Transforming Squares to Strips in Expanded Polyhedral Forms

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Abstract

An interesting connection between two polyhedra construction techniques is presented. The first method replaces edges with squares connected at corners. The second method constructs forms using strips or sticks using a strict over-under-under-over pattern. It is shown how these two techniques can be thought of as two possibilities in a continuum of edge deformations. Example constructions are shown for the five Platonic solids, the cuboctahedron, and the icosidodecahedon. A construction using a serpentine edge is also shown.

Background

Polyhedral forms can be created by replacing the edges in equilateral polyhedra and planar tessellations with flexible squares connected at their corners [2]. The squares can always be connected in such a way that there is an alternating over-under-over-under pattern. Both the vertices and faces are replaced with open spaces in the resulting forms. A rhombic tile was used in a similar construction process [3].

Others have shown polyhedral forms built using a reciprocal building structure that uses sticks or long strips using a strict over-under-under-over pattern, starting from a symmetric pattern on a polyhedral surface [4]. One of the first documented instances of this process related to planar tessellations is due to Leonardo da Vinci in his *Codex Atlanticus* notebook [1]. Applying this reciprocal structure to construct a sphere has been called a rotegrity or a nexorade, especially when applied to a geodesic sphere (n-frequency icosahedron). As with the replacement of edges with squares, these structures result in an open lattice structure.

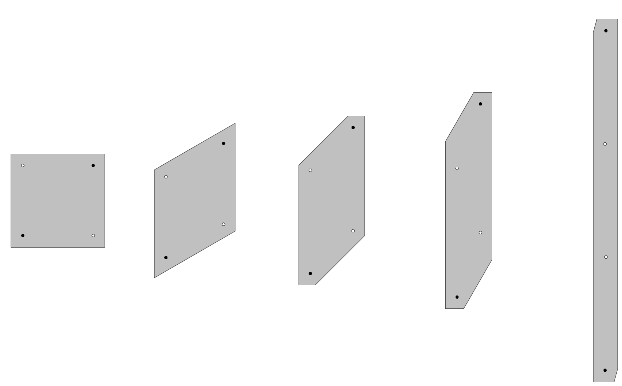

Figure 1: *Continuous deformation of a simple building unit. The square (far left) is sheared ($60°$, $45°$, $30°$, and $15°$ from left to right), then stretched, and then clipped (for small angles). The amount of stretch applied preserved the total area of the building unit. The circles represent connection points, differentiating over and under connections with open and solid circles.*

A Surprising Connection

In both of the construction methods, the building units each have four connection points, with two over and two under connections. A continuous transformation can be applied to transform the square into a strip while preserving the over/under connection patterns as shown in Figure 1. Starting with the square, a shearing and stretching transformation can be applied, resulting in a parallelogram. Clipping is done to avoid an excessively long point.

Results and Discussion

The process for constructing these polyhedral forms can be seen as starting with a polyhedral form, replacing each edge with a square, then transforming it into a strip. Constructions were made using paper-backed wood veneer (bamboo, cherry, walnut, and pine) and heavy paper. These constructions are shown in Figure 2 and Figure 3, where this process is illustrated with polyhedra where the edge lengths are equal.

Because the edge expansion using a square leaves the vertices and faces as open spaces, polyhedra duals have essentially the same form; the preferential bending direction of the wood veneer results in slightly dissimilar dual forms. For elongated building units, a clear difference between the dual forms can be seen. While not shown here, the duals of the cuboctahedron and icosidodecahedron, respectively the rhombic dodecahedron and rhombic triacontahedron, are quite different with each having large rhombic openings.

This process is also not restricted to convex building units. The deformation process can be continued beyond the simple shapes shown in Figure 1. Furthermore, asymmetric building units can be used with the cuboctahedron and icosidodecahedron (along with some of the other Archimedean solids), and still maintain a highly symmetric structure, unlike constructions with the Platonic solids. For example, 60 snake-like units were used to construct the sculpture shown in Figure 4 that is based on the icosidodecahedron. Here the hole patterns of the snake form a parallelogram, while the connections along the snake body are made following an over-under-under-over pattern as in the strip constructions. The sharp polygonal open spaces have been replaced by piecewise curved regions punctuated by snake heads and tails.

References

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[2] David A. Reimann. Nonplanar expansions of polyhedral edges in Platonic and Archimedean solids. In Kelly Delp, Craig S. Kaplan, Douglas McKenna, and Reza Sarhangi, editors, *Bridges Baltimore: Mathematics, Music, Art, Architecture, Culture*, pages 143–150, Bridges Conference, Baltimore, Maryland, 29 July–2 August, 2015.

[3] David A. Reimann. Snub polyhedral forms constructed from flexible 60-120 degree rhombic tiles. In Eve Torrence, Bruce Torrence, Carlo Séquin, Douglas McKenna, Kristóf Fenyvesi, and Reza Sarhangi, editors, *Bridges: Mathematics, Music, Art, Architecture, Culture*, pages 443–444, Bridges Conference, Jyväskylä, Finland, 9–13 August, 2016.

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Figure 2: *Edge expanded polyhedra forms starting with the Platonic solids using the five building units shown in Figure 1. From top to bottom the forms use the following base polyhedra: tetrahedron, cube, octahedron, dodecahedron, and icosahedron.*

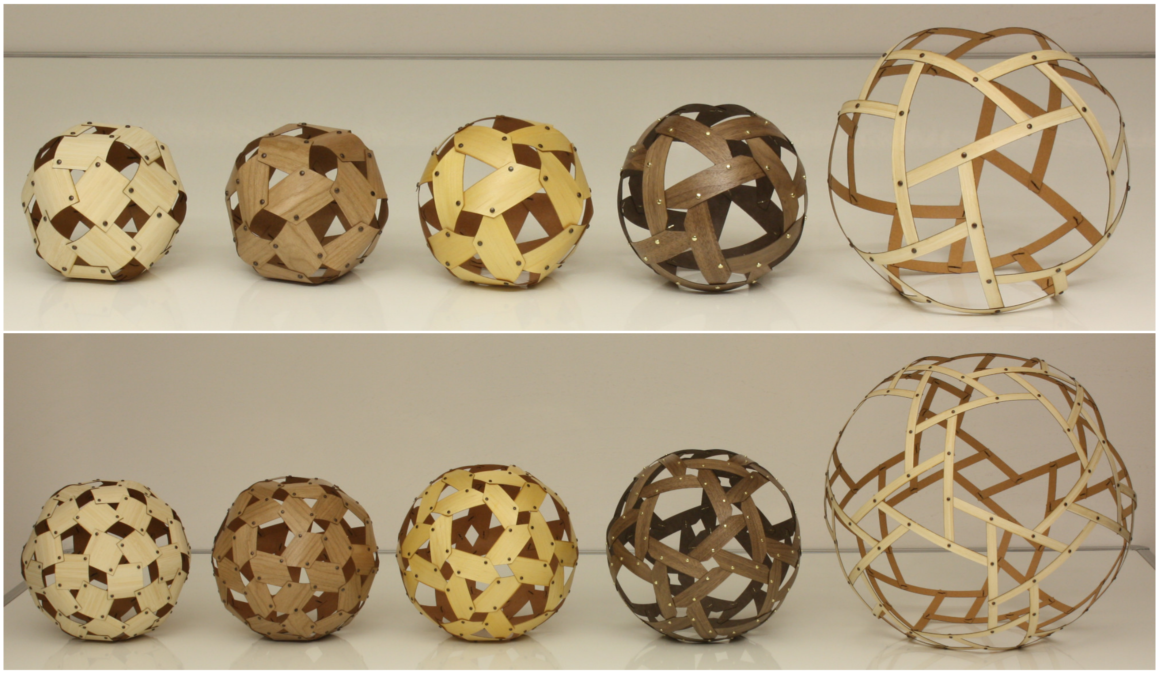

Figure 3 : *Edge expanded polyhedra forms starting with two of the Archimedean solids using the five building units shown in Figure 1. The forms use the cuboctahedron (top) and icosidodecahedron (bottom) as base polyhedra.*

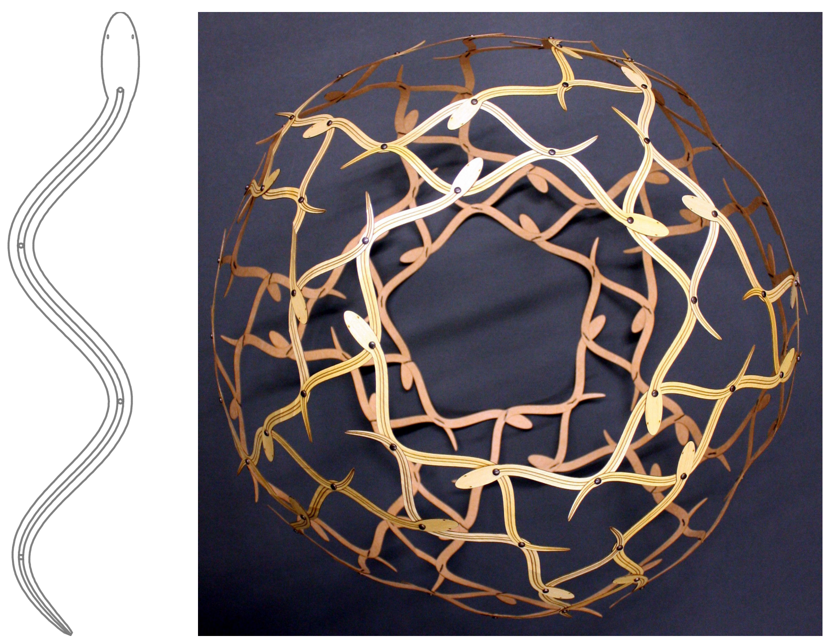

Figure 4 : *Snake edge unit (left) and an edge expanded icosidodecahedron form (right). Edges of the icosidodecahedron were replaced with a snake shaped building unit. The hole pattern forms a parallelogram similar to the walnut edge unit (second from right) as seen in Figure 3. The snake unit is essentially a curved strip, yielding a form similar to the far bottom right form in Figure 3, but using snakes instead of plain rectangular strips.*