DNA-inspired Basketmaking: Scaffold-Strand Construction of Wireframe Sculptures

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Abstract

A recent achievement in nanotechnology has been self-assembly of wireframe models from *scaffold strand* DNA—long single strands of DNA encoded with information that will become folding and gluing instructions when the strand hybridizes with shorter strands called *staple* DNA. In the assembled complex, the scaffold strand traces a non-crossing Eulerian circuit (an *A-trail*) over the surface of the model (that surface being limited to a topological sphere.) I present a larger scale—human-mediated—version of this technique for making wireframe sculptures and armatures. Using triple strands (as opposed to the double strands of DNA) is advantageous with this technique. The information that needs to be imprinted on the scaffold strand amounts to merely some pre-bends and dot markings. Having weaving instructions marked on one of the elements of the weaving may in some cases be faster and less confusing for the weaver than transcribing from symbols on paper.

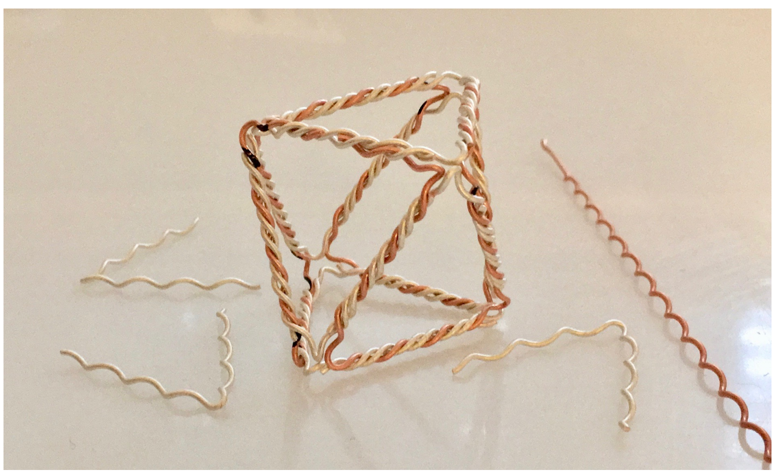

Figure 1 : *A funky, oblique octahedron assembled by following information imprinted on the long scaffold strand (copper.) The shape is held together by the short staple strands (silver plate.)*

Introduction

A recent achievement in nanotechnology [2] has been the self-assembly of what might be called DNA baskets—wireframe models that are topological spheres (including a simplified version of the standard 3-D model known as the Stanford Bunny.) The technique encodes information onto a flexible single strand of DNA called the scaffold strand. When the scaffold strand hybridizes with shorter complementary strands of DNA, called staples, the scaffold strand is rigidized along certain lengths that become struts, but remains flexible at small gaps that become folds. Furthermore, the two ends of each staple bond to different areas of the scaffold strand, thus pulling certain folds together to form nodes. We are going to make a macro-scale wireframe sculpture that goes together the same way (Figure 1.)

Eulerian Circuits and A-Trails

Since the scaffold strand supplies a part of each edge (strut) in the model, it must fold in a way that covers every edge once. If we consider the model as a graph embedded in a sphere, the scaffold strand follows an *Eulerian circuit* of the graph. Euler proved that a graph has such a circuit if and only if all its vertices are even-valent, meaning that are incident to an even number of edges. We call such graphs *Eulerian.* (If a particular model we want to make is non-Eulerian, it will be necessary to modify the model, for example by doubling edges.)

It is simplifying, and may prevent tangles, if we carry the planarity of the model right through to its construction: thus we do not allow the scaffold strand to cross over itself. That demands a special sort of Eulerian circuit; in particular, we require an Eulerian circuit that makes a 'hard right' or a 'hard left' turn at each vertex. This kind of Eulerian circuit in an embedded graph is called an *A-trail* [4] (a mark like the horizontal stroke in a capital 'A' can be used to indicate pairs of edges adjacent in the trail, as in Figure 2.) Andersen et al. [1] demonstrate some sufficient conditions for the existence of A-trails in spherical embeddings of Eulerian graphs, and give algorithms for finding them.

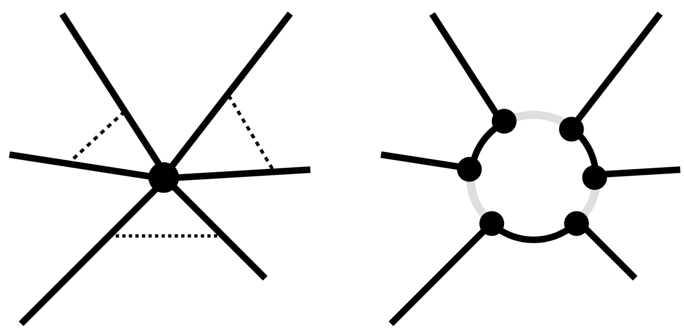

Figure 2 : *Left: edges that are adjacent in an A-trail can be marked with A-like hashes (dashed lines). Right: vertex truncation converts the A-trail into a Hamiltonian circuit (un-grayed lines) in a 3-valent graph. The grayed lines will be seen to represent the bonding action of the staples.*

A-Trails to 3-valent Hamiltonian Circuits

A simple construction called *vertex truncation* turns any A-trail into a Hamiltonian circuit in a 3-valent graph embedded in the same surface (right side of Figure 2.) Vertex truncation simply replaces each vertex with a loop, the original edges that were incident to the vertex are now separately incident to the loop at new vertices in the same cyclical order.

The A-trail in the original graph now needs to use some of the new loop edges (skipping others), and in this way visits every vertex in the new graph exactly once—in other words it is now a Hamiltonian circuit in the new, 3-valent graph rather than an Eulerian one.

An advantage of this new graph is that it explicitly shows the bonds the staple strands need to form; each skipped edge (gray in the Figure) represents one staple.

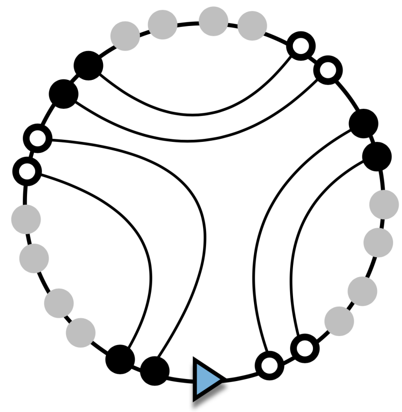

Figure 3: *A polar view of one hemisphere of 3-valent graph on the sphere after its (normally serpentine) Hamiltonian circuit has been unfolded/stretched to constitute the equator. Grayed vertices connect to edges on the other hemisphere; the first-encountered vertex of each edge on this hemisphere is shown white. Begining at the arrow and proceeding in the indicated direction, this hemisphere would be coded as the Dyck word '(())(())(())' or, in emoticon code, 'uudduudduudd.'*

3-valent Hamiltonian Circuits to Shuffled Dyck Words

If we allow the skipped edges to 'stretch' as the underlying, topologically spherical, surface is 'inflated', the serpentine Hamiltonian circuit gradually unfolds itself into a belt around the equator of the sphere (Figure 3.) Each skipped edge now lies entirely in either the Northern or Southern hemisphere; though they have stretched, the inflation process has not caused the skipped edges to cross.

Starting from anywhere on the equator (i.e., the Hamiltonian circuit) proceeding around in a chosen direction, we can encode the graph as a *parenthesis word*—in effect simulating the weaving process—in the following way. We use a different style of parentheses for each respective hemisphere: say ()-style for the upper hemisphere shown in Figure 3, and []-style for the lower hemisphere (not shown in the Figure, which diagrams the skipped edges joining the grayed vertices.) We write down an open parenthesis when a skipped edge is first encountered, and a close parenthesis when the edge is encountered the second time. The result is *shuffled Dyck word* [3] (individually, each kind of parenthesis forms a proper parenthesis word—a Dyck word—but the two words are shuffled like cards in a deck.) For example: (([[[[))((]][[))((]]]])).

Readability is enhanced by this substitution of 'emoticon letters': (= **u,**) = **d,** [= **n,**] = **u.** Notice that the letters are either open or closed, and point either up or down: **u,** up and open; **n,** down and open; **d,** up and closed; **p,** down and closed.

The example becomes: **uunnnndduuppnndduuppppdd**

A weaver would read this code sideways—like an emoticon—turning 'up' and 'down' into 'left' and 'right.'

Shuffled Dyck words that correspond to A-trails, such as the example given, have a special property: up letters and down letters always come in pairs. This is because, when the A-trail makes its 'hard' left or 'hard' right turn, both skipped edges at that turn will lie to the outside of the turn (refer to the right side of Figure 2.)

Putting Shuffled Dyck Words on a Wire

We wish to substitute a length of pre-bent wire for the scaffold strand of DNA. Rather than hydrogen bonds, we will rely on entanglements of helical wires to hold strand and staples together—"woven springs" as sculptor Alexandru Usineviciu [5] calls them. We imagine the wire to be only gently bent so that it remains extended, i.e., it is not all balled up. Each bend corresponds to a pair of letters in the shuffled Dyck word. Since the bend itself 'encodes' its own location and direction of bend, the only additional information that needs to be imprinted on the wire is whether each letter is open or closed. It suffices to place a black mark at the location of all closed letters (Figure 4.) There is always a one code letter before a bend and one letter after the bend: any such location that is unmarked is an open letter.

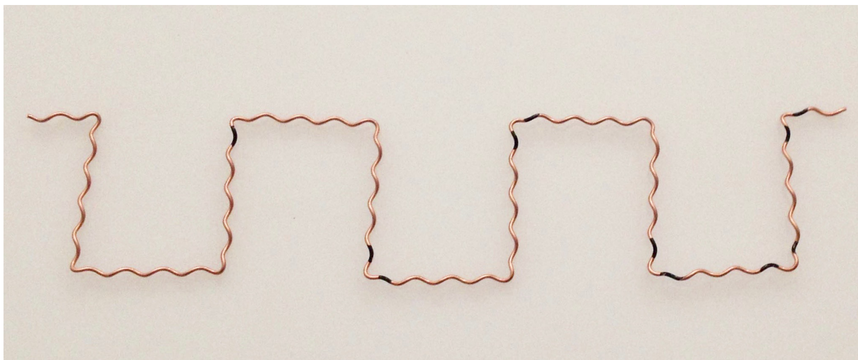

Figure 4 : *A pre-bent scaffold strand with the positions of closed letters marked with black ink. Since there is always one letter before each bend, and one letter after, the locations of the open letters are implicit. Also implicit is the direction of every letter: the non-Hamiltonian edges always lie on the outside of the bend.*

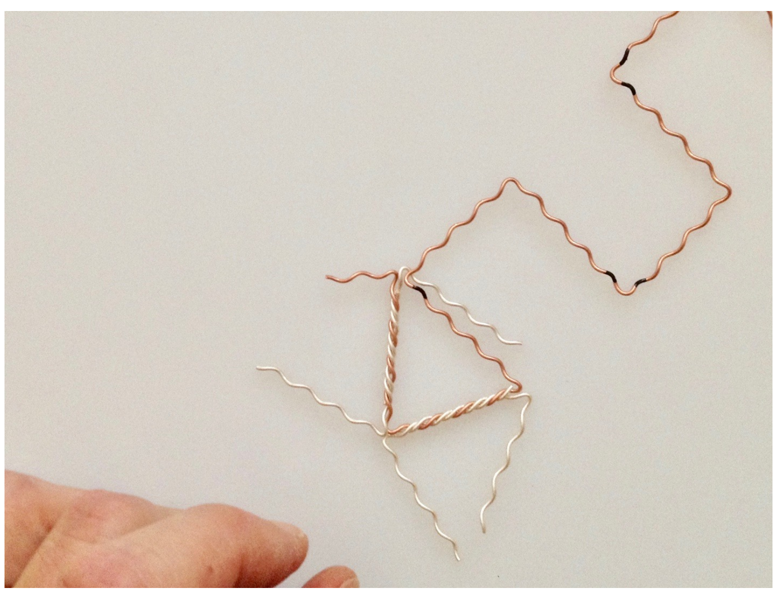

Figure 5 : *At this point in the weaving the starting letter, a 'u', has been postponed because there is only half an edge to wrap it on, and these letters have been acted, 'unn'. At the current bend, another 'n' will be acted, then the first closed letter, a 'd' will be acted by winding the free end of the first-added staple around the scaffold strand to form a 3-ply construction on that edge.*

Weaving

Once the code is learned, the weaving is straightforward.

An open letter means add a new staple.

A closed letter (i.e., a location marked with ink) means attach the free end of staple that is already in the work. Which staple? Very simply, the most recent staple on that side that still has a free end—in other words, the one closest to hand—reflecting the condition that defines a Dyck word.

Staples are added to the work by winding them around other wires. All the wires are shaped in a helix capable of combining with two other helices to make a 3-ply 'cable' (as opposed to the 2-ply construction of DNA.) This allows staples to bond with the entire length of the edge they wrap around, forming a strong, integral strut rather than a strut that has a weak point in the middle where two staples meet and are unable to proceed past each other because of the 2-ply limitation.

The scaffold strand begins and ends with a half edge. Since a half edge is not enough to wrap a staple around, acting on the first letter in the code is postponed till the very end when the other half of the same edge can be brought into play.

Figure 6 : *Free ends of the two staples wrap around the two half edges to form the final strut.*

Figure 5 shows an early stage in the weaving. By the end, Figure 6, only the two half edges and two free staple ends are left; these wrap together to form the last strut. The completed model can be seen in Figure 1.

Building with Bent Helices

Building with bent helices forces a certain quantization and incongruity in lengths if wish all to make all the bends equivalent. Consider the analogy to walking: it is difficult to make a right turn off of your right foot—

therefore we almost always take an even number of steps between turns of the same direction. For the same reason helical bends of the same sense ideally are separated by an even number of helical half-wavelengths; and bends of opposite sense ideally are separated by an even number of helical half-wavelengths. The quantization, the incongruity, and as well the need for an Eulerian model, make it difficult to order up an arbitrary shape with this technique, but the shapes this technique naturally makes are interesting and fun to make.

Summary

The scaffold strand technique being used in nanotechnology to self-assemble DNA wireframe models can be adapted for macro-scale, human-mediated construction of wire baskets. Provided the model is a topological sphere, encoding the assembly information on the scaffold strand is easy. In some cases it may be faster and/or less error prone for a weaver to decode information imprinted on a continuous element of the weaving, as opposed to working from written instructions. 3-ply construction avoids creating a weakness at the center of the struts (At nano-scale, DNA is limited to 2-ply construction.) Quantization and incongruity of lengths, as well as the need for an Eulerian model, make it difficult to produce arbitrary shapes with this technique.

Some Notes on the Model

The scaffold strand and the staples were formed from 18 gauge (1.0 mm) copper wire, both natural and silver-plated, coiled on a 1.8 mm diameter carbon-rod mandrel turned in a variable-speed electric drill. (1.8 mm diameter piano wire would serve just as well.) Commonly available copper wire is not springy enough, so the wire must be work hardened first by twisting it with a variable-speed electric drill before winding the coil. The wire should be twisted until examination under a magnifier reveals striations on the wire running at about a 45-degree angle. Stress concentration at the drill chuck may cause the wire to a break there once or twice before adequate twisting is achieved. Use no more speed or tension than necessary.

Once the coiled wire comes off the mandrel (a few twists in the uncoiling direction may be needed to free,) it needs to be stretched to a precise helical wavelength of 8.2 mm. Stretching too little will result in a loose 3-ply construction, too much will make the construction too tight to assemble without irreversible bending of the wire. Optical comparison with a printed template helps to get the wire stretched to just the right wavelength. The cut lengths and bending lengths, in terms of wavelengths, are easily gleaned in the photos.

References

[1] Andersen, L. D., Fleischner, H., and Regner, S., “Algorithms and outerplanar conditions for a-trails in plane eulerian graphs.” *Discrete Applied Mathematics,* v. 8, n.2, pp. 99-112.
[2] Benson, E., Mohammed, A., Gardell, J., Masich, S., Czeizler, E., Orponen, P., and Höffberg, B., “DNA rendering of polyhedral meshes at the nanoscale.” *Nature,* vol. 8, n. 2, pp. 441-444, 2015.
[3] Cori, R., Dulucq, S., and Viennot, G., “Shuffle of parenthesis systems and Baxter permutations.” *J. Combinatorial Theory Series A*, vol. 43, n. 1, pp. 1-22, 1986.
[4] Fleischner, H.,“A-Trails in Plane Graphs.” chapter VI.3, in Eulerian Graphs and Related Topics, *Annals of Discrete Mathematics,* vol. 45, North-Holland, 1990.
[5] Tucker, P., “Moorish Fretwork Revisited,” *Proceedings of Bridges 2012,* pp. 223-230, 2012.