Designing Modular Sculpture Systems

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Abstract

We are interested in the sculptural possibilities of closed chains of modular units. One such system is embodied in MathMaker, a set of wooden pieces that can be connected end-to-end to create a fascinating variety of forms. MathMaker is based on a cubic honeycomb. We explore the possibilities of similar systems based on octahedral-tetrahedral, rhombic dodecahedral, and truncated octahedral honeycombs.

Introduction

The MathMaker construction kit consists of wooden parallelepipeds that can be connected end-to-end to make a great variety of sculptural forms [1]. Figure 1 shows on the left an untitled sculpture by Koos Verhoeff based on that system of modular units [2].

MathMaker derives from a cubic honeycomb. Each unit connects the center of one cubic face to the center of an adjacent face (Figure 1, center). Units connect via square planar faces which echo the square faces of cubes in the honeycomb. The square connecting faces can be matched in 4 ways via 90° rotation, so from any position each adjacent cubic face is accessible. A chain of MathMaker units jumps from cube to adjacent cube, adjacent units never traversing the same cube.

A variant of MathMaker called "turned cross mitre" has units whose square cross-sections are rotated 45° about their central axes relative to the standard MathMaker units (Figure 1, right). Although the underlying cubic honeycomb structure is the same, the change in unit yields structures with strikingly different visual impressions. In general, various aspects of the geometry of a structure can be emphasized or deemphasized by judicious choice of unit.

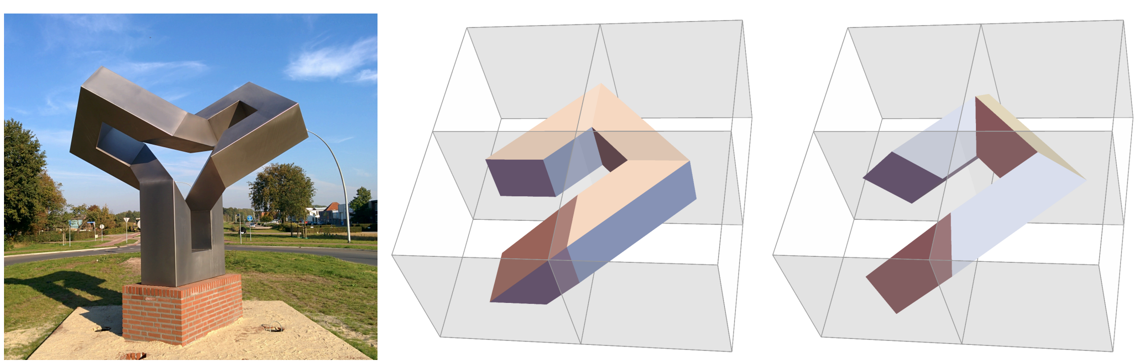

Figure 1: *Sculpture by Koos Verhoeff; MathMaker units in a cubic honeycomb; MathMaker turned cross mitre units in a cubic honeycomb.*

We are interested in analogous systems of modular units that offer similarly interesting sculptural possibilities. Any polyhedral honeycomb can serve as the underlying structure of such a system, with various choices of units giving various sculptural possibilities. We present the results of our investigations of structural systems based on octahedral-tetrahedral, rhombic dodecahedral, and truncated octahedral honeycombs.

To discover new sculptural systems, we begin by describing the adjacent face relationships of a honeycomb as "moves"—geometric transformations that abstract the connecting face relationships in a unit from the particular geometries of the unit. Using a Wolfram Language package, we enumerate closed chains of moves. We arbitrarily choose to enumerate chains which, like the Verhoeff sculpture, have twofold or greater rotational symmetry. By focusing on symmetric chains, we more efficiently generate visually interesting structures. Then we interactively explore realizations of the enumerated chains with units whose connecting faces are scaled and rotated regular polygons derived from the faces of the polyhedra in the honeycomb. The Wolfram Language package code and rotations of figures in this paper can be found at https://wolfr.am/ljb6fFln.

Octahedral-Tetrahedral Honeycomb

The octahedral-tetrahedral honeycomb has two species of polyhedron, hence there are two modular units in a structural system based on it, one that traverses octahedra and one that traverses tetrahedra. Two basic units are shown on the left side of Figure 2; these are simply the convex hulls of pairs of adjacent faces of the polyhedron. The octahedral unit is ¼ of an octahedron, and the tetrahedral unit is an entire tetrahedron.

Alternative units are obtained by taking the convex hulls of the face vertices scaled by s and rotated by r about their centers. If we denote such units by (s, r), the units on the left of Figure 2 are $(1, 0)$ units. Further units $(1/\sqrt{3}, \pi/6)$ and $(1/2, \pi/3)$ are shown in the center and on the right. We found these three sets of units by interactively manipulating the scaling and rotation of units in various structures while keeping an eye out for coincident vertices, parallel edges, coplanar faces, symmetric structures and overall visual impression.

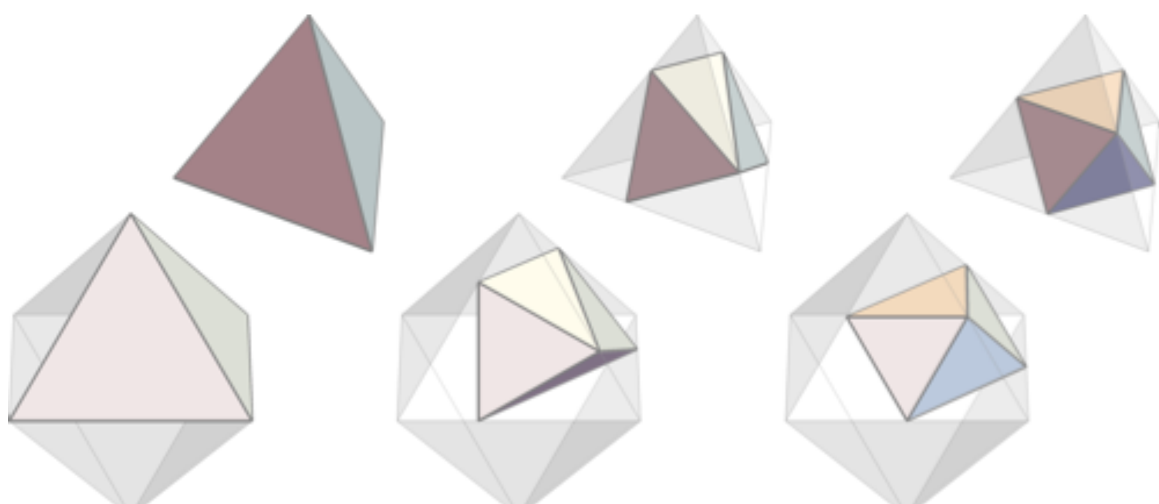

Figure 2: *Octahedral-tetrahedral honeycomb units:* $(1, 0)$ *(left),* $(1/\sqrt{3}, \pi/6)$ *(center), and* $(1/2, \pi/3)$ *(right)*

Figure 3: *Octahedral-tetrahedral structures:* (1, 0) *units,* (1/√3, π/6) *units, and* (1/2, π/3) *units*

(1, 0) units give dense, crystal-like structures (Figure 3, left). The same structure is shown in the center of Figure 3 constructed from (1/√3, π/6) units, which give an asymmetric cross-section and where coplanar faces of adjacent units often merge into larger faces. On the right is the same structure again, this time constructed from (1/2, π/3) units. This structure is reminiscent of Verhoeff's sculpture shown in Figure 1, but cross sections are triangular rather than square, and successive units are related by 120° rather than 90° rotations.

Rhombic Dodecahedral Honeycomb

Units that derive from a rhombic dodecahedral honeycomb are asymmetric and can be attached at either end to give left-bending or right-bending paths (Figure 4, left). The asymmetry arises because adjacent faces are not mirror symmetric about the perpendicular plane through the midpoint of their common edge. A natural alternative to the straight, convex hull units is the solid of revolution obtained by rotating one connecting face to the other about the edge shared by the corresponding faces of the rhombic dodecahedron (Figure 4, center). Figure 4 shows on the right a structure built from 18 such (.385, π/2) arc units, the scale factor chosen so that edges of the structure just graze each other. The π/2 rotation gives a unit with an asymmetric cross section and two planar faces, which makes for simple, bold structures.

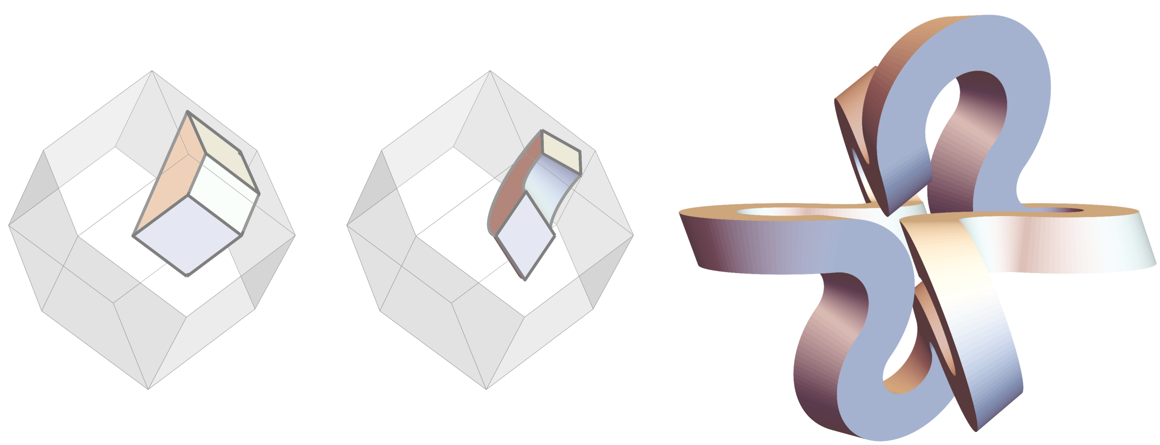

Figure 4: *Rhombic dodecahedral honeycomb* (1/2, 0) *unit; rhombic dodecahedral honeycomb* (.385, π/2) *arc unit; structure constructed from eighteen* (.385, π/2) *arc units*

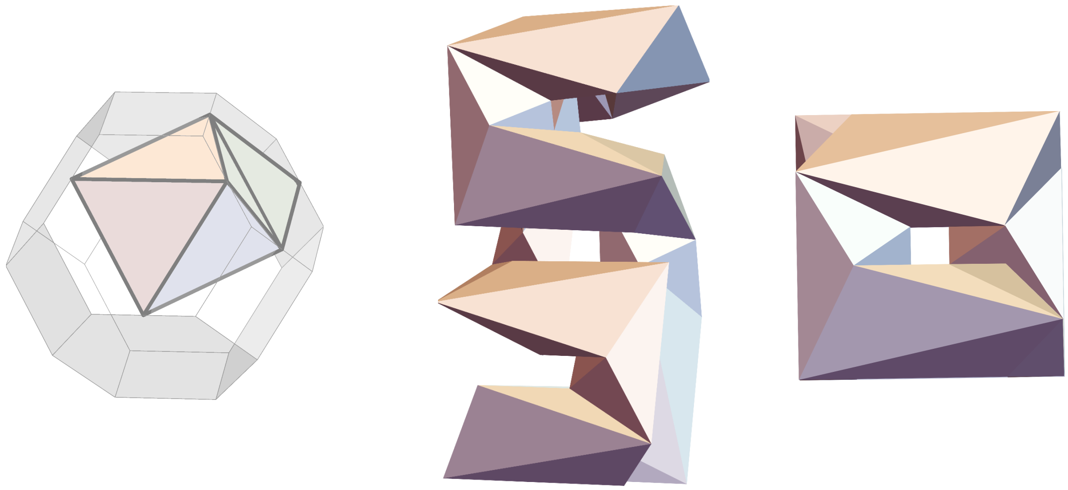

Figure 5: *Truncated octahedral honeycomb* ($\sqrt{3}/2$, $\pi/6$) *unit; structure constructed from sixteen* ($\sqrt{3}/2$, $\pi/6$) *units; end-on view of the same structure showing its square envelope.*

Truncated Octahedral Honeycomb

In a honeycomb of truncated octahedra, we traverse from hexagonal face to adjacent hexagonal face. We have chosen to make connecting faces triangles rather than hexagons, skipping every other vertex, since each hexagonal face has only three adjacent hexagonal faces. Due to the orientations of those faces in adjacent polyhedra, taking the convex hull of the connecting faces of units necessarily yields an irregular antiprism rather than a prism (Figure 5, left). The abundant facets of such a unit give visually complex structures that at higher unit counts are overwhelmingly busy. At lower unit counts, structures can be visually compelling, with interesting geometric relationships in the apparent disorder. The structure shown in Figure 5, center, is constructed from sixteen ($\sqrt{3}/2$, $\pi/6$) units. The end-on view on the right of Figure 5 reveals order in the apparent chaos: at $\pi/6$ rotation, the units neatly fit within a square envelope.

Conclusion

Our method of discovery has led us to several sets of modular units from which a large variety of interesting structures can be built. The combination of enumeration with interactive exploration has allowed us to efficiently explore how a given unit geometry behaves as a sculptural system, and conversely, how a given structure is articulated by various choices of unit. Our results hint that there are likely additional compelling modular sculpture systems to be discovered in the many honeycombs that we have not yet explored.

References

[1] MathMaker construction kit, https://www.mathmaker.nl/

[2] Untitled Koos Verhoeff sculpture, http://wiskunst.dse.nl/agenda.html. Photo courtesy of Tom Verhoeff

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