The Mathematical Center of Attention, its Attributes and Motion Analyses in Dance Choreography

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Abstract

In the arts, qualitative usage of the center of attention (CA) has long been successfully employed by visual artists. For dance, a center of attention was defined by Kasia Williams in a paper at the 2012 Bridges conference as a means for mathematically studying how choreographers and dancers tend to create and manipulate the CA in dance works. It is quantitatively calculated by an interpreter of video frames assigning weights to the dancers based on the choreographer's intent as applied to dancer configurations and locations on stage as the performance proceeds. Using Excel and Mathematica, the authors have been exploring methods of calculating and displaying the CA as a trajectory along with statistical and other quantifiers from analyses of a variety of dance performances. The hope is that the center of attention might emerge as a new useful tool in understanding, classifying, and improving audience enjoyment. Once quantified, there is the possibility that other dance parameters may also be calculated and provide a means for comparing dances. We will present our latest results and suggest ways how these ideas might be adopted beyond dance performances. These might include classroom activities and perhaps related attention-producing technologies dominated by configurations changing in space and time.

Introduction

At the 2012 Bridges conference Kasia Williams (née Wasilewska) presented a paper that included the proposal that dances might be analyzed in terms of a "center of attention mass," capturing and quantifying information on the choreographer's and audience's focus during the dance performance. She suggested that dancers' locations on stage in time and space be quantified, with numerical weights assigned to the dancers, noting that,

> "… We would assign the weights based on the type of movement performed and how likely the moves are to attract the audience's attention. We might want to assign zero weight to the dancers that are off stage. A dancer leaping across the stage would carry more weight than a dancer frozen in a pose somewhere stage left. Or, depending on the atmosphere of the dance, a dancer crouching down and being still center stage left could have more weight than dancers moving around him."[6]

The hope is that the center of attention (CA) might emerge as a new useful tool in understanding, classifying, and improving the choreographic process as well as dance audience enjoyment. This might parallel the use of similar concepts in the visual arts, for example the "rule of thirds" in photography, which essentially positions the CA at one intersection point of two equally spaced horizontal lines and two equally spaced vertical lines. It might help clarify often unstated or intuitive concepts used by choreographers and audiences in creating and responding to dance performances. In addition, once digitized various statistical properties of dances might be easily examined, such as how often certain types of movement appear, how closely linked pairs of dancers are in time or space, or the speed and directional characteristics of individuals or groups of dancers.

Initial efforts by the authors has been a close examination of the center of attention and related parameters in the dance "Apollonian Circles," from the concert The Daughters of Hypatia by Karl Schaffer [4]. We have used Mathematica 11.1 and Excel 16.0 for most of our calculations and are planning the implementation of software which will be easily usable by choreographers and others for the examination of dance parameters as described above. Using these calculations, we have created animations which model the dance and provide an ellipse centered at the CA with major axis determined during the calculations of fitting a regression line approximating the dancers' positions.

Calculation of the Center of Attention and Associated Ellipse

"Apollonian Circles," the brief 70 second dance we have initially used as a template for our development of the CA and other quantitative analyses, is a quartet from the dance concert *The Daughters of Hypatia: Circles of Mathematical Women* [3], which celebrates women mathematicians throughout history. The title refers to the mathematical concept attributed to Apollonius of Persia concerning two orthogonal families of circles. Hypatia is known to have written treatises on the works of Apollonius, and a circular sub-theme in the concert refers to circles of community and support that have nourished women in the field of mathematics. Figure 1 shows a screen shot from the dance next to a frame containing a circle for the position of each of the four dancers from the accompanying Mathematica animation. The open circle in the animation is the CA at that time and line segments connect each dancer's circle to the CA. The ellipse has size and major axis determined by the weighted regression line through the four dancers' positions. The ellipse might be thought of as describing the focus of attention, and perhaps might be useful at some point by the lighting designer. (The graphic in the background of the video screen shot is actually an Apollonian gasket, a family of mutually tangent circles in which the numbers show the curvatures of the circles.)

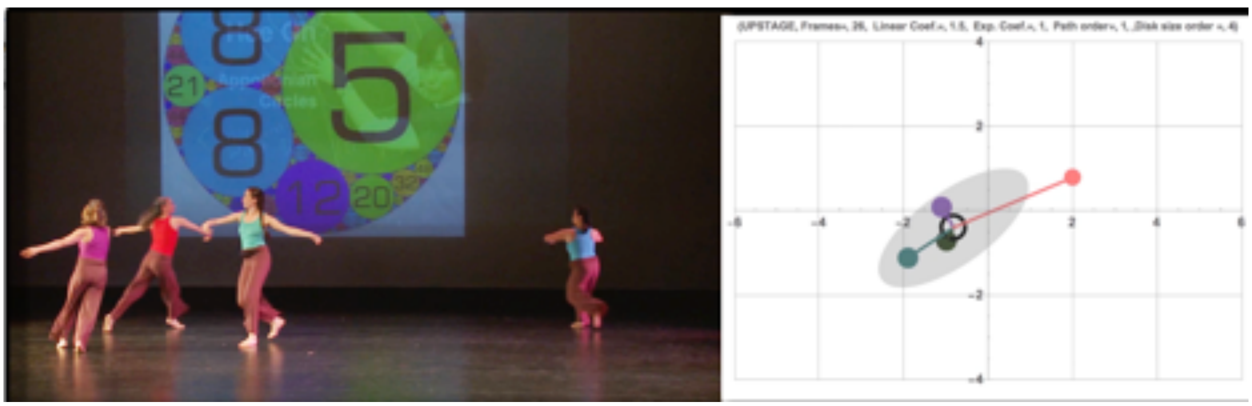

Figure 1: *Frames from the dance "Apollonian Circles" and the accompanying Mathematica animation.*

In order to construct the animation we sampled the dancers' locations 21 times, approximately every 3 seconds, and assigned weights 0, 1, or 2 to each dancer, with higher weights indicating that the choreographer meant for the dancer to take more of the audience's attention. Locations were calibrated according to "stage units," described below, in which the center stage point is assumed to have coordinates (0,0). Also, intermediate frames of the animation between the sampled points were determined by functions derived using Mathematica's interpolation command. Linear interpolation has thus far been more successful than quadratic and cubic polynomials or splines, but is used here for illustration purposes only. In practice, functional forms other than linear would be considered. Additionally, observational judgments as to the desired effect of weights and rules to assure consistency of usage would need to be made. The CA at time t is defined as $c(t) = (\bar{x}(t), \bar{y}(t))$, where $(x_i(t), y_i(t))$ and $w_i(t)$ are the location and weight respectively of the ith dancer at time t, where i takes on the values 1, 2, 3,… d, and d is the total number of dancers. Here $\bar{x}(t)$ and $\bar{y}(t)$ are the weighted means:

$$\bar{x}(t) = \frac{\sum_{i=1}^{d} w_i(t) x_i(t)}{\sum_{i=1}^{d} w_i(t)}, \quad \bar{y}(t) = \frac{\sum_{i=1}^{d} w_i(t) y_i(t)}{\sum_{i=1}^{d} w_i(t)}$$

The orientation of an ellipse indicating spread around a center of attention point $c(t)$ depends on the directions of major and minor axis lines that result from the line that is the best weighted fitting of its associated data points. The standard linear regression approach assumes there is no randomness in the x coordinates and finds the line minimizing the standard deviation of the weighted vertical distances of the points from the line. However if there is uncertainty in both x and y coordinates, as is the case with the locations of dancers on stage, then a preferred but little used method called orthogonal or total least squares regression finds the line that minimizes the standard deviation of the weighted orthogonal distances of the points to the line, see Figure 2. This weighted orthogonal regression gives both the slope and intercept of an ellipse axis along with the standard deviation of points from it. The other axis is then a line perpendicular to it and both axes pass through the CA. (See supplementary Appendix file for equation details.)

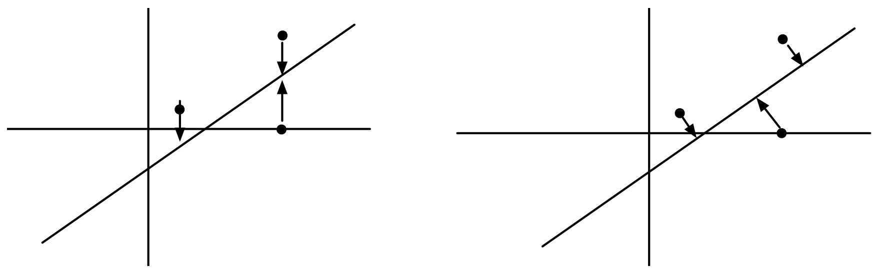

Figure 2: *Standard linear regression (left) and orthogonal linear regression (right) minimize vertical and orthogonal distances of data points, respectively.*

In reconciling standard weighted least squares and orthogonal regression, one finds the former giving a line through the data that depends on the coordinate system orientation. If the axes are iteratively re-oriented from -90° to 90° for calculation of the standard linear regression there will be a best fit among all regressions when its x-axis is parallel to the best orthogonal regression fit line. Its summed weighted errors are the same as that of orthogonal regression and hence both methods agree. Details and derivations of orthogonal regression are clearly set out in a 2014 publication by Munoz et al [1], which aims to popularize and make the method accessible to undergraduate statistics and linear algebra students (see Appendix).

Coordinates

How we decide to set coordinates for the dance stage is important if we are to do quantitative comparisons between dances, choreographers, or dance forms. This is an example of the challenges that often occur when coordinating an art form with mathematics. Dancers and choreographers tend to use two somewhat inconsistent ways of naming coordinates for the typical proscenium stage. Names for stage areas tend to divide the stage into nine sections, with stage right referring to the dancer's right as he or she faces the audience (Figure 3(a)), and upstage being furtherst from the audience.

On the other hand, dancers often place small tape markers called "spikes" at half or quarter points on the stage, easily glimpsed in the performers' peripheral vision. The shapes are often as shown, either X- or T- or inverted T-shapes (Figure 3(b). Normally only a few of those shown here are used, as otherwise the stage becomes too cluttered with tape! Most common are the center center spike and the quarter and center inverted Ts at the downstage lip of the stage (the downstage spikes are easily noticed by dancers facing the audience without being apparent to the audience.) Sometimes glow tape is used, if the dancers will need to find their places in blackout. Also, if stage sets are used, then spikes indicating their locations will be added as necessary. Note that these points and lines essentially divide the stage into sixteen rather than nine areas. Sometimes these areas are referred to, for example, as "slightly up left of center."

A numerical coordinate version of these stage divisions would probably place the origin (0,0) at the center center spike, and divide the four quadrants of the stage in the usual manner of coordinatizing the plane. Most proscenium stages used for dance are wider than they are deep, and at least somewhat rectangular in shape. For this study we have placed the origin at center stage and quadrant one at upstage left, so that the traditional planar orientation is from the point of view of the audience. The stage area is set from –4 to 4 horizontally and from –2 to 2 vertically.

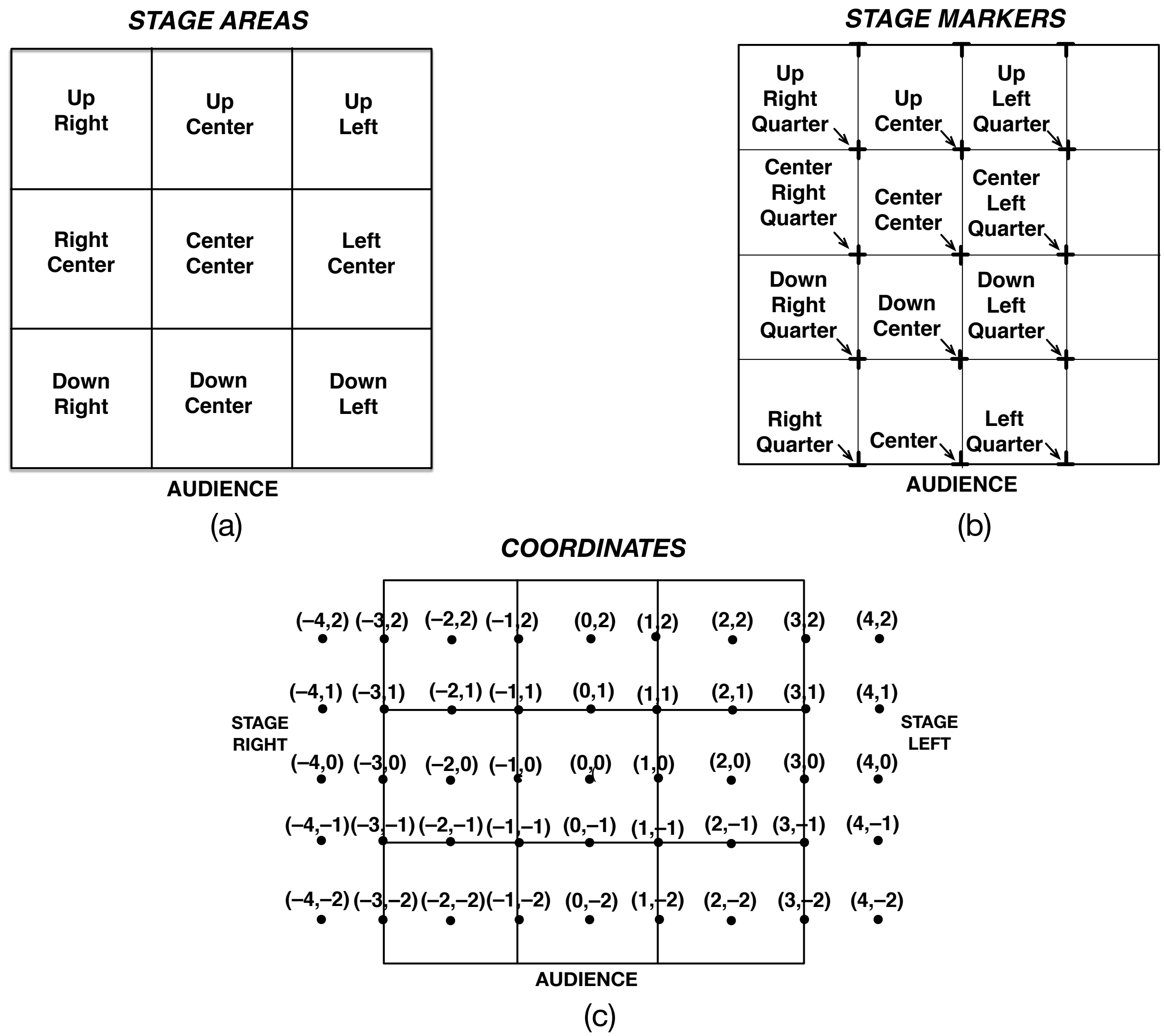

Figure 3: *Three systems for designating locations on stage: (a) Stage areas. (b) Stage markers. (c) Coordinates used in this study.*

Movement analysis of Apollonian Circles

Once we have collected the data points $(x(t),y(t))$ and associated weights $w(t)$ for each dancer in a particular dance we can run the Mathematica animation model side by side with a video of the dance. If the animation and dance match closely then other movement analyses can easily be done using these same data points. For this preliminary study, and as a model for future work, we have done a number of analyses of the dance 'Apollonian Circles," primarily using Excel spreadsheets. The side by side video and animation are at [4].

From times and coordinates for each of four dancers over an encompassing 18 frame subset used in this part of the analysis it is possible to calculate 17 or less sequential distances for each. These are measured in Euclidian stage units (SU) moved. Each such distance for a dancer is the square root of the sum of the squares of x coordinate differences and y coordinate differences. Associated with these inter-frame movement distances are their directions $\theta(t)$, ranging between -180 and 180 degrees, referenced in calculations to the positive x-axis. There are also associated velocities $v(t)$, each one calculated as (inter-frame distance moved)/(time interval between these adjacent frames). However, data is only from adjacent frames between which a dancer is moving. These fractions of the 17 inter-frame motions are indicated in the first row shown in Table 1. As an example, the 0.63 SU/sec for dancer 1 is the average of 0.59 × 17 = 10 separate velocity calculations. There is seen to be little variation here among dancer average velocities. The average of the four tabulated values, namely 0.63, 0.80, 0.61 and 0.59, is 0.66 SU/sec.

Table 1: *Motion data analysis from Apollonian Circles*

	Dancer 1	Dancer 2	Dancer 3	Dancer 4
Fraction of intervals moving	0.59	0.71	0.76	0.82
Average velocity, SU/sec	0.63	0.80	0.61	0.59
Coefficient of variation of velocity	0.60	0.72	0.42	0.48
Standard deviation of θ, degrees	94	109	91	106

It is interesting to identify to what extent lines of dancers appear, and what are their associated orientations. These can show contrast with other spatial distributions of dancers. Fitted ellipse quantifiers in frames having more than two dancers on stage were therefore examined. Results are:

Fraction of frames with all dancers in-line (minor axis = 0)	0.4
Remaining fraction of frames with ellipse describing CA	0.6
Average θ_e	-11^o
Standard deviation of θ_e	41^o

Here θ_e is the angle of the ellipse major axis with respect to the positive *x*-axis. It is significant that within half of the in-line 0.4 fraction, i.e. 0.2, are found to have a line with a slope –1. This orientation is from upstage right towards downstage left. Choreographers often employ such a diagonal line of dancers as a strong element in a dance work.

We might also look for correlations between any pair of dancers, for example the set of values for the pair of velocity angles $\theta_1(t)$ and $\theta_2(t)$ of dancer 1 and dancer 2, as these change among inter-frame intervals in which there is movement. For the six possible pairings of the four dancers, this circumstance enabling additional types of correlation calculations occurs in the majority of the entire 17 intervals. Results for the absolute values of the correlation coefficients between directions within a pair are:

Highest correlation coefficient: ~0.75 between dancers 2 and 4; and between dancers 1 and 3
Lowest correlation coefficient: ~ – 0.3 between dancers: 1 and 2; 1 and 4; 3 and 4

Somewhat related to the preceding is to compare how θ changes in sequential intervals when measurable from moving pairs. Thus, it was found that values of $\theta_{i+1} - \theta_i$, related to angular velocity $d\theta/dt$, are always much larger in a clockwise direction (i.e. such as the extent of relocating from upstage towards extreme stage left and then towards the audience) for dancers 2 and 4 compared at the same times with dancers 1 and 3.

Another correlation coefficient we checked is that between a motion state variable defined by assigning 0 or 1 to intervals in which the dancer is remaining at a stage location or is moving, respectively. Correlation coefficients for any pair of dancers are found to be positive, i.e. tend to be in the same state in any interval, more often than being in opposite states. Highest correlations are near 0.85 for both the pair 1 and 2 and the pair 1 and 4. Lowest are near 0.51 for both the pair 2 and 3 and the pair 3 and 4. An observation related to these findings is that between 76% and 94% of the time any selected pair of dancers will be in phase, i.e. either both moving or both remaining in place. This is a significant departure from a 50% expectation from random dancers, and is one measure of the extent to which choreography has introduced order into the piece.

Another aspect of analysis is to find probabilities of events that can be used subsequently in simulating a dance having "Apollonian Circles" characteristics. One set of 16 probabilities, tabulated by counting events, indicate finding a state of movement M or stationarity S for each dancer during inter-frame intervals. Designating this as XXXX for four dancers, each X may be a M or S in the 16 instances. Results for these state probabilities are given below the XXXX state description. For example, in 9 of 17 inter-frame intervals a state of MMMM occurred, the probability being 9/17 = 0.5294.

There is another set of probabilities measured by counting inter-frame events that involve what value of θ_{i+1} occurs next after θ_i among the dancers and frames. In the lower half of the Table 2 some simplification when counting events was achieved by grouping similar angles to the nearest integral multiple of ±45°, thus reducing table size. Its headings have either the prior frame's θ_i or stationarity S and then the subsequent frame's θ_{i+1} for which the probability is also noted. The subsets of probabilities having a common originating θ_i or S add to 1.0.

Results as calculated in Table 2 may have various purposes. These stem from their characterization of certain stylistic features a choreographer is using in a particular piece. Possibly in instances where substantial revisions are being made the choreographer might wish to have added insights from knowledge of how probabilities change. Quite distinct from this there may be special usage of probability tables such as this in future. This might be computer-assisted choreographed dance synthesis. Here a Monte Carlo like simulation program uses probabilities in calculating sequential frame coordinates that serves as a framework for creating a desired style given to it by tabulated probabilities.

Table 2: *Probabilities of patterns in "Apollonian Circles"*

MMMM	SMMM	MSMM	MMSM	MMMS	SSMM	SMSM	SMMS
0.5294	0.0294	0.0294	0.0294	0.0294	0.0196	0.0196	0.0196
MSSM	MSMS	MMSS	MSSS	SMSS	SSMS	SSSM	SSSS
0.0196	0.0196	0.0196	0.0294	0.0294	0.0294	0.0294	0.1176

S,180	S,0	S,–90	180, 90	180,–90	135,–90	90, 135	90,–90	45, 90	45, –90	0, 90	–90,45
0.47	0.33	0.2	0.71	0.29	1	0.5	0.5	0.5	0.5	1	1

A visual display of how the CA moves around may be seen in the trajectory in Figure 4. This analysis uses a subset of 15 of the data frames. The points plotted are the weighted averages of the x and y locations of dancers where t_i is the time at the *i*th data point, with *i* indicated in *italics*. Each point thus incorporates the four dancers' weights and coordinates rather than simply the coordinates. Additionally, the average weight is coded for each frame. Some quantitative features of this trajectory may be readily obtained from its CA values and associated times. These features are the properties of the distribution of time intervals Δt between frames chosen where significant data descriptive changes occurred. Another distribution of trajectory features in the table below is from gross tracking of trajectory points movement by their radial distances from the average CA (which is at x = – 0.162 and y = – 0.131), irrespective of angular direction. Also path-lengths of CA motion seen within Figure 4 have features of their statistical distribution calculated from individual radial distances and summarized in Table 3.

The interpreter's average time interval, 4.6 sec for the 15 frames indicates how often significant location or weight changes occur. Figure 4's radial CA quantifiers in Table 3 show it mostly confined to within the central stage area. This weighted average location of the four dancers remains somewhat centralized in spite of individual dancers having x and y coordinates up to 2. Examination of distances the CA moves between frames shows most of these being well under 1 unit. (These calculations are shown in detail in [5].) However one occurrence of 1.58 units in this 69 seconds (frequency = 1/69 sec^{-1}) appears between frames 3 and 4 - perhaps more meaningful than distribution skew (i.e. asymmetry in occurrences above vs. below the median). This extent and frequency of large CA movements in a piece can be a useful supplement to skew.

A CA trajectory such as Figure 4 gives the choreographer a combined space- time overview that provides a different perspective than what is seen, spread out in time, in a video. It is beyond the scope here to investigate how its properties, only a few illustrated in Table 3, may be an aid. It is likely that a learning process would be needed to uncover these, such as the choreographer observing trajectory properties change as changes are made in a development of a particular work. It would be a goal here to discover any particular impact that detailed knowledge of the CA trajectory might have.

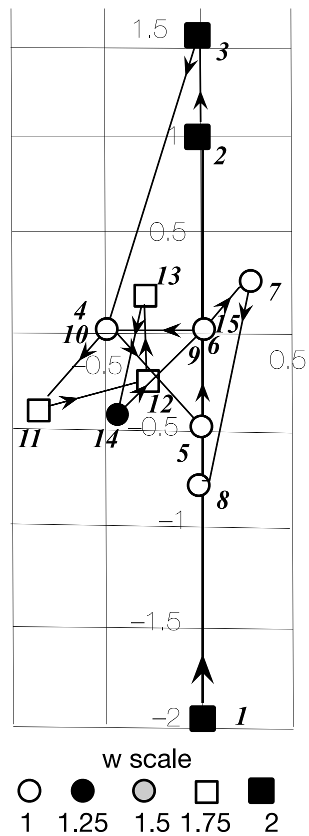

Figure 4: *Trajectory of the CA*

Table 3: *Trajectory features*

	Δt	radial distance	path-length
median	4.00	0.37	0.56
average	4.60	0.49	0.66
SD	3.50	0.38	0.30
skew	2.46	1.40	2.40
Largest	16.0	1.51	1.58
Smallest	2.00	0.14	0.35
Range	14.0	1.37	1.23
Largest change	13.0	1.14	1.08

Summary and Conclusions

We have demonstrated a variety of measurements that can be derived from a quantitative focus on the CA. However, a better sense of the possibilities may become apparent when many more dances are digitized in a similar way, and we search for quantitative characteristics between works by a single choreographer, between choreographers, or between dance forms. A related next step will be to prepare software that allows choreographers or dance viewers to digitize dances from videos. We are experimenting with software that corrects non-ovehead video calculations to overhead video for ease of tabulation. Classroom applications might include having students do similar analyses of dances performed at their school, as well as including orthogonal regression as a topic in statistics or linear algebra classes,

It might also be that this work relates to interests well outside those of the dance. The nephew of one of the authors, researching leopards in Africa, has affirmed that studies of these and American mountain lions' defined activity center may have attributes and analyses akin to those of the CA [2]. Another example, somewhat speculative, is assisting in strategizing and training of American football quarterbacks who must make optimal use of a few seconds to 'read' defenders' locations with a few sequential CA's before making his pass decision. Data might be acquired from existing expertly interpreted pass plays, or even Monte Carlo simulations having varieties of CA sequences, given the statistical behavior of specific players and configurations encountered. Such would show best pass completion percentages achieved with their optimal accompanying CA strategizing.

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