Gödel, Escher, Bach: Just Another Braid

Hans Kuiper *
Rietdekkershoek 21, 3981 TN Bunnik, The Netherlands
E-mail: hans.kuiper@net.hcc.nl

Walt van Ballegooijen
Parallelweg 18, 4261GA Wijk en Aalburg, The Netherlands
E-mail: waltvanb@xs4all.nl

Abstract

This paper describes the synthesis of two completely different minimal art techniques applied in a new type of mathematical art. The portraits of Gödel, Escher and Bach are chosen to demonstrate this. The combined techniques are minimal art objects and optical minimal art graphics.

Minimal Art Objects

Our earlier contributions in creating minimal art objects are described in [1] and [3]. An example is shown in Figure 1.

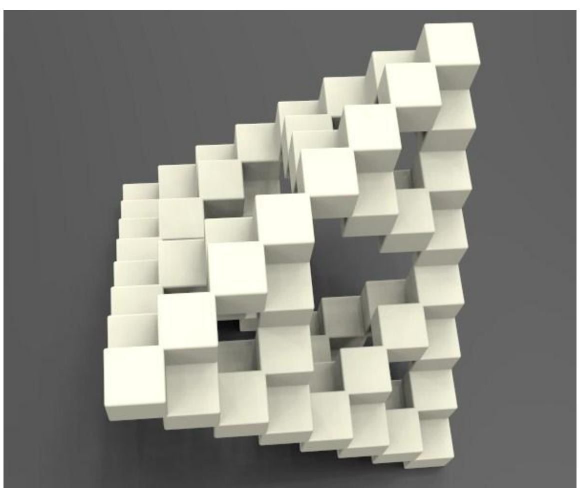


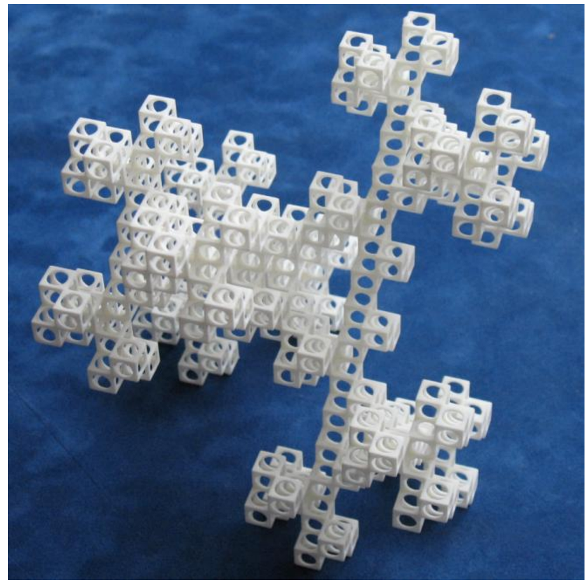
Figure 1: *Minimal Art Object.*

Such a minimal art object consists of $n \times n$ elements (cubelets) filling a $n \times n \times n$ space in such a way that it has 3 projections of $n \times n$ squares. These 3 projections are perpendicular to each other. The design rules to create such an object obey the rules of the Latin square. A Latin square is a $n \times n$ array filled with n different numbers, each occurring exactly once in each row and exactly once in each column. In our object each number is represented by a cubelet, which position in space is determined by its position in the Latin square (in x and y) and the number itself representing the height level (z).

The cubelets are connected on the edges into one object. This connectivity of the cubelets asks for further constraints of the Latin square.
For this project we use an object with 256 connected cubelets in a $16 \times 16 \times 16$ space, based on the 16×16 Latin square of Figure 2.

6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
8	7	6	5	4	3	2	1	16	15	14	13	12	11	10	9
7	8	5	6	3	4	1	2	15	16	13	14	11	12	9	10
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
4	3	2	1	8	7	6	5	12	11	10	9	16	15	14	13
3	4	1	2	7	8	5	6	11	12	9	10	15	16	13	14
14	13	16	15	10	9	12	11	6	5	8	7	2	1	4	3
13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4
16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
15	16	13	14	11	12	9	10	7	8	5	6	3	4	1	2
10	9	12	11	14	13	16	15	2	1	4	3	6	5	8	7
9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8
12	11	10	9	16	15	14	13	4	3	2	1	8	7	6	5
11	12	9	10	15	16	13	14	3	4	1	2	7	8	5	6

a

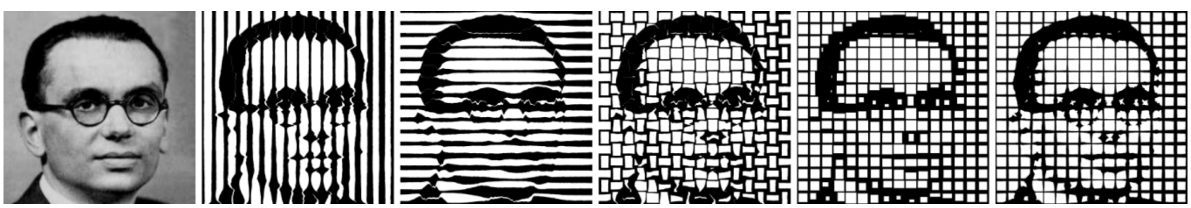


b

Figure 2: *The 16×16 Latin square and its derived 16×16×16 Minimal Art object.*

Optical Minimal Art Graphics

Optical minimal art graphics can be created by reducing the amount of RGB-colours within a row of pixels from 256×256×256 RGB to 2×2×2 [2].



a *b* *c* *d* *e* *f*

Figure 3: *Portraits of Gödel, from photograph to 5 different types of Optical Minimal Art Graphic.*

For this project we use images in grey colours. The image's height and width are the same size (Figure 3a). We divide each image into 16 × 16 squares. We reduce the 256 grey-colours to two colours, black and white. Not in one dimension such as within a pixel row (Figure 3b and c) but in two dimensions, within a square (Figure 3e and f). Figure 3d is a combination of b and c.

A new element in this artwork is how we calculated the shape of the white spot in the squares. Imagine each square is a cake. The thickness of the cake varies. The thickness at any place is represented by its pixel value of the white intensity. This varies from 0 to 255. We can now calculate the centre of gravity of the cake. Then we divide the cake into pieces as shown in Figure 4. Notice that $\triangle x$ is a very small, in practice the dimension of a pixel. We calculate the total amount of white values of each piece of cake. This total amount of white is concentrated (value 255) within the piece to the centre of gravity of the square. This calculation determines the radius of the white spot in every direction.

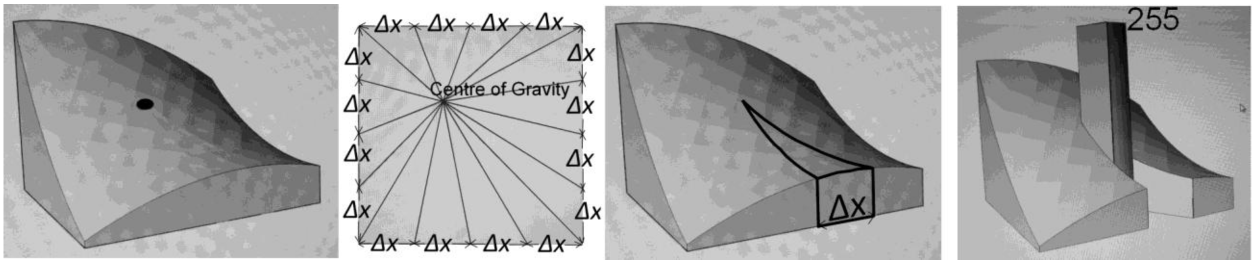
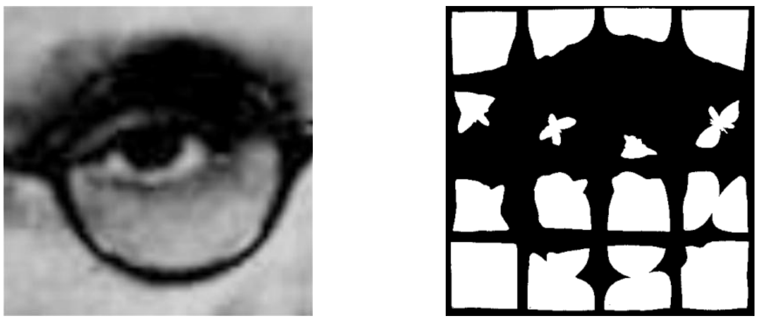


Figure 4: *Calculation of the shape of the white spot*



So the white spots are not circular but with a varying radius in every direction, to obtain a good fit with that particular part of the original image (Figure 5).

Figure 5: *Detail of Gödels left eye represented in 16 white spots*

Combination

The idea of combining both techniques occurred by chance, a kind of serendipity. By seeing the shadow of the minimal art object of Figure 2b. This is an object with as little material as possible, an extra dimension in minimal art and a manner to get a cheaper 3D print.
We used elements by 'subtracting' a sphere from a cube, causing holes in the cube. The shadow of this object showed a 16×16 grid with 256 spots of perfectly equal circles.
Seeing that, it was not difficult to imagine that, with some changes, that shadow could also be an optical minimal art graphic by using spots with different size and shape. We can use three different images, one for each direction. It was not difficult for us to find an interesting combination of persons to be part of our piece of art: Gödel, Escher and Bach.

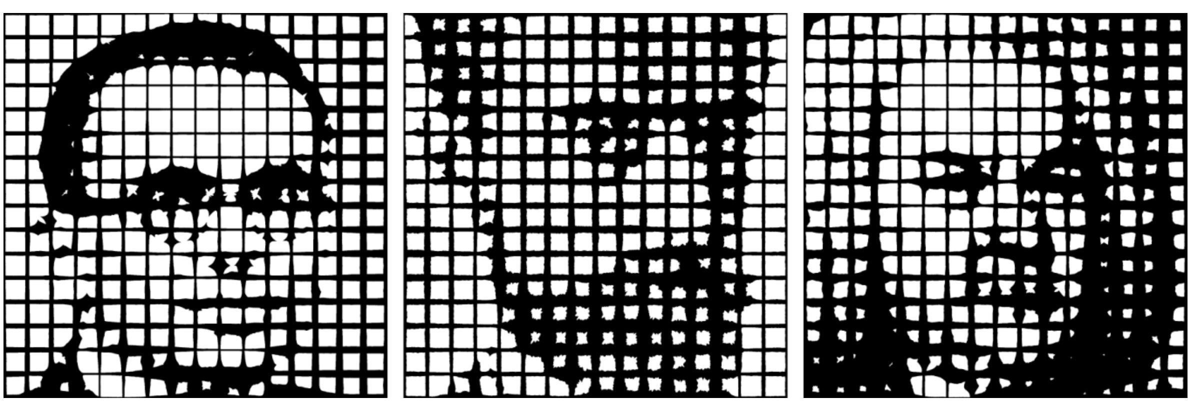


Figure 6: *Optical Minimal Art portraits of Gödel, Escher and Bach used in this project.*

The cover of Hofstadter's book "Gödel, Escher Bach, an Eternal Golden Braid" [4] shows an object with the shadows of three letters: a G, an E and a B. Our new object could show the shadow of their faces!

Creating the shape of the holes

It was not easy to design the object with 3×256 different shaped holes. Hans created for each portrait an image output file in `.bmp` format. Walt used those files as input in Rhinoceros [5] to create 3 sets of 256 polylines, extruded them each in an orthogonal direction and used them as “hole-makers”. The starting structure was made of hollow cubes with filleted edges, and with a little oversize in order to make one connected solid of all of them. The solid difference of this solid and the hole-makers gave the final model. This model was 3D-printed by Shapeways [6].

Black object, white background

We were caught in our thoughts that the shadows of our object was the output to be seen. (Figure 7). Walt even built a ‘piece of furniture’ with four mirrors and a projection screen to show the three shadows next to each other.

Later we ordered the object as a black 3D print. To our surprise, now our object itself catches the attention. A projector is not needed anymore, a white background is enough to show its properties (Figure 8).

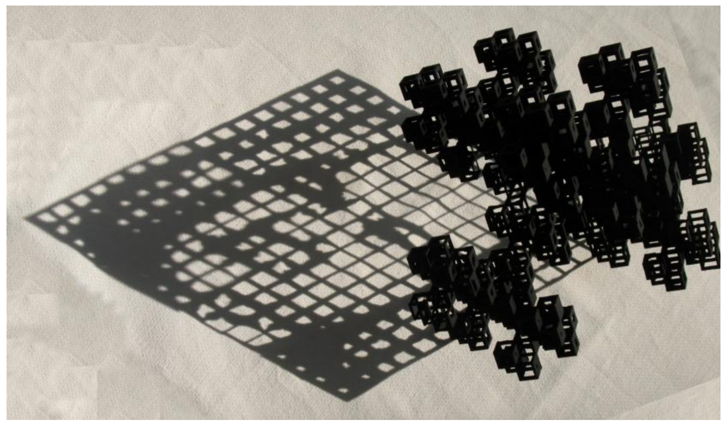


Figure 7: *Object with Bach’s shadow*

Figure 8: *Black Minimal Art Object with a white background; 3 perpendicular views.*

References

[1] Hans Kuiper, *A SUDOKU puzzle generates a Minimal Art Object*, 2008 Bridges Conference, Leeuwarden. http://archive.bridgesmathart.org/2008/bridges2008-475.pdf

[2] Hans Kuiper, *Optical Minimal Art*, 2012 Bridges Conference Towson. http://archive.bridgesmathart.org/2012/bridges2012-405.pdf

[3] Hans Kuiper & Walt van Ballegooijen, *Connected Sudoku's*, 2013 Bridges Conference, Enschede. http://archive.bridgesmathart.org/2013/bridges2013-407.pdf

[4] Douglas R. Hofstadter, *Gödel, Escher, Bach: een Eeuwige Gouden Band*, 1988 Uitgeverij Contact, Amsterdam

[5] Rhinoceros, *Modeling Tools for Designers*. https://www.rhino3d.com/

[6] Shapeways, *3D printing service*. http://www.shapeways.com/