

Snub Polyhedral Forms Constructed from Flexible 60-120 Degree Rhombic Tiles

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Abstract

We construct snub polyhedral forms using rhombi made from paper-backed wood veneer which are connected pairwise at corners using split-pin fasteners. Combining pairs of adjacent triangular faces into roughly 60° - 120° rhombi result in polyhedron with even degree vertices. The flexibility of the materials allows the final closing of the object into a roughly spherical shape, despite eliminating the dihedral angle between the adjacent paired triangles. The open lattice construction yields elegant chiral symmetric forms with visual movement.

Introduction

Platonic solids are some of the most basic bounded three-dimensional objects, having equal regular (equilateral and equiangular) polygon faces with vertices all lying on a sphere. There are five such solids: the tetrahedron, cube, octahedron, dodecahedron, and icosahedron. The Archimedean solids are polyhedra in which the faces are regular polygons and the vertices are surrounded by the same number and type of regular polygons [1]. There are thirteen such solids. Two of these are chiral (having no mirror symmetry), namely the snub cube and the snub dodecahedron. Placing equilateral triangles around the faces of a cube and dodecahedron results in the snub version of each. There are many other nearly uniform polyhedral forms, such as near-miss Johnson solids, that exhibit a high degree of symmetry [3].

I previously described constructions where the edges of Platonic and Archimedean solids were replaced by squares [2]. These objects map both the faces and vertices of a base polyhedron to openings, resulting in open lattice forms that are roughly spherical. Outward bumps or inward craters accommodate the additional material. I constructed physical models using squares made from paper and paper-backed wood veneer which were connected at corners using split-pin fasteners. The flexibility of the materials allows the final closing of the object into a roughly spherical shape. However, in the square expanded snub Archimedean solids, the bumps and craters result in a puckered surface that does not closely resemble a sphere.

Rhombic Constructions

A replacement process can be used to construct polyhedral forms using pairwise corner connected polygons. However, this only works when the vertex degree is even. The octahedron is the only Platonic solid with even degree vertices. The cuboctahedron, small rhombicuboctahedron, icosidodecahedron, and small rhombicosidodecahedron are the only Archimedean solids with vertices having even degree. However, combining pairs of adjacent triangular faces into roughly 60° - 120° rhombi in the snub cube and the snub dodecahedron result in polyhedra with even degree vertices. Similarly, the snub rectified truncated icosahedron (a near-miss Johnson solid) can be transformed into a polyhedron with even degree vertices. Physical models of these three polyhedra were constructed using rhombi with side length 5.5 cm made from paper-backed cherry wood veneer; these rhombi are connected at their corners using split-pin fasteners as shown in Figure 1.

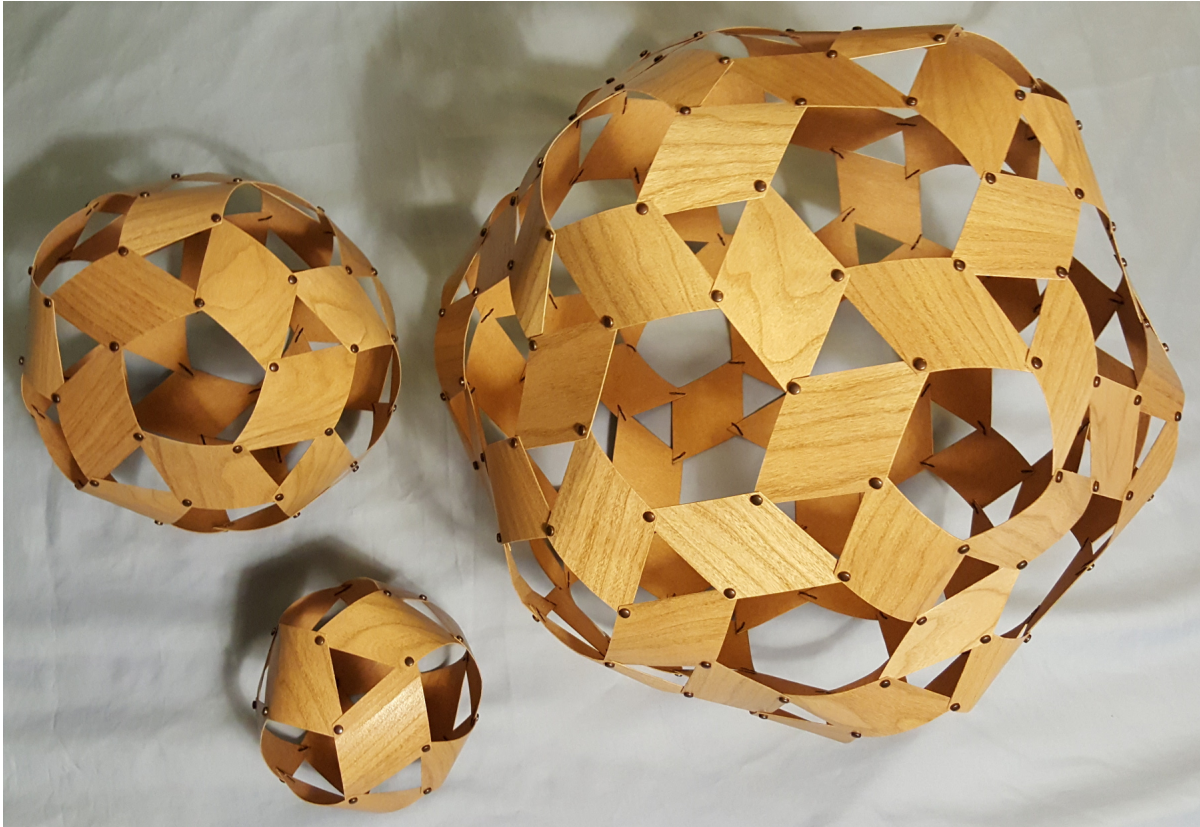


Figure 1: Rhombic polyhedral forms. Three models constructed using rhombi made from paper-backed cherry wood veneer connected at their corners using split-pin fasteners. Each rhombus replaces two adjacent triangles in a base polyhedron. The two smaller forms are based on the snub cube and snub dodecahedron (two of the Archimedean solids), and use 12 and 30 rhombi, respectively. The larger form is based on the snub rectified truncated icosahedron (a near miss Johnson solid) and uses 90 rhombi.

The open lattice construction yields elegant chiral symmetric forms with more visual movement than a similar process involving edge replacement with squares. These forms are surprisingly lightweight and rigid. Future plans include constructing other forms such as the snub rectified truncated dodecahedron, snub rhombicosidodecahedron, and snub rhombicuboctahedron, and identifying other snub forms that could be amenable to this construction technique.

References

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