Planes and Frames: Spatial Layering in
Josef Albers’ Homage to the Square Paintings

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Abstract
The later paintings and prints of Josef Albers, known collectively as the Homage to the Square series, are the culmination of the artist’s investigations of the power of color to create illusions of depth in abstract painting. The compositional framework of the Homage series is comprised of only four nested squares of diminishing size, yet this simple arrangement yields a multitude of possible interpretations of space. The author makes a systematic examination of this compositional framework and introduces a system for classifying (1) composition types, (2) plane and frame combinations, (3) near-to-far layering orders, and (4) opaque and translucent orders. The range of combinatorial possibilities is reduced by perceptual “limiting rules,” yielding a total of 171 distinct illusory spaces and showing the considerable compositional potential in this simple arrangement of four nested squares.

Introduction
Josef Albers (1888-1976) is known as one of the 20th century’s most important colorists, an artist who put color at the forefront of his exploration of spatial illusion in painting. Color, even when playing the principal role in a painting, nevertheless depends upon composition to reveal its power and potential. Albers understood this, and throughout his life he was both deliberate and inventive in developing compositional organizations that would permit colors to display their complex effects. Albers was strongly committed to a creative economy of means, believing that the artist should achieve maximum effect with minimum means: “Do less in order to do more” ([2] pp. 19, 42). He espoused this principle in his teaching, and he practiced it as an artist. Perhaps the best evidence for the latter is the Homage to the Square series of paintings that Albers pursued from 1950 until his death 26 years later. The “maximum effect” of colors in the Homage to the Square series was made possible by a compositional framework of four nested squares (Figure 1), from which either three or four squares are included in any given composition (Figure 2). This seemingly simple framework is carefully designed to yield a vast array of illusory spaces. In this paper, we shall (1) examine the features and variables of the compositional framework, (2) survey the full range of layered spaces made possible by combinations of the variables in the compositional framework, and (3) show how the constraints of illusion and perception reduce the greater number of variations to a smaller set of distinct spatial orderings.

For efficiency, it will be useful to establish standard words and phrases for some recurring references. While Albers worked in lithography, serigraphy, gouache, and oil paint, we shall use the word paintings when referring to Albers’ art works, whether prints or paintings. The Homage to the Square paintings, whether individual art works or the entire series, will be referred to by Homage or Homages. We shall employ the word ambiguity to describe the multiple and shifting spatial readings of the Homages—illusory spaces that won’t settle into a single state, where planes of color seem to shift back and forth among the spatial layers. In short, ambiguity refers to Albers’ goal to create maximal effect with minimal means. This paper also introduces a nomenclature for distinguishing and classifying the spatial features of the Homage series. We shall refer to the base structure of four nested squares as the compositional framework, from
which three or four squares are employed in any given Homage composition (Figures 1 and 2). The analyses that follow will include references to four variable characteristics of the squares in the Homage compositions, and we shall employ the following terms and abbreviations to distinguish these variables: (1) Type I, II, III, and IV will refer to the four arrangements of squares used by Albers in his Homage paintings; additionally, we will refer to the squares within the composition types with a number, from the smallest, 1 to the largest, 4 (Figure 2). (2) Squares may be understood as either planes or frames (these terms derive from the vocabulary of the visual arts; more precise mathematical terms are topological disks for the former and annuli for the latter), and since either or both may be present in a given Homage, we shall categorize compositions as plane-only, frame-only, or plane-and-frame. A plane will be identified by its specific numbered square (e.g., plane-1) and, since a frame is made of two squares, we shall indicate frames with the two square-numbers joined by a hyphen (e.g., frame-2-4) (3) The squares will be interpreted as occupying one of four layers in depth, and we shall use a place-value system for conveying this layer-order of the numbered squares, where the leftmost position is nearest in illusory space and the rightmost position is farthest (e.g., 1,2,4,3 means that plane-1 is nearest and plane-3 is farthest in the layer-order). (4) Planes and frames may be interpreted as either opaque or translucent (abbreviated o and t), and this opaque/translucent-order, or o/t-order, will use the same place-value system as the layer-order; thus, t,o,o,o means that the nearest plane is translucent and the farthest is opaque. Here is a random example that joins these four variables using our abbreviations and terms: Type I//plane-only//1,2,4,3//t,o,o,o. And, here is an example with a frame: Type II//plane-and-frame//2,1-4//t,o. Taken together, these descriptors constitute a classificatory system that gives order to the hundreds of possible spatial interpretations of the Homages.

The Homage to the Square Compositional Framework

The Homage to the Square series was the culmination of what was a nearly lifelong effort by Albers to bring color into an equal or dominant position with form in creating illusory spaces in abstract painting. As an indication of Albers’ priorities and purposes in developing the Homage compositional framework, we need only look at the compositional differences between the Homages and the paintings that preceded them, the Variants. The most important difference, and a clue to Albers’ concerns in the Homages, is that the Variants show planes intersecting at various points along their edges ([1], pp. 104-5, 115-18). These intersections are of two kinds: T-intersections, which convey opaque overlapping, where the nearer plane obscures the farther plane; and X-intersections, which convey translucent overlapping, where the nearer plane partially reveals the farther plane. It seems likely that Albers omitted these edge-intersections from the Homages because they limit the flexibility of perception to shift the planes nearer and farther in the illusory space—the aforementioned ambiguity. The graphic intersections limit spatial ambiguity in three ways: (1) T-intersections confine the planes to their respective locations—there is no opportunity to perceptually shift the nearer opaque plane to the farther location or the farther plane to the nearer location; (2) T-intersections are interpreted as opaque overlaps only and cannot convey translucency; (3) X-intersections are interpreted as translucent overlaps only and cannot convey opacity. When Albers eliminated the graphic intersections by nesting the squares, he opened the way for each square to be interpreted by the viewer as both opaque and translucent, and as both nearer and farther.

Albers determined the sizes and positions of the squares of his Homage compositions by an underlying grid (see [1] pp. 137-9 for Albers’ 10 × 10 grid). In descending order of scale, the largest square is 10 × 10 grid-units, the next is 8 × 8 units, the next 6 × 6 units, and the smallest square is 4 × 4 units (Figure 1). The four squares are horizontally centered on a common vertical axis, yielding one grid-unit between each square at left and right. The four squares are vertically eccentric, each smaller square lower than the next larger square by half a grid-unit. Albers describes these vertical displacements: “The downward shift gives additional weight, but also enhanced movement. This semi-concentric configuration avoids complete four-sided symmetry which would result in static fixation” ([1] p. 137). This arrangement of four squares, seemingly limited in its capacity for compositional variety, is in fact supremely flexible for creating a wide
array of layered illusory spaces—fulfilling the aforementioned ambiguity of spatial readings. Albers eventually established four compositional matrices for his Homage paintings, one with all four squares, and three versions with three of the four original squares ([1], p. 139). Figure 2 identifies these compositions as Types I, II, III, and IV, and it identifies the squares within the compositional framework, from smallest to largest, with numbers 1, 2, 3, and 4. Figure 3 illustrates one of the many Type I Homage paintings.

Given the four types of Homage composition, how many spatial readings are possible, and how might we bring order to the multiplicity of interpretations? Aside from the complex and subtle influence of colors (an important topic in its own right but beyond the scope of this paper), the spatial readings depend upon the interactions of three sets of variables implicit in the compositional framework: (1) Each square may be seen as lying on a different layer in illusory space, thus an Homage painting can display up to four layers of depth. (2) Each square may (especially when aided by color) be interpreted as opaque or translucent—an opaque square obscures shapes that lie behind it, and a translucent square reveals shapes that lie behind it. (3) Each square may be seen as the edge of either a plane or a frame—a plane is a continuous square surface; a frame is a square plane with a central square hole, thus a frame requires two squares from the 4-square compositional framework. Each square in any given painting displays all three variables, so we shall begin with combinations of the above three variables to discover the maximum number of spatial interpretations possible for each of the four compositional types. We will proceed in our investigation with the assumption that each square in any of the composition types can serve only a single purpose at a time and cannot function simultaneously as the edges of two planes, or two frames, or both a frame and a plane. This dual function is not out of the question for some spatial interpretations, but it will often make improbable spatial interpretations (such as a plane and frame “intarsia” on the same layer sharing the same edges) whose implications exceed the length of this paper. For our investigation, then, we will assume that for each spatial interpretation, each square will be assigned a single role as a plane or part of a frame.

At this juncture, then, consider some possibilities for spatial interpretation in Homage to the Square: Ascending (Figure 3). One might initially see a stack of planes where yellow is nearest, white is beneath yellow, gray is beneath white, and finally the large blue square is in the farthest position. Using our classification system, this would be designated as Type I / plane-only / 1,2,3,4 / o,o,o,o (Figure 4a.). Yet, if we allow the colors to suggest alternatives, we can imagine that there is a translucent blue frame two units wide on the sides, which partially overlaps a more distant white plane (the area of overlap is gray), and in the space between these two floats a yellow plane; this is classified as Type I / plane-and-frame /
2-4,1,3 / t,o,o (Figure 4b). Look again at the yellow square and imagine it floats closer to us, in the nearest position in space; the new description is Type I / plane-and-frame / 1,2-4,3 / o,t,o (Figure 4c). And yet another might be a 1-3 translucent frame overlaying a yellow plane-2 (white area of overlap), overlaying a blue plane-4: Type I / plane-and-frame / 1-3,2,4 / t,o,o (Figure 4d). These varied interpretations, and more besides, constitute perhaps the most important purpose of the Homage paintings. The viewer activates the painting’s potential spaces by imagining its spatial scenarios; the painting is not a demonstration but an evocation of space. Albers describes such perceptual dynamics: “[W]e see the colors as being in front or behind one another, over or under one another, as covering one or more colors entirely or in part. They give the illusion of being transparent or translucent and tend to move up or down…. Thus the intentional interaction of colors keeps on yielding renewed or different three-dimensional effects” ([1] p. 138).

Figure 3: Josef Albers, “Homage to the Square: Ascending,” oil on panel, 1953, 110.5 × 110.5 cm., Whitney Museum of American Art.

Figure 4: Alternative interpretations of layered spaces in Figure 3.

Plane-Only Variations

We shall begin our examination of the plane-only variations by considering all spatial possibilities first, and later we will prune away those versions that are compositionally impossible and perceptually improbable (and we will do the same for the plane-and-frame and the frame-only variations to follow). Considering the plane-only version of the Type I composition, we shall assume that each of the four planes occupies one of four distinct layers in the illusory depth. The permutations of layer-order for four planes is 4! = 24. Additionally, each plane in the 4-square composition is mapped to either opacity or translucency, yielding 16 possible o/t-orders (see Figure 8 for an example of the 16 o/t-orders for one of the 24 layer-orders). The total possible arrangements of four planes on four layers, where any of the four planes could be translucent or opaque, in any order, is 384 (24 × 16). Composition Types II, III, and IV are structurally the same as one another when we consider layered arrangements of opaque and translucent planes, so the following numbers apply to each of the three composition types. The total number of possible arrangements of three planes on three layers is six; each plane is mapped to either opacity or translucency, yielding eight possible o/t-orders. The total possible arrangements for Types II, III, and IV is 48 (6 × 8) for each type. These and the previously mentioned Type I totals must be considered as preliminary, “raw” totals because they are not as yet constrained by the requirements of spatial illusion and perception; these numbers will be significantly reduced as we proceed. For now, the grand total for the plane-only versions (i.e., not including frames) of Composition Types I, II, III, and IV is 528 (see Figure 7, “plane-only” row).
Plane-and-Frame and Frame-Only Variations

Frames are produced by two squares from the compositional framework; the larger square defines the outer boundary and the smaller square defines the inner boundary (the opening). A frame, like a plane, may be opaque or translucent. A frame may also vary in the width of its border; that is, the outer and inner boundaries may be constituted of any combination of two squares from the compositional framework. Type I compositions can include 6 possible kinds of frames (in order of increasingly wide borders): 1-2, 2-3, 3-4, 1-3, 2-4, and 1-4; Type II compositions can include 1-2, 2-4, or 1-4; Type III can include 1-3, 3-4 or 1-4; and Type IV can include 2-3, 3-4, or 2-4 (see Figures 5 and 6; dark lines indicate frames).

Given one frame and two planes in Type I compositions, there are three layers possible in illusory depth, and the frame may reside on any one of those three layers. The number of possible layer-order arrangements of each frame in Type I compositions is six. Since there are six frames possible in Type I compositions (Figure 5), there are in total 36 layer-order versions of Type I plane-and-frame compositions. Given three layers of planes and frames in these Type I compositions, there are eight mappings to opacity and translucency (see Figure 9 for an example of the eight o/t-orders for one of the 36 layer-orders), yielding a total of 288 (36 × 8) opaque-translucent permutations of the Type I plane-and-frame Homage compositions. Types II, III, and IV are limited to one plane and one frame, and therefore have two layers of depth. With three kinds of frame and two layer positions possible for the frames and planes (Figure 6), there are six arrangements of one frame and one plane in two layers for each of the Type II, III, and IV compositions. Given one plane and one frame, there are four mappings to opacity and translucency, yielding a total of 24 (6 × 4) opaque/translucent variations for Type II, III, and IV plane-and-frame compositions. The grand total of plane-and-frame variations of Type I, II, III, and IV compositions is 360 (see Figure 7, “plane-and-frame” row). As with the plane-only compositions discussed above, this total will be significantly reduced when we consider the limitations and requirements of perception and illusory depth.
A final set of variations is possible in Type I compositions only, where the four squares can be considered as two frames (no planes). In these frame-only compositions, we can have the following three combinations of frames (partitions of four squares into two sets of size 2): 1-2 & 3-4, 1-3 & 2-4, and 1-4 & 2-3, and each pair has two possible layer-orders. Each of these has 4 possible mappings to opacity and translucency, yielding a total of 24 (6 × 4). The numbers of possible versions for all composition types are shown in Figure 7.

<table>
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<tr>
<th>RAW TOTALS</th>
<th>Type I</th>
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<th>Type III</th>
<th>Type IV</th>
<th>Totals</th>
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**Figure 7**: Raw totals resulting from layer-orders multiplied by opaque/translucent-orders.

### First Reductions: Opacity Prohibitions

In generating the above raw numbers, we did not consider that some of the configurations create spatial contradictions. We must disallow a significant number of configurations on the basis of what we will call the **opaque-larger-nearer prohibition rule**. Any larger and nearer opaque plane or frame obscures any smaller planes or frames behind it, and thus reduces the number of squares visible in the composition. This results in two problems regarding the composition types: (1) when a single square is obscured in a Type I composition, it must be re-categorized as a Type II, III, or IV composition; (2) when two or more squares from a Type I composition are obscured, or when a single square from a Type II, III, or IV composition is obscured, the result would yield only two squares and thus not be a valid composition type. We need not be concerned about nearer translucent planes or frames here, since translucency reveals the square boundaries behind them. And we need not be concerned about whether the farther planes or frames are opaque or translucent, since it is the nearer that obscures the farther and not vice-versa. For example, Figure 8 shows the problem in a Type I composition with near-to-far order of planes as 3,1,2,4 (see Figure 2 for plane-number labels). In Figure 8 the diagram at left shows the near-to-far order as thick to thin lines. The table at right in Figure 8 shows the near-to-far layer-order from left to right, and the columns beneath each plane-number shows opacity (o) and translucency (t), while the rightmost column shows allowed (+) and disallowed (-) designations. The 8 disallowed versions here result entirely from an opaque plane-3, which obscures both planes 1 and 2; this results in eight disallowed compositions where only two squares (planes 4 and 3) are visible. Each of the 24 Type I, plane-only, layer-order variations is affected by this rule, although the number of disallowed versions varies for each. The same conditions hold true for Type II, III, and IV plane-only compositions.

Frames are subject to the same **opaque-larger-nearer prohibition rule** (although it must be remembered that a plane smaller than the frame opening is not obscured). Figure 9 shows the problem in a Type I plane-and-frame composition. The frame is formed by squares 2 and 4, while squares 1 and 3 are planes. Frame 2-4 is nearest, followed by plane-3 in the middle-ground, and plane-1 farthest in space. The frame overlaps the square boundary of plane-3. While it might seem that we can see plane-1 through the frame-opening, plane-1 is obscured in those versions where plane-3 is opaque. So we must consider multiple levels of the **opaque-larger-nearer prohibition rule**: we must disallow all versions where frame 2-4 is opaque (because it obscures plane-3); and we must disallow all versions where plane-3 is opaque (because it obscures plane-1), even those versions with a translucent frame-2-4. The multi-leveled application of this rule is relevant to all composition-types, all layer-versions, and all opaque-translucent-variations. When these reductions are applied, the revised totals (Figure 10) are considerably smaller than the raw totals (Figure 7).
Further Reductions: Translucency Problems

The above reductions are necessitated by the logic of opaque occlusions, but there are further reductions to consider from the standpoint of perceptual principles, and we shall apply these in the interests of paring the totals down to a minimal set of perceptually plausible spaces. There are two problems concerning the plausible recognition of translucency. The first is what we will call the global-translucency problem. Variations that include a translucent plane-4 in the nearest layer of space are unlikely to be recognized as such. Since plane-4 is the largest square (i.e., the entire compositional field), its translucency modifies all other planes and frames in the composition—a global color translucency. Global translucency is comparable to chromatic color constancy, whereby human vision discounts color changes in illumination ([3] p. 133). In short, the perceptual effect of global translucency is akin to no translucency at all. This is a generalization, of course, for there may be special color relationships that can overcome this perceptual tendency. However, on the basis of this global-translucency problem we will disallow those variations with a translucent plane-4 in the nearest layer. The second problem will be called the farthest-defaults-to-opaque problem. Since the illusion of translucency depends upon recognition that the nearer color modifies the farther by overlap, it is very unlikely that any farthest plane or frame will be recognized as translucent (since there is nothing farther to be modified); it will instead default to the most generic interpretation, opacity ([3] p. 299). On the basis of this farthest-defaults-to-opaque problem we will disallow variations with a translucent plane or frame in the farthest position. For example, while the o/t-order variations in Figure 8 would not be affected by the global-translucency problem, the farthest-defaults-to-opaque

<table>
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Figure 10: Revised totals after application of the “opaque-larger-nearer prohibition rule.”

Figure 8: Type I, plane-only composition, 3,1,2,4. Left: near-to-far shown as thick-to-thin lines. Right: 16 permutations of opaque (o) and translucent (t) layering, with designations of allowed (+) and disallowed (−) at far right.

Figure 9: Type I plane-and-frame composition, 2-4 (frame), 1,3 (planes). Left: near-to-far shown as thick-to-thin lines. Right: 8 rows of opaque (o) and translucent (t) layering, with designations of allowed (+) and disallowed (−) at far right.
problem would further reduce the allowed variations (+) from 8 to 4; in Figure 9, the two allowed variations would be reduced to one. Although the variations eliminated by these two problems are logically possible, they are nevertheless perceptually improbable. The revised totals, after the above reductions, appear in Figure 11; we may consider these as the minimal set of distinct possibilities of the layer-order, opaque/translucent-order, and plane/frame combinations for all four composition types.

<table>
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<tr>
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<tr>
<td>Totals</td>
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Figure 11: Revised totals after application of the “global-translucency problem” and the “farthest-defaults-to-opaque problem.”

Conclusion

The purpose of this paper has been confined to showing how we may understand both the combinatorial expanse and the perceptual constraints implicit in the Homage compositional framework. But it should be acknowledged that much could not be included in this short analysis, especially the nuances in the perception of color and depth, and these would greatly re-expand the possibilities far beyond the final, reduced totals just discussed. For example, there are very interesting color effects that can float planes relatively nearer and farther even within a given layer of space; and there are color effects that can create different degrees of translucency, from very thin films to denser, semi-opaque screens. Probably more important, there are figurative associations and poetic allusions that can arise, and these lend layers of meaning to the above-mentioned spatial variations. In addition to these aesthetic considerations, there are also more complex variations of layering, such as the possibilities for planes and frames to share edges and/or to coexist on the same layer, and there might emerge exceptions to our assumed transparency reductions. Although the aforementioned characteristics exceed the scope of this paper, they suggest the possibilities for further analysis. Nevertheless, the definition and classification of spatial possibilities inherent in the Homage compositional framework provide us some insight into how Albers could actualize his maxim of “do less in order to do more” in a large series of paintings. From the establishment of this compositional framework in 1950 until his death in 1976, Albers created over 1000 individual paintings in his Homage series, each a unique combination of colors, and each a unique expression of space.

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References