The Golden Ratio and the Diagonal of the Square

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Abstract

An elegant geometric 4-step construction of the Golden Ratio from the diagonals of the square has inspired the pattern for an artwork applying a general property of nested rotated squares to the Golden Ratio.

A 4-step Construction of the Golden Ratio from the Diagonals of the Square

For convenience, we work with the reciprocal of the Golden Ratio that we define as: $\varphi = \sqrt{(5/4)} - (1/2)$. Let ABCD be a unit square, O being the intersection of its diagonals. We obtain O' by symmetry, reflecting O on the line segment CD. Let C' be the point on BD such that |C'D| = |CD|. We now consider the circle centred at O' and having radius |C'O'|. Let C'' denote the intersection of this circle with the line segment AD. We claim that C'' cuts AD in the Golden Ratio.

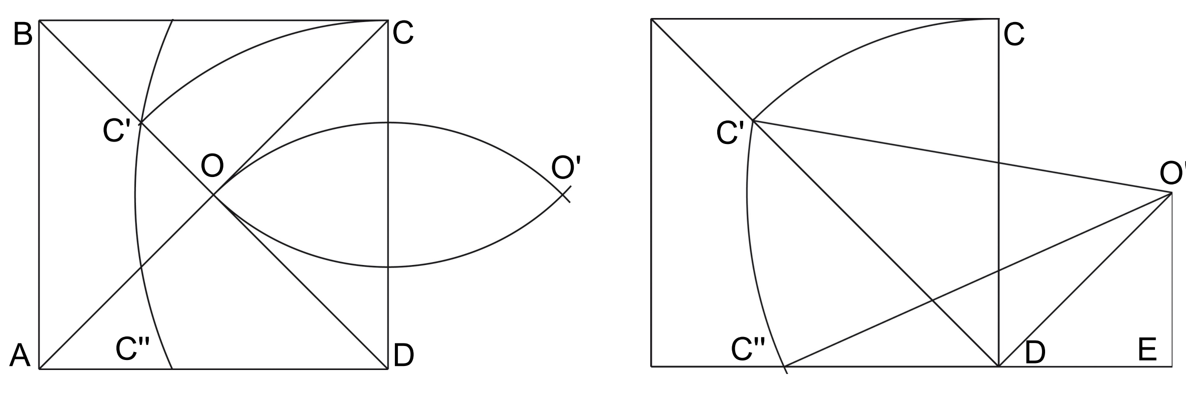

Figure 1: *Construction of φ from the diagonals of the square and demonstration.*

Demonstration

In Figure 1 since |CD| = 1, we have |C'D| = 1 and $|O'D| = \sqrt{(1/2)}$. By the Pythagorean Theorem: $|C'O'| = \sqrt{(3/2)} = |C''O'|$, and |O'E| = 1/2 = |ED|, so that $|DC''| = \sqrt{(5/4)} - (1/2) = \varphi$.

Golden Ratio Pattern from the Diagonals of Nested Squares

The construction of the Golden Ratio from the diagonal of the square has inspired the research of a pattern of squares where the Golden Ratio is generated only by the diagonals. In Figure 2 we can visually demonstrate, by the Intercept Theorem, that the sides of a smaller square nested in the points of Golden Ratio of the sides of a bigger square, are cut by the diagonals in the Golden Ratio. This general property of rotated nested squares can be generalized with any ratio, but the application to the

Golden Ratio also generates a fact of intrinsic mathematical beauty: as demonstrable by the Pythagorean Theorem, starting from a unit square, the side of the third nested square is equal to the area of the second one, and so on with smaller successive squares.

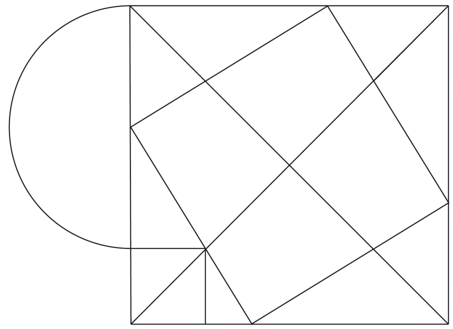

Figure 2: *Insertion of the smaller square in Golden Ratio points and visual demonstration by the Intercept Theorem.*

Project for an Artwork

We reflect symmetrically the pattern of nested squares in Golden Ratio and describe an image that displays a "perspective fugue" of pentagons. Despite their irregularity, these pentagons maintain some properties of regular ones, as the reciprocal section of diagonals in Golden Ratio. Figure 3 represents the synthetic study for a work in a series of "red and green" paintings. For the use of red and green high contrast colours we refer to our work published in Bridges 2016 Mathematical Art Gallery [1]. A pattern of this kind is also a perfect subject for the making of a floor with the antique art of pebble mosaics, that we professionally practice.

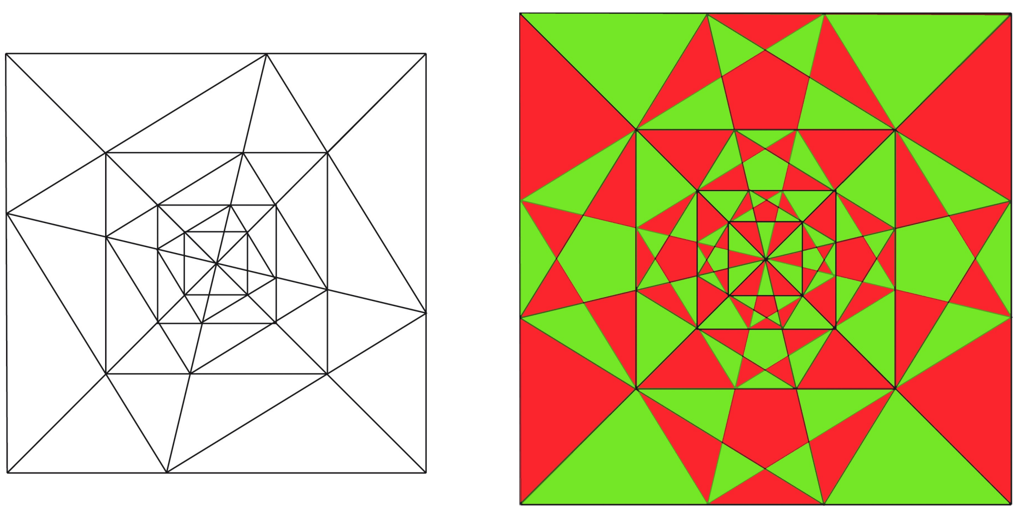

Figure 3: *Golden Ratio pattern of nested squares and the model for an artwork.*

References

[1] G.Gelatti, *The most beautiful triangle,* http://gallery.bridgesmathart.org/exhibitions/2016-Bridges- Conference