Polyhedra:
Eye Candy to Feed the Mind

Stacy Speyer
Cubes and Things
Alameda, CA 94501, USA
cubesandthings@gmail.com

Abstract

This paper presents a group of polyhedra I made for a traveling exhibition on geometry. It describes how the beautiful photographs made to document them inspired the creation of a book called *Polyhedra: Eye Candy to Feed the Mind*, and gives geometric examinations of a few forms, including two new shapes that I discovered.

Geometry Playground

In 2009, the Exploratorium hired me as an Artist-In-Residence to create an interactive exhibit for the traveling exhibition called Geometry Playground. https://www.exploratorium.edu/geometryplayground/ Located in San Francisco, the Exploratorium describes itself as a museum of science, art, and human perception. Geometry Playground gathered a collection of objects, developed a group of hands-on exhibits, and experimented with climbing structures that all engaged participants spacial reasoning and showed off the fun and accessible side of geometry. While working with the Exploratorium's excellent staff, I made fifteen metal polyhedra and they built a round room (8 feet in diameter and 8 feet tall) to hold them, Figure 1.

Figure 1: *Polyhedra exhibit from Geometry Playground*

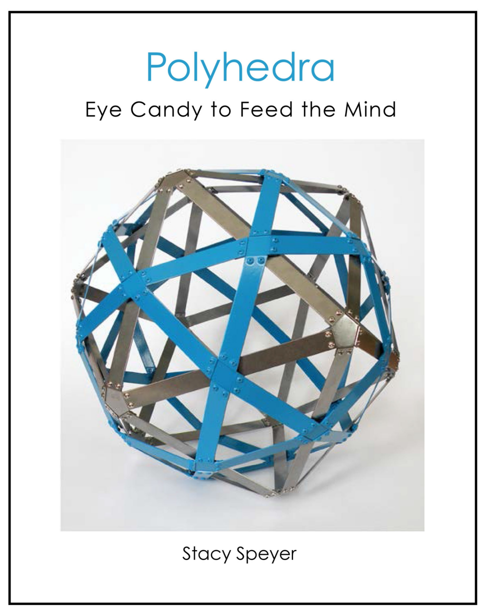

Figure 2: *Cover of book* Polyhedra: Eye Candy to Feed the Mind

Each piece was set into the cylindrical wall in its own little window. The polyhedra could be viewed and turned from the inside or outside of the space. One big label spanned the inside of the round room providing the name of each shape, the number of faces/vertices/edges, and each polyhedron had a line linking it to another form with a word describing their geometric connection. The arrangement of the forms, their ability to be turned, and the written information combined a playful experience with learning through observation and comparison.

Photographic Obsession

In art school, the importance of thoroughly documenting one's artwork is emphasized again and again, but I overdid it on this project. I photographed not only the completed forms, but I also shot the work at different stages of assembly. Watching each piece grow enchanted me. Often, after documenting one piece, I would unbolt it, attach it onto or inside another piece, and take more pictures. Occasionally, I would walk by a finished piece and a new angle would catch my eye, demanding to be captured in a photograph. Because these pieces were constructed with a minimal amount of material and the backside was visible through the front, the image of a form changed drastically if I moved the camera only a little. The perfection of aligned symmetry for one shot might fall into a curious tangle of lines from a slightly different angle. (For additional details and a big collection of color photos, see my book [1].)

In the midst of trying to pick out the best images (from the nearly 5,000 photographs), I noticed that the pictures told a story about the mathematical connections between the shapes. Just by arranging the photos in a certain sequence, I could teach concepts about polyhedra without using words. I picked out my top 200 images and organized them by symmetry type into what became the second and third chapters of my book, *Polyhedra: Eye Candy to Feed the Mind.* Figure 2 is the book's cover.

The first chapter, titled Introduction to Polyhedra, grew organically. I don't know if I could ever have begun this project had I known that I, an artist, would spend 4 years researching the math of polyhedra, writing, and making diagrams. My goal was a simple explanation that friends and family, who didn't share my enthusiasm for math, could understand. This required breaking down the geometry into bite sized pieces and using my best teaching voice to make the text fun enough to entice people to turn the page and learn another new concept.

A Little Bit Truncated

Constructing a form of only edges and vertices gives it a light, open aesthetic. Often my artwork feels like the object is just coming into being or slowly dissolving away. Like lines of calligraphy, the edges appear wide and bold in one direction only to become delicate accents from another view. A person preferring a strict definition of a regular polyhedron might not approve of the light truncation in these metal sculptures.

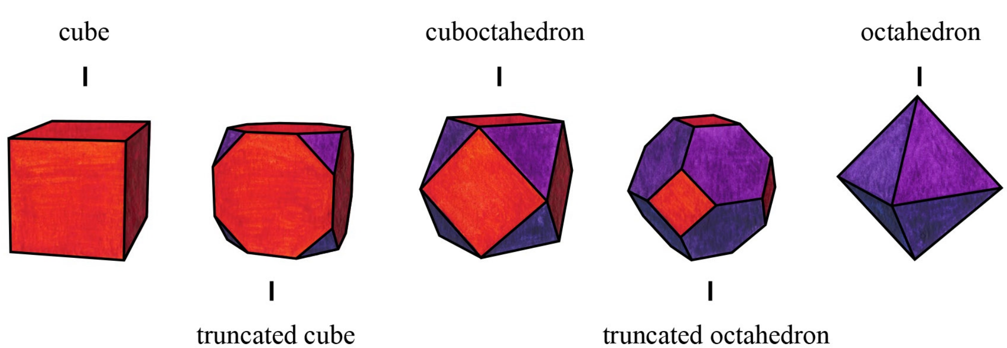

Figure 3: *Watercolor diagram of a truncating transition from cube to octahedron.*

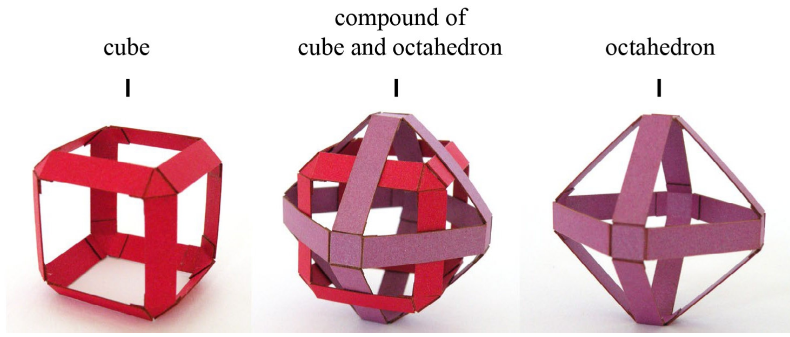

Figure 4: *Polyhedra made from glued laser cut paper pieces*

Truncating a polyhedron means to cut part of it off. To perform a basic symmetric truncation, slice away the same amount of material from all the vertices on a plane parallel to its dual face as in Figure 3. Though there are many ways to truncate a polyhedron, this series of metal polyhedra is just a little bit truncated. Only the ridges of the edges and the points of the corners have been trimmed away. Peter R. Cromwell, in Chapter 10 of his book *Polyhedra* [2], shows how these polyhedra can be seen as a stage of a metamorphic journey between forms, but for our interests, not enough material has been removed from these shapes for them to become a different polyhedron with a new name.

Truncating the cube in stages to reveal the octahedron, as in Figure 3, is one way to display the duality of these shapes. Other traits of duality are visible in the compound polyhedron in the middle of Figure 4, where all of the octahedron's edges perpendicularly cross those of the cube. Also, a small amount of truncation exposes the polyhedron's vertex figure. Do you see the little triangles on the corners of the cube and the squares on the vertices of the octahedron? An intrinsic connection between two duals is revealed when the edges of one shape frame a vertex figure of its dual.

Flipped Vertex Figures

Technically, only the point at the corner of a polyhedron is its vertex. A little bit of truncation reveals the vertex figure when, as Cromwell states, the same amount is removed from each edge [2]. For most of the metal shapes I made, the polygon at the vertex is the correct shape for that polyhedron's vertex figure, but it is not in the same orientation as the actual vertex figure. In Figure 4 the triangular vertex figure (where the edges of the cube meet) is flipped 180 degrees and the octahedron's square vertex figure is turned by 45 degrees. Compare these to the truncated cube and truncated octahedron in Figure 3. The vertex figure is a polygon dual to the sculpture's vertex piece: each vertex aligns with the center of the dual's edge.

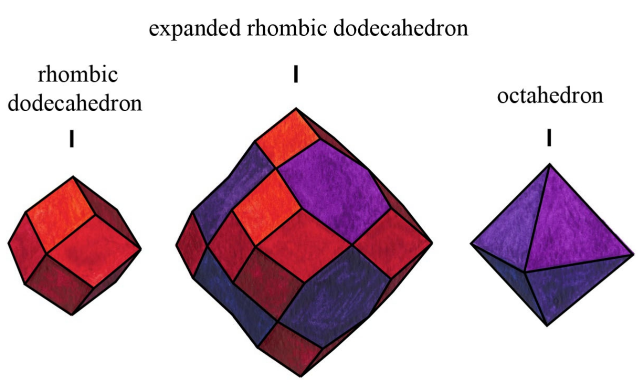

Figure 5: *Expanded rhombic dodecahedron with component polyhedra in watercolor*

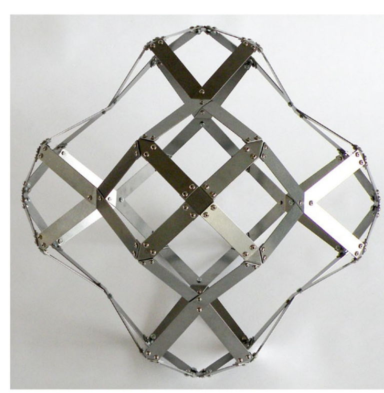

Figure 6: E*xpanded rhombic dodecahedron in metal*

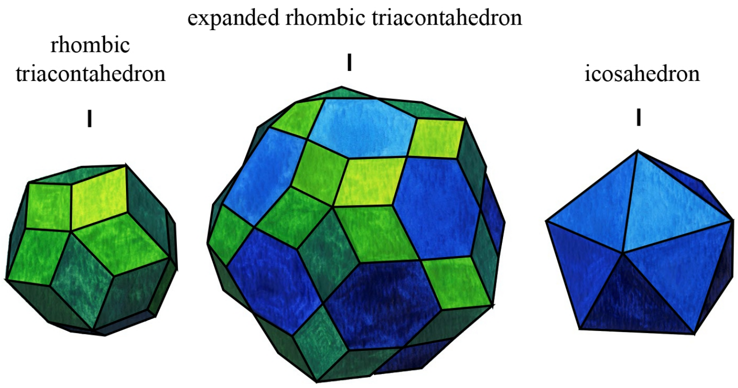

Figure 7: *Expanded rhombic triacontahedron with component polyhedra in watercolor*

Figure 8: E*xpanded rhombic triacontahedron in metal*

Expanded Vertices

To examine this pair of non-regular forms, please look at a vertex of the rhombic dodecahedron in Figure 5, where four rhombi meet. Imagine if all 6 such vertices were pushed out—expanded—yet each vertex retained its 4 diamonds. This would create 8 hexagonal spaces, arranged similarly to the faces of the octahedron. Unlike an ordinary flat polygon that stays within the two dimensional plane, these skew hexagons undulate. (Peter Pearce describes regular skew polygons in his book of polyhedral investigations *Structure in Nature is a Strategy for Design* [3].) Since I had not encountered this form (or the other below it) anywhere else, I added the word 'expanded' to their names to link them with the outward movement of the vertices, before I knew about Alicia Boole Stott's procedure for expansion [4].

I first discovered these shapes with the nodes and struts of Zometool but, as I remade them with different edges lengths in paper and metal, the convexity changed in interesting ways. The rhombi in Figures 6 and 8 are no longer co-planer. Though they are less regular than the Archimedean solids, both expanded forms conform to Euler's formula: faces + vertices = edges + 2. The smaller expanded rhombic dodecahedron (about 22 inches in diameter) has 32 faces, 42 vertices, and 72 edges. The largest metal form in this set, the expanded rhombic triacontahedron, has 80 faces, 102 vertices, and 180 edges, with a diameter of 36 inches.

Project Details

The set of fifteen forms includes familiar polyhedra (the Platonic solids, cuboctahedron and icosidodecahedron, their duals, and the stellated dodecahedron) as well as my experiments (the expanded forms in Figures 6 and 8, Figure 2 (book cover), and two shapes composed of joined tetrahedra: a cuboctahedron and a compound of two tetrahedra). All were made of water jet cut hot rolled steel with mostly 5" edge lengths. Some of the sharp metal edges were softened through machine tumbling, others I did by hand on a wire wheel. I bent the vertices on a brake at the Exploratorium. After bolting, all the forms were powder coated. My interest in these polyhedra continues. I have begun another series of shapes made in this way.

References

[1] Stacy Speyer, *Polyhedra: Eye Candy to Feed the Mind*, Cubes and Things, Alameda, 2016.
[2] Peter R. Cromwell, *Polyhedra,* Cambridge University Press, Cambridge, 1997.
[3] Peter Pearce, *Structure in Nature Is a Strategy for Design,* MIT Press, Cambridge, 1978.
[4] W. W. Ball and H. S. M. Coxeter, *Mathematical Recreations and Essays*, Trinity College, Cambridge, 1987.