Self-Similarity and the Tumbling Square

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Abstract

This hands-on workshop will present the "Tumbling Square" paper sculpture. I will explain how I developed the form, offer detailed instruction on how to create I, and guide the participants in making their own Tumbling Squares. I will also provide guidance for creating new patterns that will serve two functions: as guidelines for construction, and aesthetic content. After thorough instruction on the single sheet element, I will explore more elaborate sculptures requiring multiple sheets of paper. This project offers opportunities to introduce Mathematical topics including self-similarity, diagonal bisections of squares, and isosceles right triangles.

Introduction

I am always very excited to develop an art project that incorporates my own work into a project that teaches people a sculptural paper form, as well as some mathematics. The Tumbling Squares were the result of combining numerous sources. The mathematical themes of my "Grid Shift" drawings offer useful graphics for guiding the cutting and folding necessary for building these pop-up forms.

Grid-shift Drawings

Square grids have always played a major role in my drawing process. Over the years I have been working on a series of "counted marking" drawings. It is the number of stroke-like marks in each grid cell that holds significance. Drawing these marks, I build a visual language to express the aesthetic properties of numerical sequences and series. Sitting in a darkened auditorium at a Bridges conference, I was mesmerized by an image Jean-Marc Castera presented. It was a photo of an architectural detail of text-based Islamic ornamentation. This form of decoration involved the same text written repeatedly in rows. The size of the font became smaller with each row: full-sized, half-sized, quarter-sized, creating an example of self-similarity. I immediately realized this could have implications for my work. Markings of various sizes danced through my brain. If I vary the grid sizes (in this case to 1", ½" and ¼" squares) but kept the number of strokes per square the same throughout a drawing, I could create a similar result. I call these works "grid shift" drawings. This idea was more difficult to execute than I anticipated. Each drawing needed its own hand-drawn set of grids. It was not easy to get my hand and eyes to cooperate in constantly switching scales. I eventually was able to make numerous drawings and artist's books using this technique.

Tumbling Squares

I came across the pop-up paper folding project "Tumbling Triangles" in Esther K Smith's 2008 book "Magic Books & Paper Toys" [2]. I realized that, if I made a few adjustments, using a square sheet of paper, cutting and folding on a perpendicular plane, I could make "Tumbling Squares". This form seemed like the perfect structure for a square grid shift drawing.

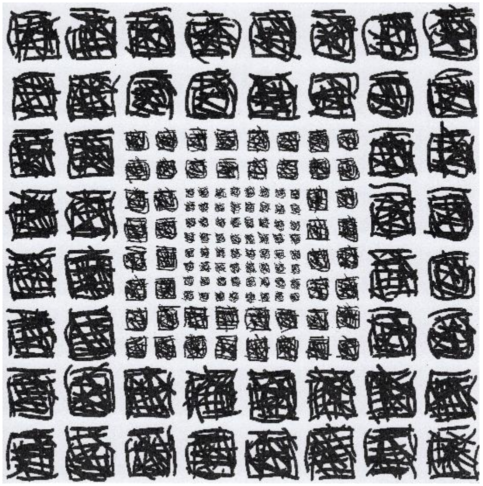

Figure 1: *Grid Shift drawing*

One 8” square sheet of cardstock paper with the drawing in figure 1 on both sides is the starting point for a tumbling square. The only tools needed are scissors and, if available, a bone folder (a basic tool for book and paper artists). The cutting and folding diagram is in figure 2.

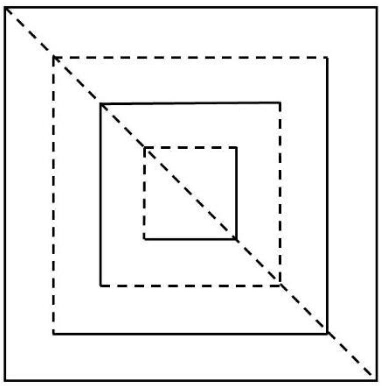

Figure 2: *Cutting and Folding Pattern*

The solid lines are cuts and the dashed lines are folds. The best way to begin is to make a diagonal fold as in figure 3. To facilitate the pop-up action, you need to crease the fold in both directions to loosen up this hinge. Once the paper is folded in half you can follow this schematic drawing to make the correct cuts and folds.

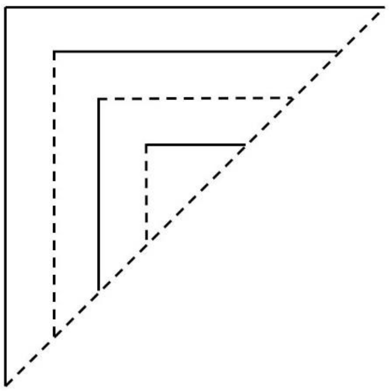

Figure 3: *Diagonal Fold*

Start with the longest outermost cut, then make the fold perpendicular to the cut. Next, crease in both ways. Now make the second cut parallel to the first fold, followed by the second fold perpendicular to the second cut. Continue this process for one more iteration. Unfold the paper to make sure your folds and cuts match the diagram. Refold the square along the diagonal while at the same time pressing along the outermost perpendicular fold in the opposite direction of the diagonal fold, pressing down to create a pop-up. Repeat this process for each of the other fold lines. When you open and close the square along the diagonal the interior squares rotate. The resulting Tumbling Square is in figure 4.

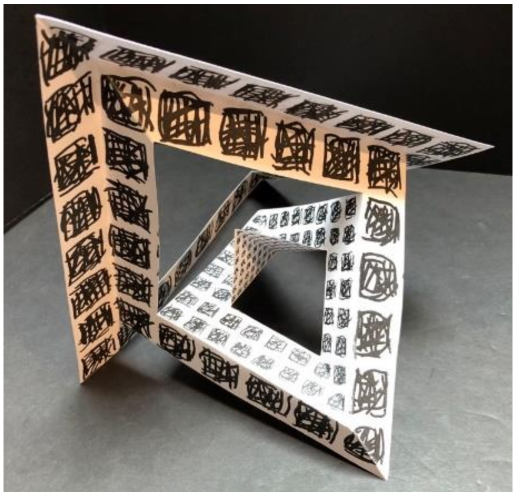

Figure 4: *Tumbling Square*

Tumbling Squares as Building Blocks

The cutting and folding guide for the tumbling square looks suspiciously similar to the folding guide "Hypar crease pattern" Erik and Martin Demaine published in their paper "Mathematics is Art" [1]:

> *We were fascinated by a known geometric origami model, the "pleated hyperbolic paraboloid" or hypar (...). The folder makes a simple crease pattern—concentric squares and diagonals alternating mountain and valley—and then the model almost folds itself into a striking saddle surface.*

Both patterns feature a set of concentric squares. The main difference is the addition of paper cuts. These cuts relieve the tension in the paper, allowing the paper to rotate, instead of creating a saddle. It is this kinetic nature of the tumbling square that makes it so much fun to make. It is quite impressive how much is happening with this form after only three cuts and four folds.

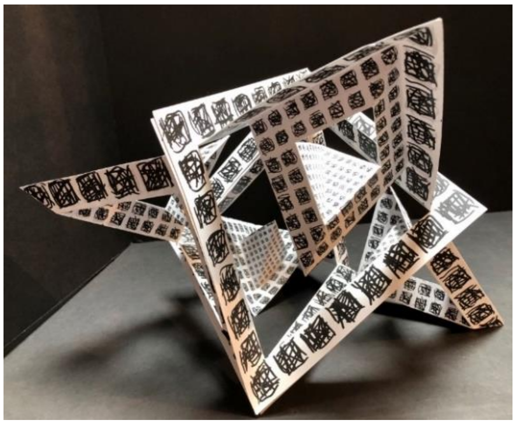

Figure 5: *Sculpture with 4 Tumbling Squares*

Erik and Martin Demaine show in their paper that multiple hyperbolic paraboloids can be combined to make paper sculptures. Similarly, I decided to make geometric structures using tumbling squares as building blocks. The result of combining four Tumbling Squares into a new form can be seen in figure 5. The original diagonal folds for each square are positioned in the center of the multi-element form. The outer inch of each square has been glued to another square, alternating a one-inch strip with a folded strip.

I feel this sculpture is very architectural, alluding to walls, corners, windows, and walls. By fabricating these structures my grid shift drawings are returning to the root of my original inspiration.

Teaching Tumbling Squares

There are numerous ways to incorporate the tumbling square into a math and art curriculum. Middle school students will be able to cut and fold the project if they are given the double-sided photo copies using my graphics. Geometric topics include: perpendicular, parallel, diagonal bisections, isosceles right triangles, and self-similarity. After building a pop-up with existing graphics, high school and college students can make their own self-similar repeating patterns. Figure 6 shows the grid pattern to be used as a guide.

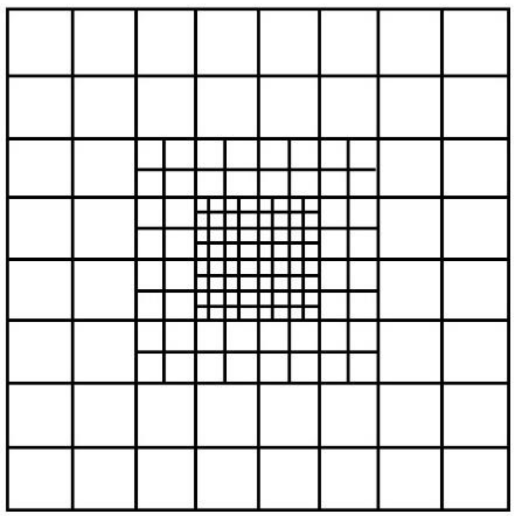

Figure 6: *Grid Pattern*

More advanced students can also explore the possibilities of using multiple squares to build new structures. In “Magic Books and Paper Toys” Esther K. Smith offers a disclaimer with the instructions for “Tumbling Triangle”, “It's tricky. (Don't hurt yourself!)”. The mathematics in my drawings has facilitated the production of the form by offering structured grid geometry. The architectural and kinetic nature of the tumbling square has enhanced the aesthetics of the mathematics.

References

[1] E. D. Demaine and M. L. Demaine . “Mathematics Is Art.” *Bridges Conference Proceedings*, Banff, Canada, Jul. 26–30, 2009, pp 1-10. http:// http://archive.bridgesmathart.org/2009/bridges2009-1.html.

[2] E. K. Smith. *Magic Books and Paper Toys*. Potter Craft, 2008, pp 16-17.

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