Folding the Dragon Curve Fractal

Natalija Budinski
School "Petro Kuzmjak"
Rusinska 63, 25233 Ruski Krstur,
SERBIA
nbudinski@yahoo.com

Miroslav Novta
Schneider Electric DMS NS,
Narodnog Fronta 25A-D, 21000 Novi Sad,
SERBIA
miroslav.novta@gmail.com

Abstract

This workshop will introduce participants to fractal properties through the exploration of the fractal called "the dragon curve". This approach can be applied in teaching in order to introduce students to new directions in mathematics. The workshop will have three parts, the first part will be an introduction to fractals through paper folding, the second part will be exploring mathematical properties and the last part will be exploring the infinite world of fractals in other disciplines besides mathematics.

Introduction

When fractals are mentioned, they usually suggest modern and advanced mathematics to us. The father of fractals, Mandelbrot, described them as: "beautiful, damn hard, and increasingly useful" [2]. They can be found everywhere, not just in mathematics and science, but in nature, which is practically full of fractal patterns. Fractals have become an inspiration for mathematical art [4].

On one hand, there is a rising interest in fractal geometry in the world of science, but on the other hand, lessons such as those here cannot be found in the curricula of most schools in countries worldwide, [6], [8], even though there are many ways to bring aspects of fractal geometry into the classroom. This paper presents a new one. By including fractals into regular lessons new directions in mathematical education might appear that bring together mathematics, art and science. Changing the way of teaching geometry can contribute to educational transformation in general.

Fractals can easily be applied in mathematics teaching, particularly for visualization. Appealing patterns, based on mathematical expressions, can contribute to many aspects of mathematical education, even on a basic level. Geometric shapes, geometric structures, symmetry, iterative and recursive forms can be taught by modeling and explaining fractals. Although fractals are very complex, they are easy to make through a repetition of forms. Researchers have found many excellent educational benefits by teaching fractals, [6], [7], [8], and this workshop will contribute to the inclusion of fractals into mathematics teaching through a combination of simple mathematics and hands-on activities.

The Dragon Curve Fractal

The workshop gives instructions on how to teach fractals and their properties, which are advanced mathematical concepts. It will demonstrate how to introduce this topic to students with no previous mathematical knowledge of fractals. Good results can be achieved during the teaching process by using hands-on activities and paper folding, connecting aesthetic and mathematical aspects of fractals. The workshop will have three parts: an introduction to fractals, discovering their mathematical properties and finally enjoying their universal natural and artistic beauty. As an illustration we have chosen the dragon curve fractal.

The dragon curve was discovered by John Heighway, a NASA scientist, in 1966 [9]. This curve consists of series of right angles. It can be generated by starting with a right angle at the segment base, and replacing each segment with two right angles, rotating at 45° to the left or to the right. It can be seen that each segment is replaced by two smaller segments scaled by $1/\sqrt{2}$. The additional segments represent the legs of an isosceles right-angled triangle with the original segment as hypotenuse. In this part of the workshop, participants will be challenged to make short drawings of the dragon curve in a standard iterative way with a series of successively more complex drawings [11], as shown in Figure 1.

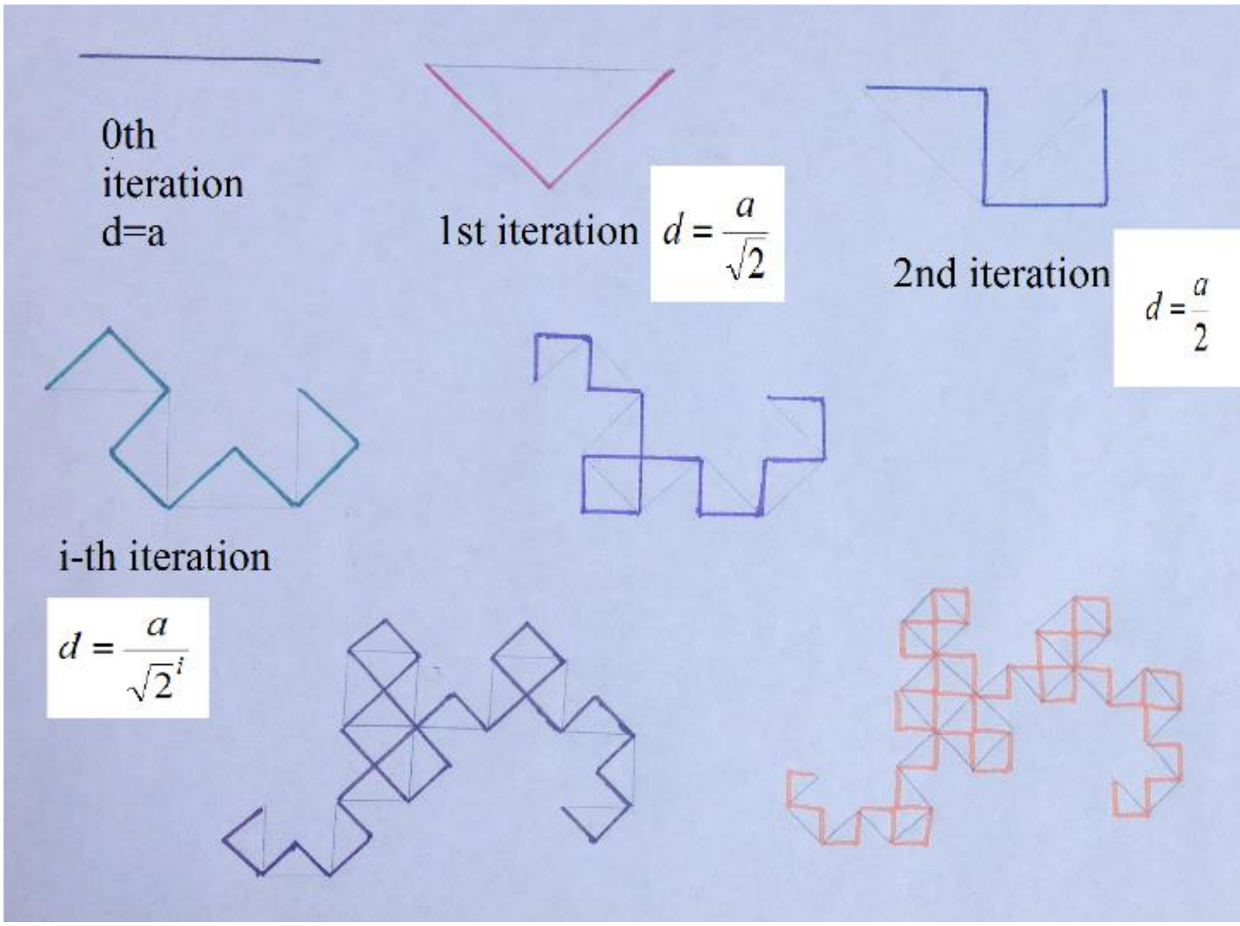

Figure 1: *Dragon curve iteration drawn by hand*

Dragon curve made by paper folding. The next part of the workshop is based on the fact that the visual representation and hands-on activities can aid the participants in discovering fractals without formal definitions and concepts [3]. For that purpose, the participants will be given paper strips. The first step will be to fold a strip of paper in the same way four or five times from left to the right. Folding the paper in half increases its thickness, so a paper strip folded more than five times is harder to manipulate. After unfolding the paper, so all creases are opened up to 90° angles, the participants will be asked to observe the apparently jammed piece of paper and find a pattern as in Figure 2.

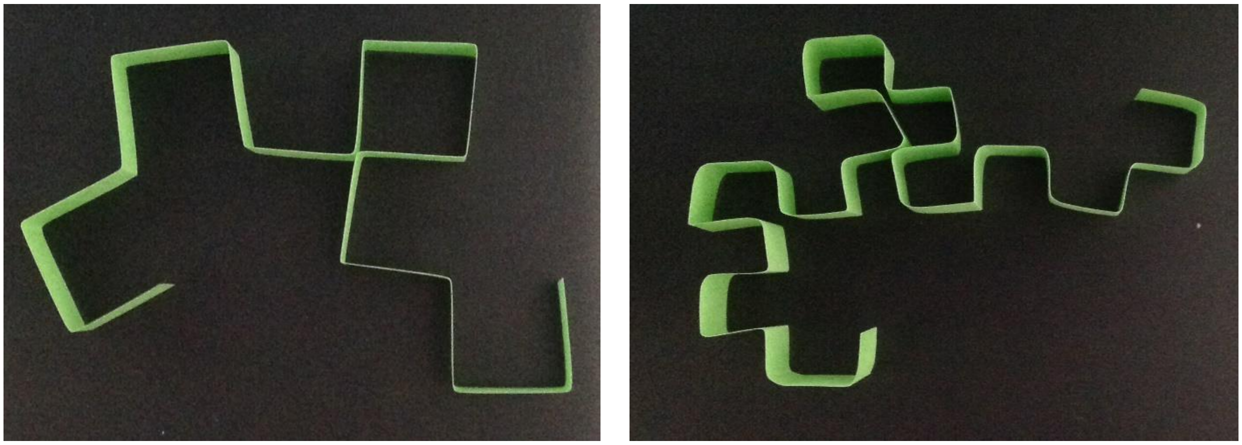

Figure 2: *The result of folding the paper strip four and five times, respectively*

After folding and unfolding a paper strip, we will use origami and paper folding language to explain it. Each paper fold is either a valley fold or a mountain fold, as shown in Figure 3. Participants will be told that the folds always halve the strip with the same crease distance, and have the same fold angle as well. If we fold the paper strip for the first time, we have one valley fold. The second fold results in a valley, a valley and a mountain, and the third fold would be VVMVVMM, where V stands for valley and M for mountain fold, respectively. Also, during this part the participants will be asked if they can further predict the pattern of the mountain and the valley fold. Figure 4 shows several iterations of the dragon curve achieved by folding the strip of paper progressively more times and then unfolding to 90 degrees.

Figure 3: *Valley and mountain folds*

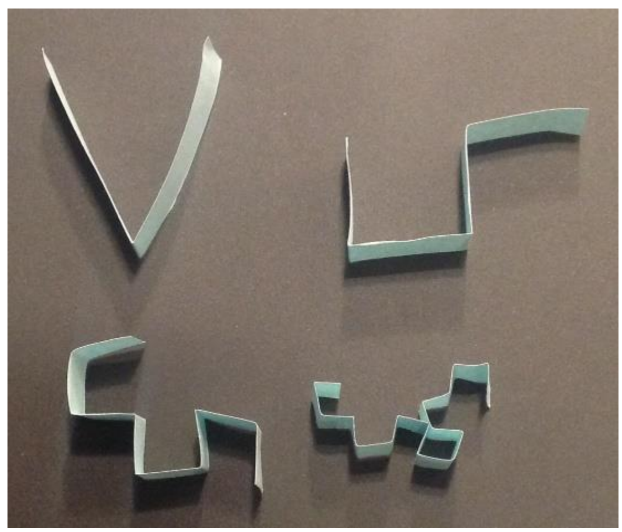

Figure 4: *Patterns of valley and mountain folds*

We will ask a hypothetical question: “If we could keep folding an infinite amount of paper strips, what would be the result?” The participants should be motivated to fold as many paper strips as they can and then join them together [10]. To achieve more, the participants should work in pairs or groups. For example, if they have eight dragon curves received by folding the paper strip four times, then their combination would follow the steps shown in Figure 5.

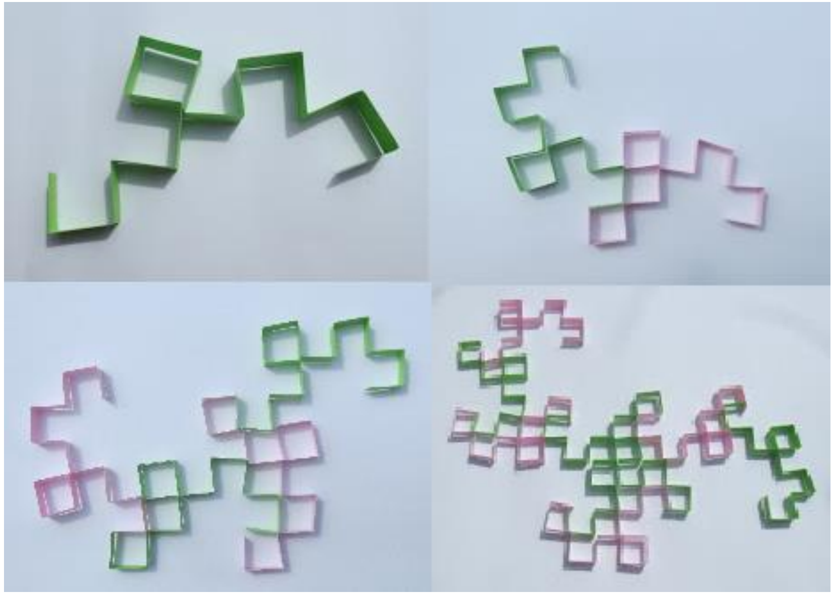

Figure 5: *Dragon curve as a result of joining folded paper strips*

Even though the dragon curve has "nice" mathematical properties, it is not sufficiently familiar to the wider audience [9], and this workshop will try to reveal many dragon curve facts to the participants. This section of the workshop should conclude by introducing the participants to a computer generated dragon curve as in Figure 6. This activity will last approximately 40 minutes.

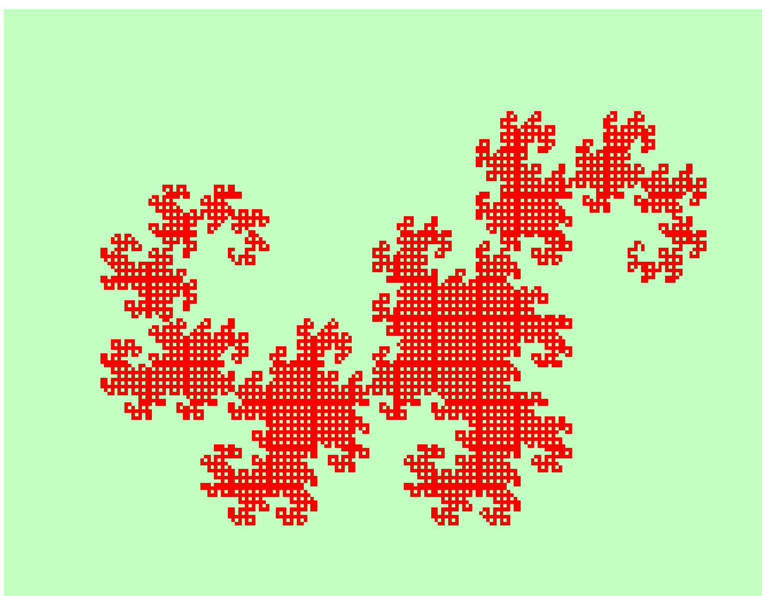

Figure 6: Dragon curve generated by a computer

Mathematical properties of the dragon curve. In the second part of the workshop, fractals will be analyzed in the light of mathematics, and the participants will be given an insight into modern geometry. Fractals can be defined as geometrical curves consisting of identical shapes that are repeating infinitely on a decreasing scale [5]. An important feature of fractals that makes them beautiful and eye-appealing is self-similarity. Self-similarity is a property when a part of an object is exactly or approximately similar to the whole object. This is a common fractal property. The dragon curve has many self-similarities as it is shown in Figure 7. It is compelling fact that self-similarity distinguishes the dragon curve and the fractals in general from Euclidean figures [6].

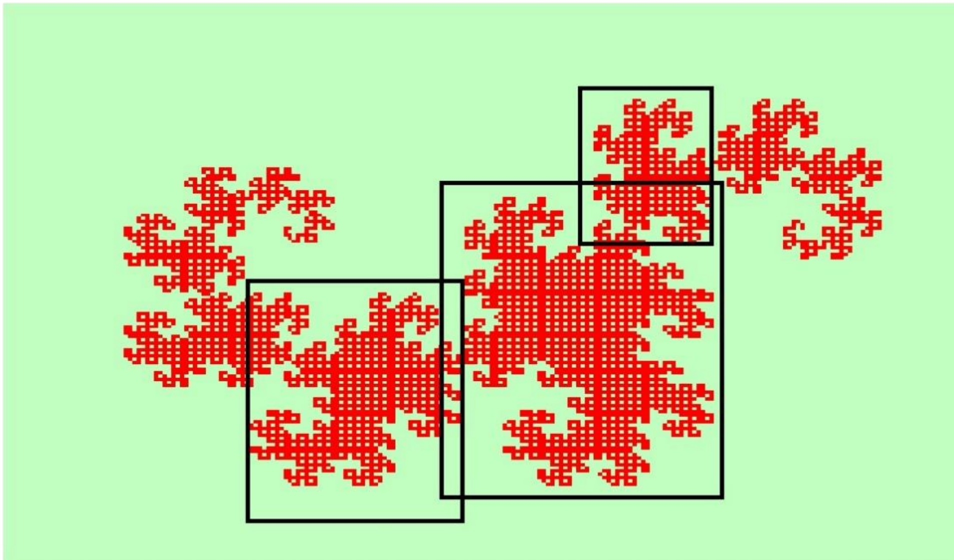

Figure 7: *Self-similarities of the dragon curve*

The fractal dimension is also an important property and interesting because the dimension value can be a fraction. That is how fractals got their name. At first sight, explaining the dimension which is not necessarily a natural number, such as 1, 2 or 3 as we are used to, might seem a hard task, but the participants will grasp the idea in just a few steps, as explained in Figure 8. If we halve a segment (dimension 1) we get two self-similar copies from the segment. In this case the scale is 1/2. If we do the same with a square, or a cube, which have dimensions 2 and 3 respectively, we will get four and eight self-similar copies respectively. Mathematically, this can be noted as $n=s^d$, where 1/s is scale, n is the number of self-similar copies, and d is dimension.

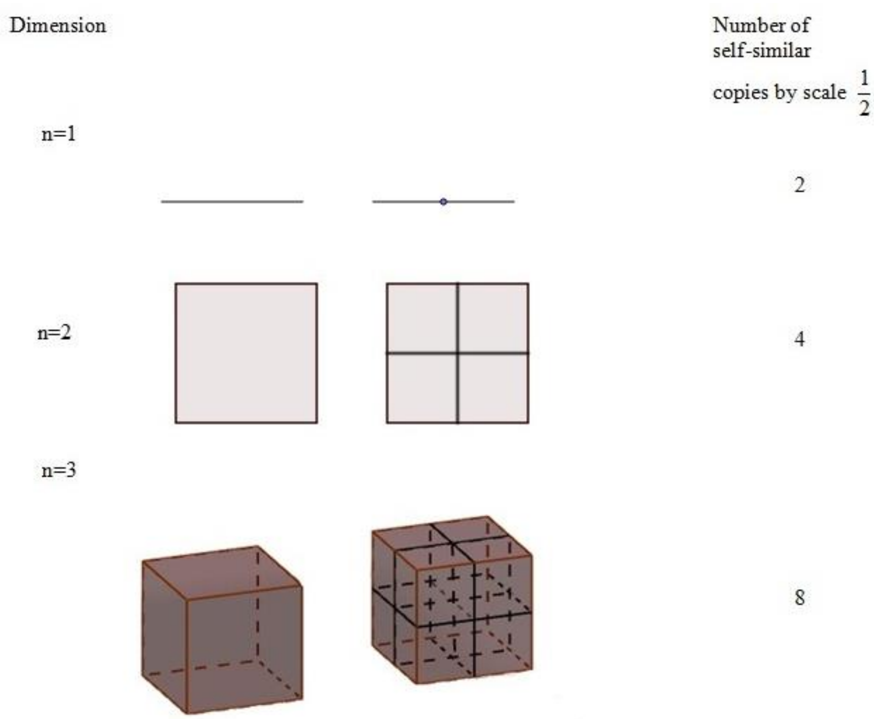

Figure 8: *Traditional way of defining dimension*

The workshop participants will be encouraged to discover the dimension of the dragon curve, which is 2. It is the result of the equation $2 = \left(\sqrt{2}\right)^d$, and the consequence of the fact that we have two self-similar copies at scale $1/\sqrt{2}$. Also, as a final conclusion for this part of the workshop the formula for calculating the fractal dimension will be derived as formula d=(ln n)/(ln s). The mathematical explanation will last 30 minutes.

Fractals Around Us

The third part of the workshop will be dedicated to fractals and their universal aesthetical beauty. Since nature performs on different levels of complexity, Euclidean geometry lacks the ability to describe certain instances such as clouds, mountains or other shapes that exist in nature [1]. The workshop ends by showing some examples of fractals in nature, art, or architecture in a variety of forms such as cauliflower, lightning or clouds. The last part will be 20 minutes.

Conclusion

There are various ways to start the educational journey of exploring connections between mathematics and art. This workshop describes how to investigate fractals with the help of hands-on activities and paper folding. This kind of workshop can be implemented in regular school lessons in addition to classical geometry, which undoubtedly plays a significant role in mathematical education and shapes human logical thinking. In this workshop the participants will be guided from simple paper folding to surprising complexity, which might inspire them to teach fractals to their students.

References

[1] B. Mandelbrot. *The Fractal Geometry of Nature*. W.H. Freeman and Company, San Francisco. 1983.

[2] B. Mandelbrot. *24/7 Lecture on Fractals.* Ig Nobel Awards. Improbable Research. 2006.

[3] D. Hilbert and S. Cohn-Voss. *Geometry and the imagination.* AMS Chelsea Publishing. 1999.

[4] G. Irving and H. Segerman. "Developing fractal curves". *Journal of Mathematics and Arts, 7*(3-4), pages 103-121. 2013.

[5] L. Debnath. "A brief historical introduction to fractals and fractal geometry". *International Journal of Mathematical Education in Science and Technology, 37*, pages 29-50. 2006.

[6] M. Fraboni and T. Moller. "Fractals in the classroom". *National Council of Teachers of Mathematics, 102*(3), pages 197-199. 2008.

[7] R. Siegrist, R. Dover and A. Piccplino. "Inquiry into fractals". *The Mathematics Teacher, 103*, pages 206-212. 2009.

[8] S. Chen. *Assessing Awareness, Interest, and Knowledge of Fractal Geometry among Secondary Mathematics teachers in the United States and China*. University of Southern Mississippi, The Aquila Digital Community. Dissertations, 129. 2015.

[9] S. Tabachnikov. "Dragon curves revisited". *The Mathematical Intelligencer, 36*(1), pages 1-13. 2014.

[10] `www.cutoutfoldup.com` (as of Feb. 1, 2017).

[11] `http://vihart.com/doodling-dragon-dungeons/` (as of Apr. 15, 2012).