Don't Preach Facts – Stimulate Acts

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Abstract

The successful maths and art project "Everything is Number" goes digital. The artist Eugen Jost has created his new pictures with the help of a drawing software and we use special software to make them dynamic. Our goal always remains the same: to stimulate interest in mathematics and to improve mathematics education.

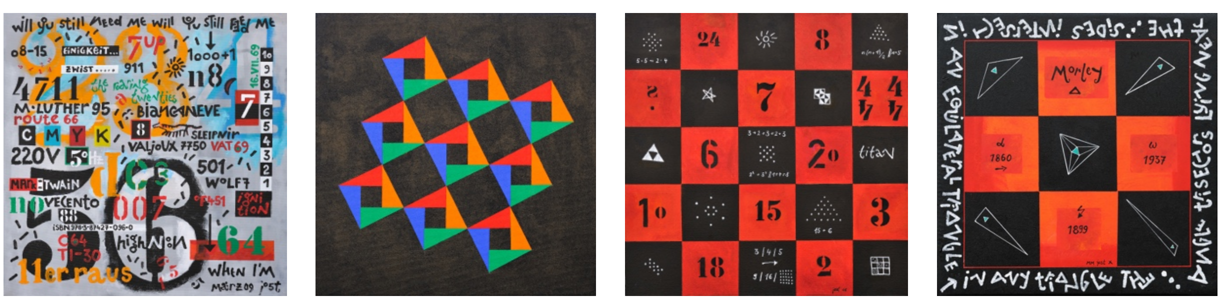
An Educational Leitmotif

Our hometown Bayreuth is famous for the composer Richard Wagner and the annual opera festival. We have the only opera house in the world that is used only six weeks a year and only for operas by Wagner. Characteristic for most of his operas are short repetitive tunes, the so-called leitmotifs.

Leitmotifs do not only make sense in music, they make sense in education as well. The American mathematician Paul Halmos (1906 – 2006) has phrased our educational leitmotif: "Don't preach facts – stimulate acts." That means: The teacher is not an entertainer, the student is not only a consumer. "Stimulating acts" means to encourage students to develop their own (informal) methods for doing mathematics. We ask them to explore, to observe, to discover, to assume, to explain, to prove.

A Visit to an Art Gallery

"Stimulating acts" means to offer students an active approach to mathematics. A visit to an art gallery, for example, is an excellent opportunity for stimulating acts. Let us present four works of art exemplarily.



(a) (b) (c) (d)

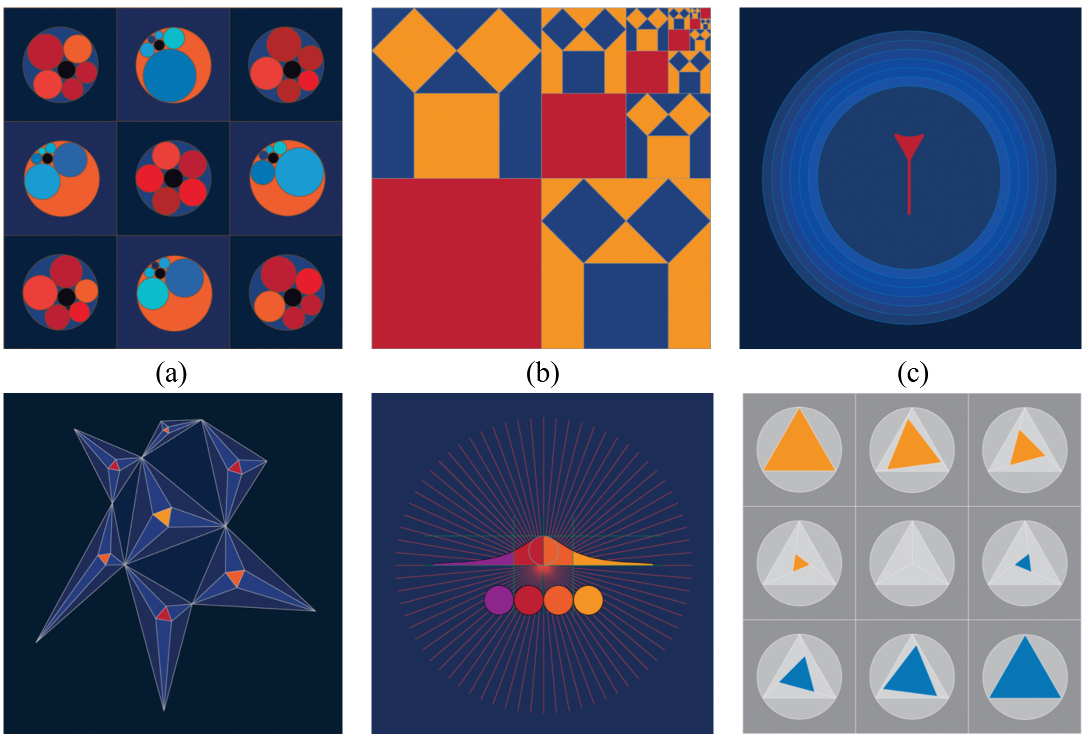
Figure 1: *(a) When I'm 2^6, (b) Quattro Stagioni, (c) Black & Red Magic, (d) Morley's Miracle.*

These pictures are given titles like "When I'm 2^6", "Quattro Stagioni", "Black & Red Magic" and "Morley's Miracle". Actually their styles are different, but they all have a common background that one doesn't suspect behind these titles: mathematics. The Swiss artist Eugen Jost has created all these wonderful paintings and many more. The pictures tell stories, they stimulate interest in mathematical results and relationships. They call attention to the persons who have created the mathematics shown in the paintings. For Jost mathematics is a beautiful, lavishly landscaped, colourful garden with manicured flower beds and wild shrubs. There are lots of paths, partly broad and even, partly narrow and winding, partly fairly steep. Jost strolls through this garden, not as a botanist or a gardener, but as a lover of flowers. On his way he goes from one flower to the next, picks a beautiful one from time to time and after his walk he has collected a magnificent bunch of flowers. In his paintings we find such bouquets.

Together with Jost we started a maths & art project some years ago. The success has really been fantastic and stunning. We have had dozens of exhibitions of original paintings and many more of digital prints of the paintings. We not only have shown pictures but have provided information, hints, and questions, to encourage reflection on mathematics. Within the project we created maths & arts calendars and books. The idea to get an access to mathematics with the help of art works is still vivid and popular. Requests for our exhibition continue unabated. Jost's paintings attract kids, students, and adults with and without mathematical knowledge.

Another Artistic Tool – Computer Software

Meanwhile Eugen Jost has laid aside his paintbrush and works with special computer software. The result is graphic arts with a strong relation to mathematics.



(a) (b) (c) (d) (e) (f)

Figure 2: *(a) Soft Touches, (b) Squares without End, (c) Sumer 60, (d) Morley's Miracle, (e) Agnesi's Witch, (f) Dynamic Triangles.*

The artist as well as the mathematician works with the same device, a computer or a tablet-PC. The artist Eugen Jost creates beautiful patterns and shapes that appeal to emotions and aesthetic feelings. The mathematician Carsten Miller animates the graphic art [5]. The animation stokes curiosity and facilitates understanding of the underlying construction. To illustrate the intention of our maths & arts project we are going to have a closer look at three of the above pictures.

Dynamic Triangles

The line, circle, and triangle are basic geometrical objects. Three lines, no two of which are parallel, either meet in a point, or the intersection points of these lines are the vertices of a triangle. These vertices lie on a circle, the so-called circumcircle. Jost plays with all these objects in his picture (Figure 2f).

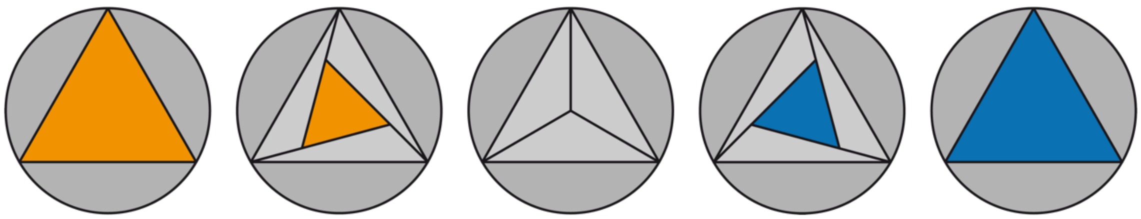


Figure 3: *Snapshots of dynamic triangles.*

The picture sequence (Figure 3) indicates a dynamic process where an additional triangle is generated in the interior of a fixed equilateral triangle. The interior triangle becomes smaller and smaller and finally disappears, but then it grows again until it is finally congruent with the fixed triangle. "Don't preach facts – stimulate acts" means to make this process alive. The visualization can be realized by an applet [5].

Interesting questions arise: How do we get the interior triangles? Which angles appear in this configuration? Is the interior triangle also equilateral? What happens if the fixed triangle is not equilateral?

In this context it is worth mentioning that elementary geometry and triangle geometry flourished especially in the 19th century and not in ancient times as often is presumed. There was a very productive community of high school teachers whose publications contributed to the reputation of geometry. Several of these teachers became well known in academic circles and received professorships at universities (cf. [2], [3]).

Witch of Agnesi

We turn our attention to another artwork (Figure 2e) by Eugen Jost. This time "Don't preach facts – stimulate acts" means to analyse the underlying geometry of this beautiful picture. We consider a fixed circle between two parallel tangents. The line through the touching point O of the lower tangent intersects the fixed circle in point A and the upper tangent in point B. The horizontal line through A and the vertical line through B intersect in point P.

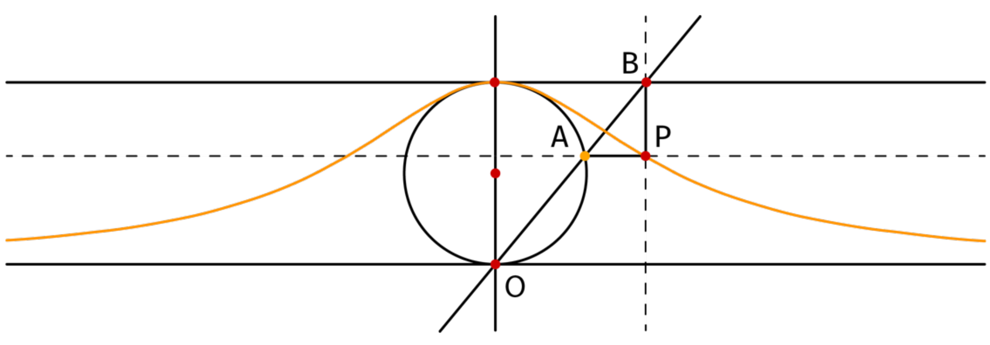


Figure 4: *Construction of Agnesi's Witch.*

As point A is moved along the circle line, the trace of point P generates a new curve, the so-called Witch of Agnesi. A construction on a tablet-PC with the gesture based software sketchometry [7] enables an intuitive approach to this interesting curve.

To find the origin of the name of this curve we have to go back into the 18th century. The remarkably gifted Maria Gaetana Agnesi (1718 – 1799) is the author of the *Istituzioni analitiche ad uso della gioventu italiana*. This excellent two volume textbook on both differential and integral calculus – at that time a new branch of mathematics – contains a discussion of the above constructed cubic curve.

The investigation of Agnesi's Witch stimulates acts by fulfilling the following tasks:

- Describe the characteristics of this curve.
- Define the parametric equation of the curve in dependence of the angle between line OA and the y-axis.
- Transform this equation into a Cartesian equation.
- Try to explain the meaning of the four circles in Jost's picture.
- Write a short biography of Maria Gaetana Agnesi.
- Find out why the curve is named Witch of Agnesi.

By the way, we must not forget to celebrate Agnesi's 300th birthday this year.

Morley's Miracle

Not all knowledge about triangles can already be found in the writings of antiquity. Thus in 1899 the mathematician Frank Morley (1860 – 1937) discovered an astonishing result that had apparently been overlooked by the ancient geometers: In any triangle the angle trisectors lying near the sides intersect in an equilateral triangle. His colleagues were so enthusiastic that they even spoke of Morley's Miracle. Using dynamic geometry software provides a first impression that the theorem holds true for all triangles. The proof itself is a bit challenging. But the beauty of Morley's theorem lies in the simplicity of the statement.

Stimulating acts means

- to try to trisect an angle with (unmarked) ruler and compass,
- to trisect an angle using other tools than only ruler and compass,
- to try to construct Morley's triangle (cf. Figure 2d)),
- to generate a dynamic Morley construction,
- to study the three famous problems of Greek antiquity.

Summary and Conclusions

"Don't preach facts – stimulate acts" allows an active access to mathematics. Our examples show that art may be an excellent vehicle for stimulating acts. We look at pictures, we discover patterns, configurations, and relationships, we do geometry with software tools and our finger, and we improve our knowledge and understanding of mathematics. Additionally the paintings often stimulate historical excursions. Thus art helps to make clear that mathematics is much more than mere computing, mathematics is part of our culture.

References

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[5] Don't Preach Facts – Stimulate Acts. http://jsxgraph.uni-bayreuth.de/~carsten/ein/.

[6] JSXGraph. http://jsxgraph.uni-bayreuth.de.

[7] sketchometry. https://sketchometry.org.