pegs in different lengths.*



(a) (b) (c)

Figure 2: *Modularity and 3D. (a) Four boards big maze, (b) Four boards form a huge coordinate system, (c) Two boards at a right angle – a pattern can be shown in an upright position.*

- *Diverse templates* – We have created a variety of additional templates (Figure. 3) that enable users and teachers to predetermine initial conditions of the building process. They are thin layers of laser cut and engraved cardboard fixed on top of the wooden board with the anchor pegs to cover some of the holes and to add markings (the templates will be made from laminated paper when mass produced). For instance the number of holes can be reduced to 5x5, 10x10, 11x11. There are different templates for a coordinate system, multiplication and division boards, clock, pie chart, chess, mazes, and many more. The templates increase tremendously the usability of the Open Geoboard and provide guidance for problem solving (coordinates, clock, percentage count, etc.). By reducing the number of holes to 5x5 even the age group 3-6 can benefit from the boards. Furthermore, the Open Geoboard enthusiasts can make their own template designs serving their needs and imagination and share them with the community.

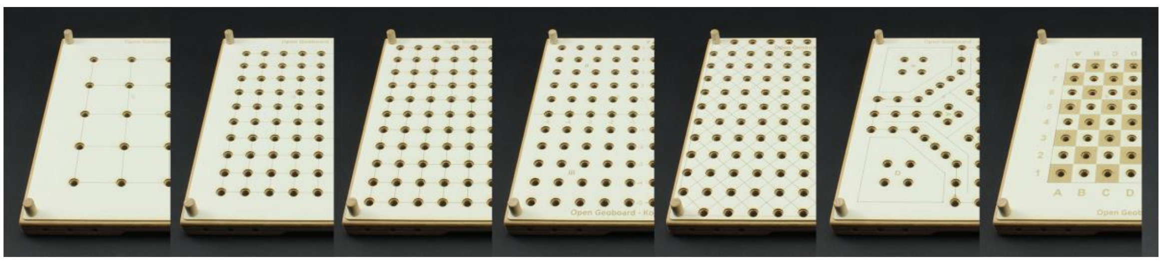


Figure 3: *Templates: 5x5, 10x10, 11x11, Coordinate System 11x11, Maze, Ludo, Chess.*

- *Colourful rubber bands*. A set of rubber bands of different size and colour gives the ability to emphasize patterns or create visual effects (Figure. 2, Figure 4).
- *Open Design* – **Fab**rication **Lab**oratory-ready designs. We are developing the Open Geoboard as an open design platform under a CC license so everyone is able to make it in a FabLab or a Maker Space using CNC milling machines, laser cutters and some hand tools. Our goal is to encourage more people to tinker around the idea of the Open Geoboard and to create an active community of teachers, students and parents.
- *Teaching* – We are developing classroom kits assembled with instructional cards, lesson plans and worksheets for kindergarten, pre-school, primary and high school curriculums, as well as after class projects and use for children with special learning needs.
- *Art creations done on the* "*Open Geoboard*" – Beyond the main focus on math the Open Geoboard platform can be successfully used as a creative 3D art canvas (Figure. 4).

Figure 4: *Creative art.*

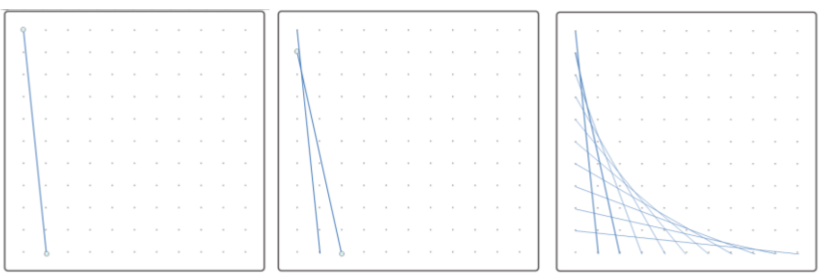
Workshop

Activity 1. Brainstorming. Our workshop is suitable for students 8+ of any educational backgrounds. First we explain how the Open Geoboard parts can be used. Then the whole group brainstorms upon designs including all components – boards, pegs, templates and rubber bands. There are no rules - only improvisation and imagination. Once all participants have agreed upon a particular design they decide how to build their patterns. They work either all together, in small teams or in pairs. The group is free to decide how to distribute the building elements among the participants.

Reflection. We end with a discussion about what was easy, what was hard, what worked and what went wrong. Participants will share what math or art concepts they came up with during the free play with the Open Geoboard. Should there be teachers and educators present this could expand into a discussion on how the Open Geoboard can facilitate teaching math concepts at primary and/or high school.

Activity 2. From Simple Steps to Complex Art. Here our goal is to demonstrate how we can create patterns by iterating simple steps without variations. Thus we will generate some repetitive structures or regularities with the Open Geoboard. Please be aware of those and observe how the shape evolves.

Study in 2D. Participants work in pairs. Each pair is supplied with a Cartesian board and numerous short pegs and rubber bands. They are asked to follow a simple sequence of steps: (1) insert pegs in the first row and in the last column; (2) make a line by wrapping a rubber band around the first peg of the column and the second peg of the row; (3) Now connect the 2nd peg of the column to the 3rd peg of the row; (4) Repeat connecting pegs with rubber bands until no peg pairs are left free (Fig. 5).



(a) (b) (c)

Figure 5: *Activity 1. How to build a parabola out of straight lines? (a) Step 1, (b) Step 2, (c) Result.*

Study in 3D. Participants work again in pairs. Now an additional rule about height is included – the pegs are getting higher while approaching one of the corners.

Reflection. Participants share their personal experiences, emotions and thoughts during the activity and share their observations from an aesthetic perspective. Finally, they can suggest how to continue the workshop by selecting a number of options: Guided Improvisation, Free Improvisation or a Group project.

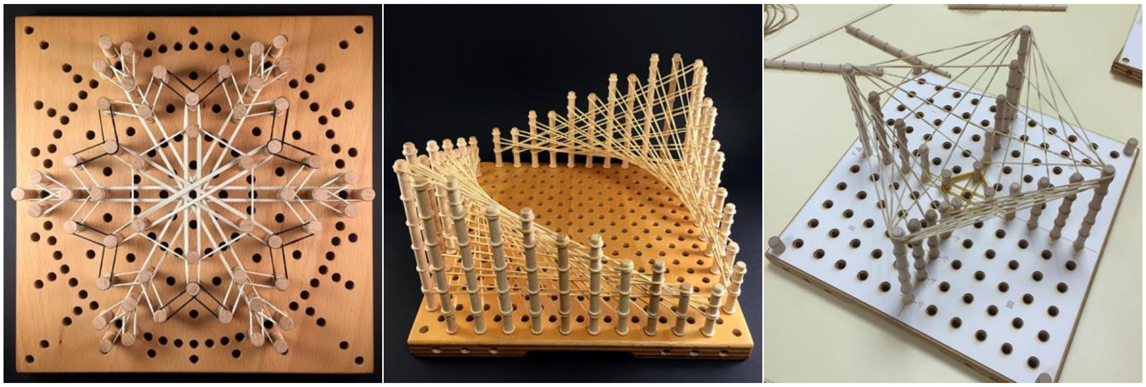
Activity 3. Guided Improvisation: Small Variations at the Beginning lead to Dramatic Changes in the Result. For the guided improvisations there are still some rules to be followed when building the patterns, however each participant has the freedom to experiment either with the position of the pegs or their height, or they can manipulate both. The groups decide how to proceed. They can either divide the board and come up with individual designs, or they can collaborate. *Please do not forget that we will continue to use steps 2, 3 and 4 from* "*Activity 1*" *no matter how we decide to experiment further* (Figure. 6).

Vary position. In this activity participants will be able to choose (1) the type of the geometric board – Cartesian or Polar; (2) the starting point and (3) the angle between the lines; (4) lines can be straight or change directions; (5) the pattern can be symmetrical or not.

Vary height. Similarly participants can vary the length of the pegs as they wish.

Vary both. This is a combination of the two previous approaches.

Reflection. Participants present their designs. The group evaluates deigns by originality and aesthetics.



(a) (b) (c)

Figure 6: *Examples for variations using the Polar and Cartesian Open Geoboards.*

Activity 4. Free Improvisation. There are no building rules for this build, only improvisation and imagination. First the whole group brainstorms upon designs including all materials - boards, pegs and rubber bands. After all participants agree upon a particular design, they decide how to build the shape. It can be either all together, by smaller teams or by pairs. The group decides how to manage materials between participants as well.

Reflection. Participants present their designs. The group evaluates deigns by originality and aesthetics. We will discuss what was easy, what was hard, what worked and what went wrong.

Activity 5. Math Reflections. Parabolic Surfaces - Curves or Straight Lines After All!? Paraboloid is a quadric surface that has (exactly) one axis of symmetry and no center of symmetry. The term “paraboloid” is derived from parabola, which refers to a conic section that has the same property of symmetry [8]. You can explore paraboloids by plotting their characteristic equation $x^2/a^2 + y^2/b^2 = z$ with a 3D graphing software like Mathematica or Matlab (Figure. 6b).

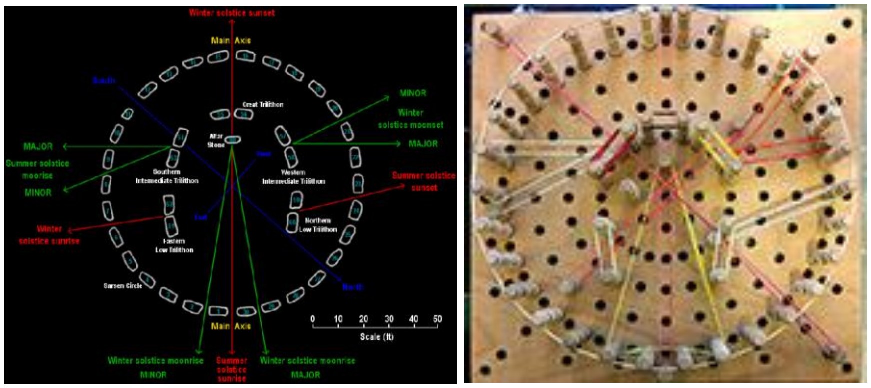
Iterations and Variations. We experienced how by repeating small sequences of steps the result of the previous iteration is the initial point for the next iteration. We saw that by using this simple algorithm we can build highly complex patterns, which might be even beyond our current understanding of math. Furthermore, when we introduce variations to our system the outcome of each build differs dramatically from the others. Nature very often combines these simple principles to generate diversity in forms and patterns. Can you give some examples? Which natural pattern resembles your design?

Combining Models. You can combine boards, pegs and modules to create a composite plane, where each board is a tile. In what other ways can you combine them?

Applications. Explore applications of the similar parabolic designs in art, architecture, engineering, etc.

Closing. Short discussion sharing opinion: How do I feel? What I liked? What I would change?

Activity 6. Extend Interdisciplinarity. Paleo Architecture and Astronomy. Stonehenge. The Open Geoboard enables us to go deeper into ancient architecture and astronomy. For example the Polar board can serve as a platform to model the megalithic monument of Stonehenge with its massive dolmens represented by the wooden pegs. Thus, a quick overview of the architecture plan, building materials and techniques can be given. Further, we can elaborate on the ancient astronomy by showing different trajectories and angles that mark the compass directions, points of the moon and sun rise and set at summer and winter solstices. This is a good example of a multidisciplinary approach where each subject can be emphasized differently depending on the context. Additionally, we can discuss that Open Geoboard the difference between a discrete set of points and a continuous space, e.g., how the discrete pattern of holes in the board forces one to make an approximation to, say, a circle. Further, how the available set of angles might not match precisely with the exact astronomical angles? (Fig. 7)



(a) (b)

Figure 7: *Stonehenge model and marks for major moon and solar astronomical events: (a) schematic plan from Astro-Archaeology at Stonehenge [6], (b) build by Open Geoboard.*

Summary and Conclusions

Altogether the Open Geoboard has proven to be a versatile platform for playful learning and creation, already successfully tested with dozens of teachers and hundreds of students. It upgrades the pioneering ideas of Gattegno giving a powerful tool for teaching and learning math, science, architecture and art. Further inspirations for developing visual thinking can be found in Nelsen's book "*Proofs Without Words*" [3].

Acknowledgements

We took our inspiration for the step by step instructions in "Activity 2." from Cory Pool [4] and for the workshop activities plan from "Making Math Visible" project [2].

References

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