Repeating Fractal Patterns with 4-Fold Symmetry

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Abstract

Previously we described an algorithm that can fill a region with an infinite sequence of randomly placed and progressively smaller shapes, producing a fractal pattern. In this paper we extend this algorithm to fill a fundamental region for the “wallpaper” group $p4$, then we tile the plane with copies of that region. This produces artistic patterns which have a pleasing combination of local randomness and global symmetry.

Introduction

In the past we have created pleasing patterns with an algorithm [1] that can fill a planar region with a series of ever smaller randomly-placed motifs. In this paper we extend that algorithm to create wallpaper patterns with $p4$ symmetry (we previously treated $p4mm$ patterns [2]). Schattschneider [3] gives a nice overview of wallpaper groups. Figure 1 shows such a random pattern of circles with $p4$ symmetry. To create our

![Figure 1: A locally random circle fractal with global $p4$ symmetry.](image-url)

patterns, we fill a fundamental region for $p4$ with randomly placed, progressively smaller copies of a motif,
possibly with different colors, such as the circles of Figure 1. This randomness generates a fractal pattern. Then copies of the filled fundamental region are used to tile the plane, yielding a locally fractal, but globally symmetric pattern. Here math provides the algorithm and art provides the coloring. In the next section we explain how the \( p4 \) algorithm works. Finally, we indicate directions of future work.

The Algorithm

The idea of the algorithm (below) is to randomly place progressively smaller motifs \( m_i \) within a region \( R \) so that they do not overlap any previously placed motif [1]. Here we show the original algorithm in normal type face with the modifications in bold that are needed to produce a \( p4 \) pattern:

For each \( i = 0, 1, 2, \ldots \)

Repeat:

Randomly choose a point within fundamental region \( R \) for \( p4 \) to place \( m_i \)

If \( m_i \) has 4-fold symmetry and overlaps a 4-fold rotation point

Move \( m_i \) to be centered on that 4-fold rotation point

If \( m_i \) has at least 2-fold symmetry and overlaps a 2-fold rotation point

Move \( m_i \) to be centered on that 2-fold rotation point

Until \( (m_i \) doesn’t intersect any of \( m_0, m_1, \ldots, m_{i-1} \))

Add \( m_i \) to the list of successful placements

Until some stopping condition is met, such as a maximum value of \( i \) or a minimum value of \( A_i \).

It has been found experimentally by the second author that this algorithm is non-halting if the areas of the \( m_i \)'s obey an inverse power law [1] (we note that exponentially decreasing areas cause halting).

Summary and Future Work

We have presented a method for creating patterns that generate global wallpaper patterns with \( p4 \) symmetry but are locally fractal in nature. Our goal is to make pleasing patterns with this kind of global symmetry but to also maintain local randomness. The methods presented here and in [2] should also work for other wallpaper groups. It would also be interesting to create corresponding spherical or hyperbolic patterns that are locally random, but have global symmetries.

References

