# Combinatorics, Probability and Choice in Music Composition: Towards an Aesthetics of Composing Systems for Non-Musicians 

Giovanni Albini<br>Conservatorio "J. Tomadini", Udine, Italy; mail@giovannialbini.it Eesti Muusika- ja Teatriakadeemia, Tallinn, Estonia


#### Abstract

The aim of this paper is to study how combinatorics and probability informed and can inform the aesthetics of composing systems for non-musicians. For this purpose, an historical example is taken into account, Mozart's Musikalisches Würfelspiel and it is compared with a contemporary digital system developed by the author with similar objectives, Op. 60 Ricercari Diatonici, for people, composing system and harpsichordist.


## Introduction

Combinatorial methods and systems for composing music meant for non-musicians have a long history, from Athanasius Kircher's Arca Musarithmica (1650) to the several systems - often referred as to 'games' - that were quite popular in the $18^{\text {th }}$ century ${ }^{1}$. According to [4],

From 1757 to 1812 at least twenty musical dice games were published in Europe, some in several editions and languages. All made it possible for the person ignorant of music to write minuets, marches, polonaises, contredances, waltzes and so forth by selecting bits of prefabricated music through the use of chance operations.

Examples include: J. P. Kirnberger, Der Allezeit Fertige Menuetten- und Polonaisencomponist (1757), the anonymous Ludus melothedicus (1758), C. P. E. Bach, Einfall, einen Doppelten Contrapunct in der Octave von Sechs Tacten zu Machen, Ohne die Regeln Davon zu Wissen (1758); M. Stadler, Table pour Composer des Minuets et des Trios à la infinie (1780), P. Hoegi, A Tabular System whereby the Art of Composing Minuets is Made so Easy that any Person, Without the Least Knowledge of Musick, may Compose Ten Thousand, all Different, and in the Most Pleasing and Correct Manner (1770), J. F. Wiedeburg, Musikalisches ChartenSpiel (1788), F. J. Haydn, Philharmonic Joke (1790), W. A. Mozart, Musikalisches Würfelspiel (1787/1792), A. Calegari, Gioco Pitagorico (1801) and G. Catrufo, Bareme Musical (1811). ${ }^{2}$ Some of them let the aided non-musician composer choose the elements that compose the final work, some others leave them up to chance. Let us consider the latter, but notice also that the choices of a non-musician could be random as well, due to his or her alleged inabilty to read and understand them - or to try to evaluate them until the composition process has ended. In this context, to consider the system fair, it can be reasonably assumed that the following requirements are fulfilled: 1) the system outputs scores that have the same probabilty to come out and 2) all the outputs have the same artistic value. Therefore, I will consider a case model, Mozart's Musikalisches Würfelspiel, and I will show that not only the two aforesaid assumptions could not be true at all, but also that as a result there are some relevant aesthetic implications. I will then present $O p$. 60 , a digital system I developed under the same aesthetic principles.

[^0]
## Mozart's Musikalisches Würfelspiel

W. A. Mozart, Musikalisches Würfelspiel (1787?), was published in 1793 in Berlin and Amsterdam by publisher J. J. Hummel and also in 1796 by Nikolaus Simrock in Bonn, [4], not long after Mozart's death. Although the attribution have never been authenticated, [3, 4], it is well known that Mozart left a draft of a musical dice game, K. 516f, [6]. The frontpage of the game, written in four languages (German, French, English and Italian), states: "Instruction to compose without the least knowledge of music so much German Walzer or Schleifer as one pleases, by throwing a certain number with two dice."

The rules are simple: the game gives 176 numbers, each of them coupled with 176 bars of music, that are arranged in 16 columns with 11 numbers-bars each. To select the first bar of a waltz two dice are thrown and, according to which of the 11 possible totals resulted (from 2 to 12 ), the corresponding bar is selected from the first column. Then the dice would be thrown again and the new total is used to choose a bar from the second column, and so on for 16 bars. Bars 8 and 16 are always the same, regardless of the rolls, so the combinations-waltzes are, totally, $11^{14}=379.749 .833 .583 .241 \simeq 379 \times 10^{12}$, almost four hundred billion. But, as has been pointed out in [7], not all the possible waltzes have the same probability to appear. In fact, according to [8], the formula to calculate the probability to obtain $p$ points throwing $n s$-sided dice is

$$
\begin{equation*}
P(p, n, s)=\frac{1}{s^{n}} \cdot \sum_{k=0}^{k_{\max }}(-1)^{k}\binom{n}{k}\binom{p-s k-1}{n-1} \text { with } k_{\max } \text { floor function of } \frac{p-n}{s} \tag{1}
\end{equation*}
$$

that in case of two 6-sided dice gets to

$$
\begin{equation*}
P(p, 2,6)=\frac{6-|p-7|}{36} \text { with } 2 \leq p \leq 12 \text {. } \tag{2}
\end{equation*}
$$

Thus, only one waltz on almost four hundred billion is the more likely to appear: the one given by a series of 7 as results of any roll of the dice. It is shown in Figure 1.


Figure 1: The most probable waltz of Mozart's Musikalisches Würfelspiel.
What is interesting is that the most likely waltz is also particularly well crafted with regard to the $18^{\text {th }}$ century taste and compared to a random one made within the game. Actually, it shows a peculiar thematic coherence: the two intertwined periods reveal clear symmetry and recurrence. It is difficult, if not impossible, to know if this was done intentionally, but indeed, after Galileo's Sopra le scoperte dei dadi (1596), the math behind dice games was well known. So, could this particular waltz be left there for a future discovery?

## A Combinatorial Aesthetics of Discovery and Participation

The most important aspect of these systems-methods-games for composing music which are meant for non-musicians is that they engage everyday people in participating the process of music composition. Combinatorics could then be not only a tool to formalize a piece of music into its structural and constituent elements, [3, 9], but it could also be an instrument to involve people who are usually restricted to the role of music listeners (or, at the very best, performers) in music composition. More than that, it would take more than a composer lifetime to explore and listen to all the combinations that the system can output. But several people playing with the system can speed up browsing its music and choosing the outputs they feel more valuable. As Italo Calvino clearly explained in [2], referring to the psychological theories of art by Ernst Hans Josef Gombrich:

> The juvenile pleasure of the combinatorial game leads the painter to experiment layouts of lines and colors and leads the poet to experiment juxtapositions of words; then something triggers, and one of the combinations mechanically and autonomously got, regardless of any meaning or effect on any other level, gains an unexpected meaning or an unforseen effect that would never have been gained intentionally: an unconscious meaning, or at least the premonition of it. ${ }^{3}$

The combinatorial game seems then to be an archetypal way to get through knowledge and to an artistic output, and only mathematics can draw its borders, or, like in the case of the most probable Mozart's waltz, can possibly lead its aim. Therefore, combinatorics and probability offer music a distinctive aesthetics of participation and discovery. This led me to the following question: how could I exploit such an aesthetics in a contemporary musical work?

## Op. 60, for People, Composing System and Harpsichordist

Designing a system for composing music today cannot fail to take into account the nowadays social and technological background. Music composition and performance is more and more confined to a professional or occasional educational experience, while most of the people are pushed to the role of casual listeners and music consumers: the audience. So, today it is difficult to imagine that someone could play a musical dice game around a piano holding pencils and music papers. But almost everybody owns a mobile device that can get around the problem. ${ }^{4}$ Therefore, I designed my $O p .60$ to be an application for mobile devices (currently it runs on iOS and Android) that can easily compose, engrave and play a massive number of short music pieces for harpsichord. The user is allowed to do only two things: 1) generate a new score (that can be viewed and listened to) and, in case he or she likes it, 2) send it to me. I can sort then the music gathered by any means and potentially feature some of it in a live concert.

Furthermore, $O p .60$ explores a specific and stylistically pretty constrained set of short pieces for harpsichord that are all generated in the context of a specific mathematically informed aesthetics of music composition. The idea behind it is that mathematics can serve shaping musical structures that grant a neat focus on traditional music elements and yet put them in a different, hopefully new, perspective, so as not to renounce the desire for novelty, nor the awe-inspiring aura of the established and intelligible material of musical legacy. More specifically, $O p .60$ focuses on diatonic trichords and explores the possibilities of composing with them abiding by several sets of rules I developed after a strict formalisation that has shown some of their inner and otherwise hidden features. To a certain extent, I can say that the users of the software help me exploring all the possibilities I set and so evaluating which ones can be considered effective and

[^1]which ones are considered insignificant, and accordingly change the software rules themselves. That is also the reason behind the subtitle of the software, Ricercari Diatonici: ricercare means to search out.


Figure 2: Four screenshots of Op. 60.

## Summary and Conclusions

Eighteenth-Century dice games, and specifically Mozart's Musikalisches Würfelspiel considered as a case model of systems for composing music meant for non-musicians, let me establish the unique charachteristics of a possible aesthetics informed by combinatorics and probability. They are 1) the participatory aspect in the process of music composition and 2 ) the participatory aspect in the discovery of artistic values. I presented then my contribution, $O p$. 60 , for people, composing system and harpsichordist, that exploits such an aesthetics in a contemporary musical work and considering the nowadays social and technological background.

## References

[1] G. Albini and P. Girol. "Mobile Devices' Idiomaticity as a Cue for Musical Creativity." 2nd Conference on Computer Simulation of Musical Creativity, CSMC 2017 Proceedings, Milton Keynes, United Kingdom, Sept. 11-13, 2017.
[2] I. Calvino. "Cibernetica e Fantasmi." Le Conferenze dell'Associazione Culturale Italiana, vol. XXI, 1967-1968.
[3] D. Cope. "Experiments in Musical Intelligence." A-R, Editions, Inc. 1996.
[4] S. A. Hedges. "Dice Music in the Eighteenth Century." Music and Letters, vol. 59, no. 2, 1978.
[5] L. Meyer. "Style and Music: Theory, History, and Ideology." University of Chicago Press, 1989.
[6] A. Poggi and E. Vallora. "Mozart - Signori il Catalogo è Questo." Einaudi, Torino, 1991.
[7] Z. Ruttkay. "Composing Mozart Variations with Dice." Teaching Statistics, vol. 19, no. 1, 1997.
[8] J. V. Uspensky. "Introduction to Mathematical Probability." McGraw-Hill, New York, 1937.
[9] P. Worth and S. Stepne. "Growing Music: Musical Interpretations of L-Systems." Applications of Evolutionary Computing. EvoWorkshops 2005., vol. 3449, Springer, Berlin, Heidelberg, 2005.


[^0]:    ${ }^{1}$ The abundance of such games in the $18^{\text {th }}$ century, in opposition to the absence in the $19^{\text {th }}$, had been well explained in [5], underlining that in the former "what constrained the choice of figures were the claims of taste, coherent expression and propriety", while in the latter it was "the inner necessity of a gradually unfolding, underlying proces", which is much more difficult to schematize.
    ${ }^{2}$ In addition, some theorists of the $18^{\text {th }}$ century based part of their composition treatises on combinatorial procedures, e.g J. Riepel, Grundregeln zur Tonordnung insgemein (1755), J. P. Kirnberger, Methode Sonaten aus'm Ermel zu Schüdden (1783), H. C. Koch, Versuch einer Ainletung zur Composition (1782-1793) and F. Galeazzi, Elementi Teorici-Pratici di Musica (1791-1796).

[^1]:    ${ }^{3}$ This author's translation.
    ${ }^{4}$ Moreover, this aesthetics of participation and discovery lets conceive a musical work idiomatically designed for mobile devices, as postulated in [1].

