# Spelunking Adventure VI: An Equal Tempered Icosahedral Scale

## Using a musical metaphor to construct and navigate icosahedral domains

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## Abstract

With 3D Computer Aided Design I have divided 12 unit radius icosahedral polyhedra into sets of 'symmetry units' represented in software as *blocks* containing graphic elements. These 12 polyhedral systems' radii are assigned a length to match a musical note's position in a chromatic scale. Combining these polyhedral system *notes* creates and names a *chord* consisting of concentric polyhedra (matroyska domes). To paint these objects I use a recursive self texturing technique. With 3D path and turntable animation tools I create movies: visual songs or intonations that document what a virtual camera sees as it transits "In, Out and Around" this menagerie of painted forms.

## Introduction

A long journey with 3D modelling software seems at last to be bearing artistic fruit in the form of dynamic colourful animations. My set of polyhedral drawings are, like an author's poems, continually reworked. Recently I modified my database of polyhedra by adding sets of paintable surfaces and vectors produced by symmetrical cutting operations that use the center angles in Figure 1. This redraw added the *internal to circumsphere* structures underlying these polyhedral forms.

| Nodes | Degrees | φ Relation |
|---|---|---|
| DOE | 20.905 | arctan ($\varphi^{-2}$) |
| BOE | 31.717 | arctan ($1/\varphi$) |
| BOD | 37.377 | arctan ($2*\varphi^{-2}$) |

**Figure 1:** *Icosahedral center angles & the golden ratio* $\varphi=(1+5^{1/2})/2$

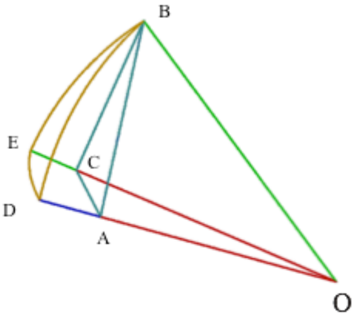

**Figure 2:** *Simplex of an Icosahedron*

| | |
|---|---|
| Altitude | OCA |
| Base | OBC |
| Hypotenuse | OAB |
| Screen<br>1/6 of an icosa face | ABC |
| Spherical Lens | BED |
| Spherical excess 1 | ACED |
| Spherical excess 2 | BCE |
| Spherical excess 3 | ABD |

**Figure 3:** *Named Layers & Surfaces in Figure 2*

Using these central angles, I defined simplexes, or orthoschemes [4], from 1/120th of the interior of each of the 12 icosahedral systems I use. See the icosahedron's simplex in Figure 2. These simplexes are encoded in block data structures that serve to minimize file size and processing loads by maximizing that which is repeatable in a drawing. Blocks are the ideal data structure with which to containerize these simplexes. A block is copied with a sequence of mirror and rotate commands to generate the polyhedron and all of its interior and exterior radii and truncation planes within its circumsphere. These 120 blocks form an icosahedral *system*.

Each block is tagged with a *transformation matrix* that informs software of its position and orientation in space. For these 12 polyhedra every one of the 1,440 blocks that define this data set has a transformation matrix that consists of numbers from the set: {0, ± {1, sin(54), sin(30) and sin(18)}}.

Any 3D modelled object could be used as a block and transformed by these matrices to create a grotesque icosahedron. Viruses do it with proteins! I keep it simple and restrict my canvas to virtual surfaces belonging to 12 icosahedral systems selected from the platonic, archimedean and catalan solids.

## Painting my Icosahedral Systems

Using blocks also simplifies how surfaces are painted. Within a block, surfaces are defined on named layers, Figure 3. Each layer is assigned a material. To create materials I began with my vector graphics derived from icosahedral solids' 2D shadows that form a unique set of spline curves I named *cyclons* [2], Figure 4, left. For this project I reformatted these 2D vector graphics into bitmaps that parameterize surface colour and transparency, Figure 4 center and right.

**Figure 4:** *Icosahedron's vertexial cyclon and texture map components: colour and transparency*

I defined three materials for each polyhedral system from my set of cyclons, one material for each symmetry group's axis, axes formed by the origin and: 1. the mid-edge tangent to the midsphere; 2. the mid-face tangent to the insphere; and 3. the mid-vertex tangent to the circumsphere. These 36 materials became the palette from which thousands of surfaces are painted when selected for display.

These materials were derived from the polyhedron that they subsequently paint. A recursive self texturing.

## Mapping Icosahedral Symmetry onto a Chromatic Scale

With the icosahedral system assemblies complete I sought to combine them in new ways. In my previous work I combined icosahedral systems bound by a common circumsphere [2] and constrained by a natural spatial scale [3] based on Hesse coordinates [1]. Within this project's manufactured chromatic domain each note has its own circumsphere, its own *personal space,* radially relative to its neighbour. These personal spaces which I call Notes, combine synergistically as chords to create unique colour fields that are derivative of both size and shape, Figures 5-8.

With chords constructed I then used simple camera animation techniques to further the experience of these spatial relations.

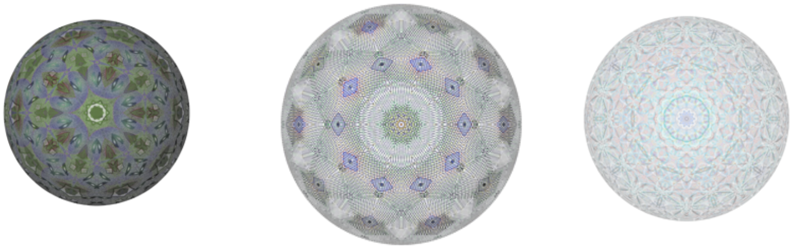

**Figure 5:** *D# Minor Notes: (root) D#, A#, F*

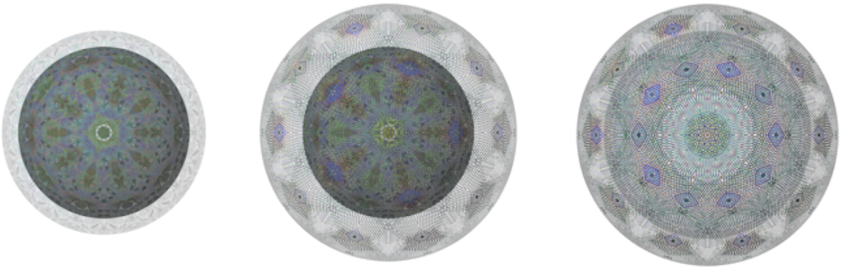

**Figure 6:** *D# Minor dyads*

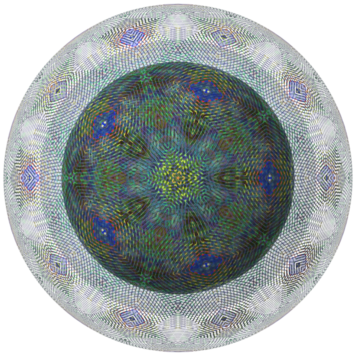

**Figure 7:** *D# Minor Triad*

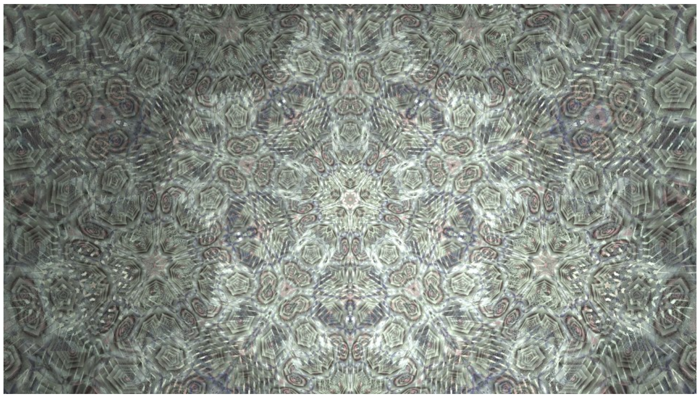

**Figure 8:** *D# Minor inside*

## Animations

When a camera transits these domains, the surfaces within the chord's colour field are seen with relative motions, informing our sense of space-time. During this work it became clear that R. Buckminster Fuller's iconoclastic take on 4D, his notions of dimension expressed as "In Out Around" had rubbed off on me. I use this muse to inspire my animations, my intonations.

To date my intonations have used linear axial paths (in/out) and conic/turntable animation techniques (around) to document my chords. I chose two animation run times of 36 and 72 seconds using 2160 cels at 60 & and 30 frames per second respectfully. In the music metaphor these durations set tempo *grave* by defining the duration of half notes and whole notes.

Depending on rendering method and scene complexity, 2160 cels will render in a few minutes for a wire frame drawing or in as much as 60 days for a full render. Not wanting to delay gratification that long, I standardized on animations in 'Render Viewport' mode when I found I could trick my system to produce detailed colourful animations that render at 1080p resolution in an hour. However, they lack full render effects like shadow and index of refraction.

Simple post processing: packaging the frames as a movie, clearing the cache of frames and setting up the next chord takes about an hour. It takes two hours to capture a 72 second visual experience, an intonation. Grave indeed.

## In Out and Around: A Conclusion

It took me a year and hundreds of animations to get a clean database. Developing logical layer names and locations of accessory files was critical. I have built a database of 12,960 surfaces spread across 12 notes. They share center angles and are easily combined concentrically to produce a vast array of 3D models. These models, or chords, have symmetries pleasant to the eye when painted and put in motion.

Recently a clean database made it possible to record the F# major scale: 7 notes*,* 7 chords*,* 7 intonations (504 seconds) in 14 hours. Imbuing these surfaces with transparency and colour and impelling a virtual camera through these spaces captures them in a way that can be shared on all the screens that bombard modern retinas. I would like to capture this process as an algorithm in order to reduce production time. With real time production these domains can be explored by virtual reality enthusiasts.

Animations of the F# major key signature are provided as an accessory file with this paper. I hope they provide a more serene and contemplative visualization than that typically found on television.

Mapping these polyhedral systems onto the chromatic scale does not infer or imply meaning to the shapes. However, by using established conventions for chord construction the intonations acquire names! This permits their collection into playlists and opens the door to scoring a visual symphony or transcribing existing opi. Of course, there is much more to do to create a functioning visual analog of music. For now, I am satisfied with the metaphor as it allows me to continually rework the contents of the frame.

To date I have four key signatures documented by these animation techniques. I have aspirations of scripting future intonations on a faster machine or in the cloud. But why do all this work? What's the point?

## The Point

Perhaps the point is to draw attention to an alternate pedagogy of space based on center angles. See Synergetics section 1050.20 about trigonometry [7].

## References

[1] H. S. M. Coxeter, *Regular Polytopes*, Dover Publications Ltd., pp. 50-54. 1973.

[2] C. L. Palmer, *Digitally Spelunking the Spline Mine*, Bridges Banff, Conference Proceedings, pp. 309-313. 2005.

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