Fostering Creativity in the Teaching of Mathematics with Project Based Learning

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Abstract

The subjects of mathematics present many times an overload of theoretical knowledge and an unreasonable stiffness in operative processes. In this paper we propose promoting autonomous student learning in order to enhance other capabilities. Project Based Learning (PBL) is intended to make use of a number of skills that students own (autonomy, intuition, creativity, etc.) but are usually masked in the teaching and learning process. An example of this type of approach has been featured by architecture students at the University of the Basque Country in Donostia-San Sebastián, Spain. Their task was to develop a project in which their mathematical knowledge should be extended to other areas of their degree; specifically, the use of a Voronoi Diagram in the field of art and architecture.

Introduction

Currently, Mathematics subjects maintain an eminently theoretical teaching and, in many cases, away from the reality in which students must cope to access the labor market. Practical exercises do not require the students to encourage certain skills that could be useful in their future career. In fact, students often does not reason themselves but try to match the exercise they are faced with a "standard problem" whose resolution process is well known for them.

The authors believe that the learner should consider challenges that require the development of those skills which possess but usually is not asked to show. We refer to the confrontation with problems that may arise in working life and not those expressly prepared by the teacher to qualify. Experience shows that many educators are reluctant to make such experiences since they involve updating their teaching methods and leave the comfort of working with known exercises which do not support different interpretations apart from the standard.

As teachers of the Department of Applied Mathematics at the University of the Basque Country we believe that mathematics should be applied. We realize that students should be able to confront a real open project and develop a range of useful skills facing the professional future: understanding and intuition, selective and efficient search for information, ability to obtain appropriate solutions, creativity, self-reliance and self-criticism.

Therefore, we pursue that the students improve these skills meanwhile the teacher gets involved in the process and evaluates it as part of the total course grade. The uncertainties that may arise must be overcome: the ones of the student before a project to be carried out by its own means and the ones from the educator facing different solutions unknown in advance, but arising from the creative process of another person.

Project Based Learning (PBL)

To encourage the development of the aforementioned skills it is necessary to complement traditional teaching of mathematics with projects of short duration. Classical education is indispensable to convey the basics of the subject but it is essential that students learn how to apply these theoretical knowledge in a real world oriented work.

Project Based Learning (PBL) is an active methodology that allows students to acquire the knowledge and specific skills and the cross competencies of the subject from a self-taught viewpoint. Educators need to capture the attention of students by presenting them an original and attractive environment to apply the acquired theoretical knowledge. This new workspace should be different from the ordinary and based on real cases with immediate practical application.

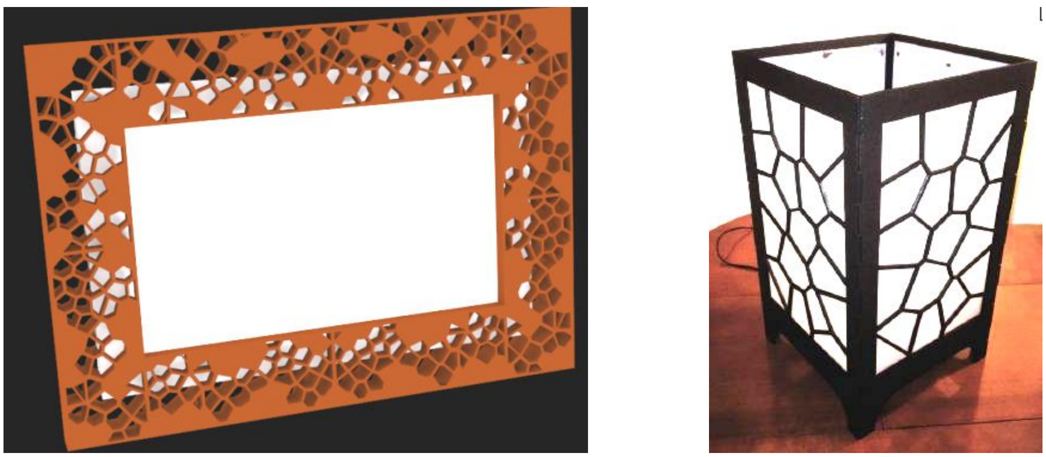

Figure 1: *Two projects made by students applying Voronoi Diagrams: photo frame (left) and umbrella stand (right)*

With these projects, pupils become protagonists in their own learning as they must plan, organize and carry out the tasks to complete successfully their assigned work. They must also be creative to develop an appropriate resolution process and critical to evaluate it.

The experience in the School of Architecture of Donostia-San Sebastián

Project Based Learning was introduced several years ago in the E.T.S. Architecture of the University of the Basque Country through individual open jobs of short duration. Architecture combines, as no other college career, technical knowledge with other ones in which prevail sensitivity with a taste for aesthetics and proportion. So, we will try to introduce and develop that beautiful relationship between art and science in our projects.

A primary goal of PBL is to propose works that appeal to all students. Another objective is to get the students perceive that mathematics may be applied in other areas of both scientific and artistic scope. Interdisciplinarity constitutes an added value for these projects in which an abstract mathematical concept is related with everyday objects such as a picture frame or an umbrella stand (Figure 1).

In short, an open project that appeals to students should make them get interested in the matter. That way, students demonstrate their knowledges and skills but also mold their personal stamp (self-reliance and self-criticism) and explore different ways of resolution without teacher interference (creativity and imagination).

The selected project: Voronoi Diagram

This diagram is named after Russian mathematician Georgy Voronoi Feodósievich (1868-1908) but it is also known as Dirichlet tessellation (Johann Dirichlet 1805-1859) or Thiessen polygons (Alfred H. Thisessen, 1872-1956). The Voronoi diagram is a partition of the plane into convex polygons such that each polygon contains one generating point and every point inside the polygon is closer to it than to any other generating point in the plane (Figure 2).

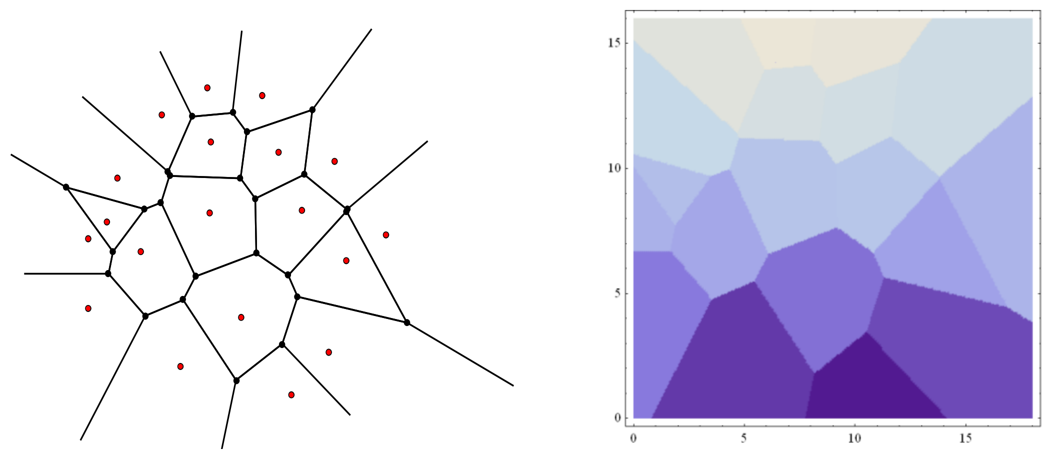

Figure 2: *Plane tessellation showing a Voronoi diagram starting from a random sequence of generation points.*

Figure 3 shows an example that captures the essence of what was sought when we proposed the project to our pupils. The generator for the points were the endpoint of the flames from a set of candles, the Voronoi tessellation was made with GeoGebra software and the final product was a lamp built from the Voronoi polygons obtained.

Figure 3: *Lamp designed and constructed starting from a Voronoi diagram.*

In this paper we want to describe one of the PBL experiences held as part of our mathematic courses. The first step is to explain some basic and general notions of the subject, the Voronoi diagrams. Students were instructed to devise one original application based on a Voronoi diagram. Although they had complete freedom, the possibility of tutorial consultation was provided during the semester to guide them in their creative and productive process. Tutorial action is important in this type of learning but always from the background to interfere the least in the creative process of the students.

One of the requests for the project was the election of a singular and imaginative sequence of generation points. After that, students could perform tessellation either by hand or with any computer tools they deemed pertinent to develop their work. Citation of the sources was required in the case they were inspired by an idea from other author.

Are our students creative?

Given the freedom provided for the work and the very few restrictions for its execution, the quality and diversity of ideas developed has been outstanding. The results shows a group of students more heterogeneous and autonomous than when they work with ordinary academic tasks. Although the level of commitment varies depending on the person, in general the result has been highly satisfactory.

We should highlight the wide variety of fields of application and the originality of many of the proposals: art (mosaic, painting, sculpture, tiling), furnishings (lamp, shelve, umbrella stand, wall clock), jewelry (pendants, rings and jewelers), architecture (facades, roofs and territorial ordination), cartography (metro stations, ski slopes, airspace management) and design (mobile case, vase, photo case).

With respect to the final production, the number of handmade projects was similar to the computer made ones. In many cases different techniques were combined, mixing handmade drawings with computer graphics or manufactured procedures with digital fabrication in 2D or 3D. And answering the initial question of this section, YES, our students are highly creative. As a sample we have selected a number of remarkable designs that stand out for its originality.

Figure 4: *Different stages in the making of a case for mobile phone based on a Voronoi diagram.*

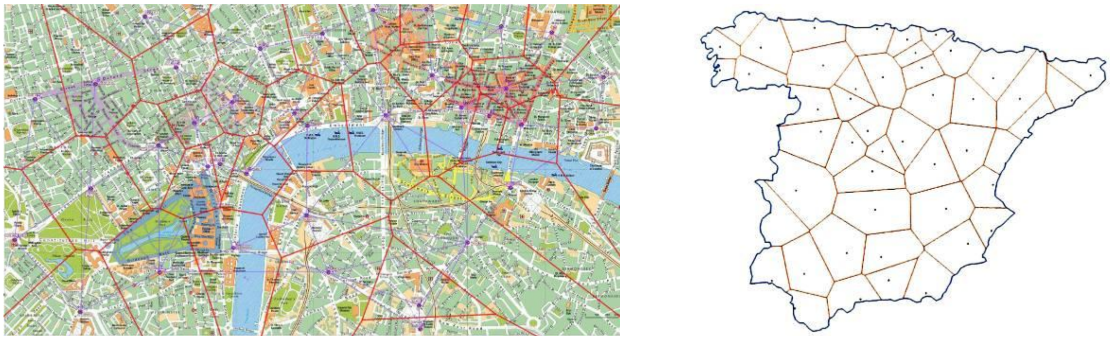

Figure 5: *Superimposed on the map of the city of London the Voronoi diagram shows the subway station nearest to any location (left). A new territorial arrangement of Spain is displayed in terms of proximity to the province capital cities (right).*

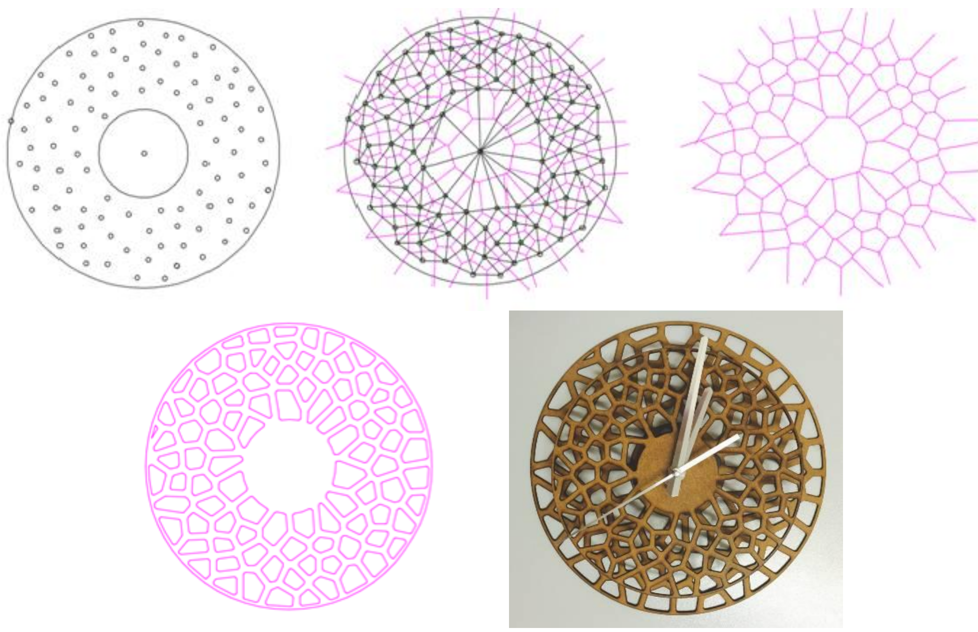

Figure 6: *Design process and final construction of a wall clock; (a) generation of points inside an annular region, (b) drawing of the segments connecting points and their perpendicular bisectors, (c) Voronoi diagram, (d) final template for the construction, (e) real working model hanging on a wall.*

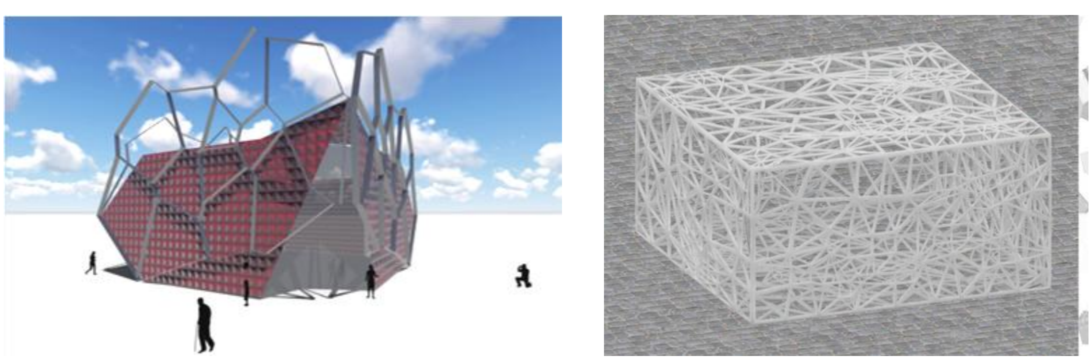

Figure 7: *Examples of application in architecture; in this case, students present two different Voronoi enclosures for exhibition pavilions.*

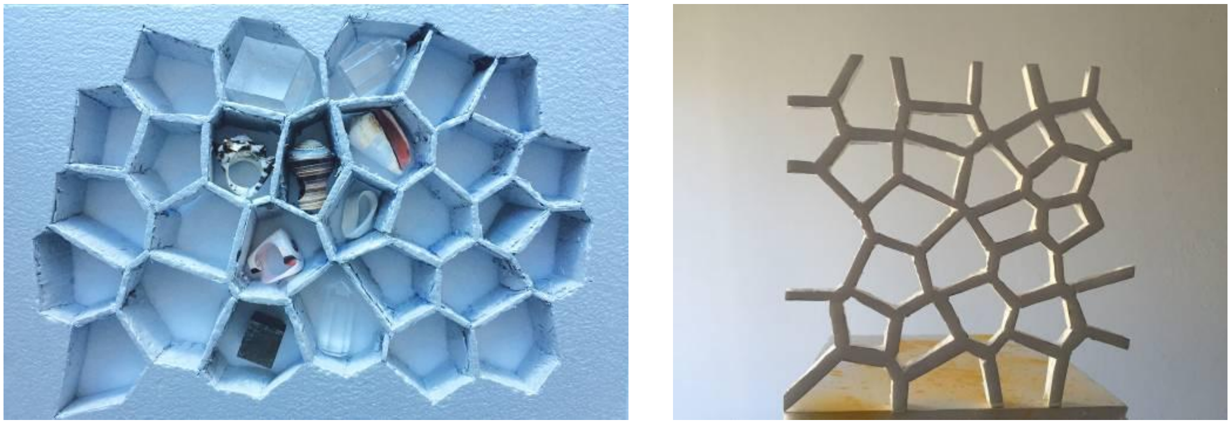

Figure 8: *Shelf for the display of small objects (left). Scale model of a Voronoi lattice wall (right).*

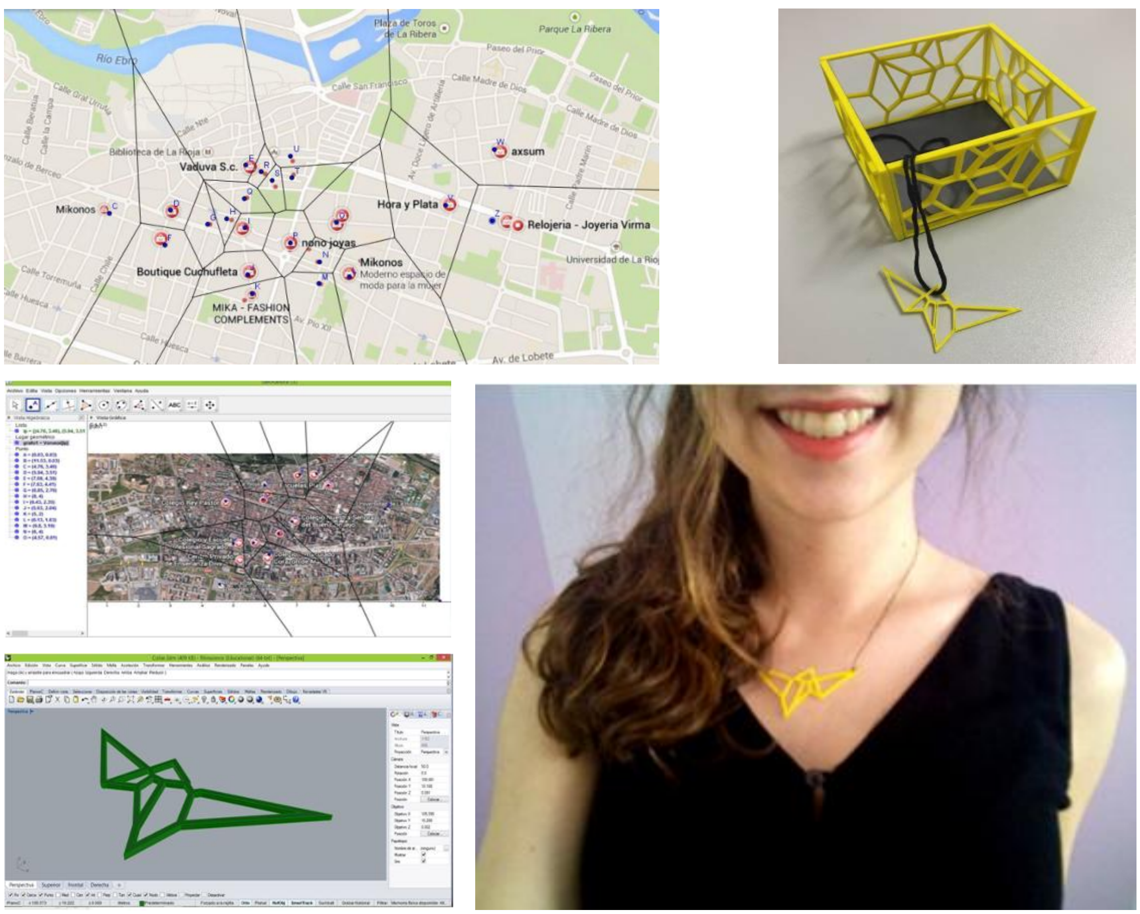

Figure 9: *This curious project of a necklace and a jewel box bases its generation points in the geographical coordinates of the jewelries from the student hometown. GeoGebra software was used to generate the associated Voronoi diagram. The resulting image was exported to Rhinoceros software to design the jewel box and the necklace. Both items were finished using 3D printing technology.*

Conclusions

After reviewing and evaluating the results obtained with this teaching experience, Project Based Learning has proven to be a suitable tool to stimulate the creative work of students from an initial mathematical concept, in our case, Voronoi diagrams. Other similar PBL have been proposed in the E.T.S. of Architecture of Donostia-San Sebastián (anamorphosis, polyhedra, mathematical sculpture, tessellations, sliceforms, op-art design, parametrization of singular buildings...) obtaining similar results to the Voronoi experience.

The objective for the students is not to generate Voronoi diagrams but to apply them in an original, creative and stimulating way. Therefore, the tutorials are minimal to avoid conditioning their cognitive process. Once the project is clearly defined almost no tutorial aid is required, except to verify that the initial idea chosen by the student is acceptable and to approve the necessary steps until its complexion.

It should be noted the high degree of interdisciplinary reached in the works. Pupils have shown great capacity to implement the initial mathematical idea into different knowledge areas. Students have achieved to establish links between art and mathematics -in their broadest sense- a capacity that could be helpful to them for their future careers as architects.

The execution of the final work was materialized in both virtual and physical models. Some jobs were assembled with craftsmanship techniques while others required different CAD and mathematical software combined with digital manufacturing. The use of mathematic in the different stages of the process provides a significant learning and the acquisition of skills to relate different areas of knowledge.

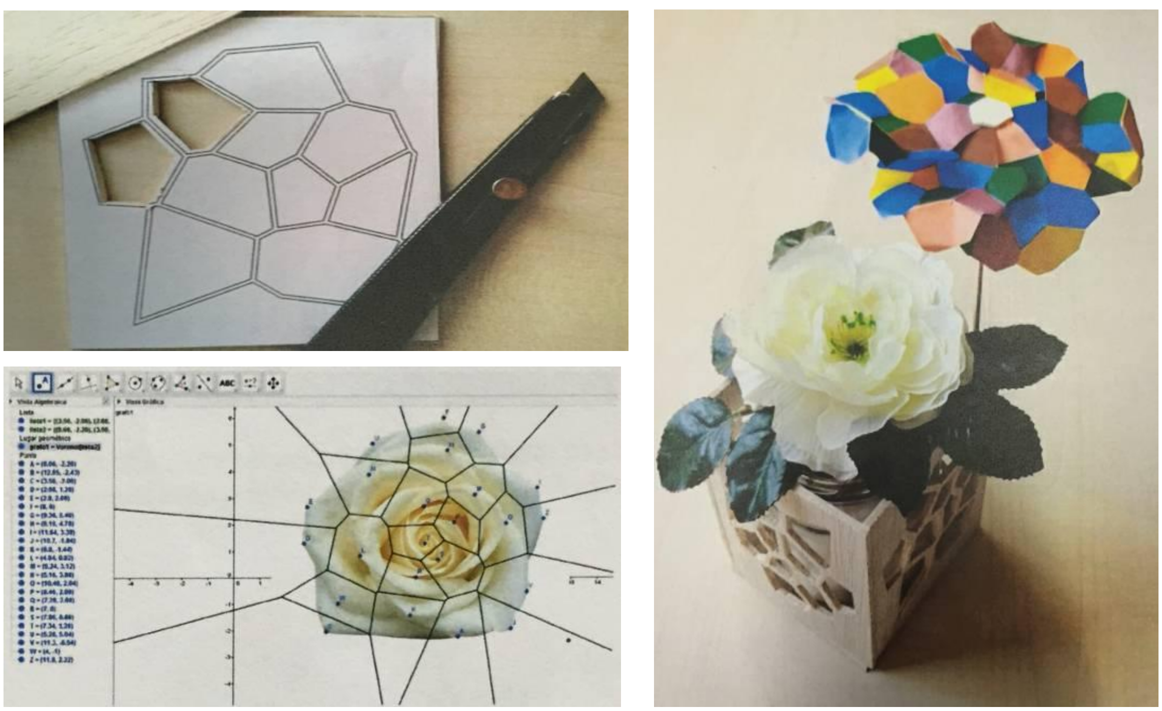

Figure 10: *Students have been able to perceive the essence of the project and the right combination of mathematics and art using both technological processes and manufacturing craft.*

Acknowledgements

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