Thinking like a Pianist/Mathematician/Potter-Designer:
Strategies for Tuning Ocarinas

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Abstract
This workshop presents three different design strategies for tuning an ocarina so that it can play a full-octave diatonic scale. The first strategy involves “thinking like a pianist,” yielding a seven-hole instrument. The second strategy involves “thinking like a mathematician,” yielding four holes (the minimum possible). The third strategy involves “thinking like a potter-designer” and introduces aesthetic preferences that may influence the final number of holes. Workshop participants will be invited to build their own clay ocarinas. Thinking like mathematicians, our goal will minimally be to use two holes to produce the first four tones of an ascending diatonic scale.

Introduction
An ocarina is a vessel flute with a globular resonating body and an airduct assembly (windway, aperture, and beveled edge). The player blows air through the windway into the resonating cavity. This increases air pressure inside the cavity, causing air to escape through the aperture. The inertia of outrushing air causes the air pressure inside the cavity to drop; this in turn allows air back in. The oscillation of air back and forth across the bevel produces a musical tone.

An ocarina needs no holes other than the aperture to produce a tone, but additional holes are added to produce additional pitches. The frequency $f$ of an ocarina pitch can be described as

$$f \propto \sqrt{\frac{A_t}{V_0}},$$

(1)

where $A_t$ is the total open hole area, and $V_0$ is the volume of the resonating cavity [1]. An ocarina is similar to a Helmholtz resonator, in that the location of holes does not affect pitch (unlike flutes and recorders). However, unlike a Helmholtz resonator, wind speed significantly affects pitch.

For a basic introduction to diatonic scales and tuning systems, see [2]. The ratio between two frequencies determines the interval between two pitches. A perfect octave, for example, has a frequency ratio of 2:1 between the upper and lower pitch. From Eq. (1), for an ocarina with a steady wind, two holes of equivalent area $A_h$ will always individually produce the same interval above any lower pitch. However, if the two holes are opened in succession, the second hole will always produce a flatter interval than the first:

$$\frac{f_h}{f_0} = \sqrt{\frac{A_0 + A_h}{A_0}} > \frac{f_{2h}}{f_h} = \frac{A_0 + 2A_h}{A_0 + A_h}. $$

(2)
Design Strategies for Full-Octave Ocarinas

Thinking like a Pianist. An intuitive way to design a diatonic 8-note ocarina is to use seven holes (excluding the aperture), opening one additional hole for every step in the scale (like a pianist playing successive keys). Scale degree $^4$ is produced with all seven holes covered; the first hole is opened for $^2$, the second hole for $^3$, etc., until all seven holes are open ($^8$). The main disadvantage to this design strategy is that tone quality becomes breathier as more holes are added. As a 7-hole single-octave ocarina has multiple pitch redundancies (multiple ways to achieve the same $A_1$), the number of holes could be reduced.

Thinking like a Mathematician. An ocarina with $n$ holes has $2^n$ unique fingerings:

<table>
<thead>
<tr>
<th># Holes</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td># Fingerings</td>
<td>1</td>
<td>2</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>Fingerings</td>
<td>(●)</td>
<td>●</td>
<td>⬤</td>
<td>⬤</td>
<td>⬤</td>
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<td></td>
<td>○</td>
<td>○</td>
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</table>

Table 1: Fingerings possibilities for ocarinas with 0-4 holes; ● = closed hole; ○ = open hole.

Table 1 shows that a three-hole ocarina offers eight unique fingerings and could thus potentially produce an eight-note scale if no two fingering combinations yield the same $A_1$. However, each fingering must yield an actual pitch in the diatonic scale. The three holes must individually produce the first three intervals above $^4$: a major second ($M2$), major third ($M3$), and perfect fourth ($P4$). Disregarding that each ocarina hole adds a flatter interval as $A_1$ increases, the maximum interval between $●●●$ and $○○○$ is already a minor second short of a perfect octave ($M2 + M3 + P4 = M7$). The major seventh range can be extended through under- and overblowing, but altering wind speed across an entire octave can be unnecessarily challenging and impractical. A diatonic scale requires a fourth hole.

Adding a fourth hole doubles the number of unique fingerings, allowing ample opportunities to attain a well-tuned diatonic scale. British mathematician John Taylor was the first to recognize, in 1964, the full-octave potential of the four-hole ocarina [3].

Thinking like a Potter-Designer. A full-octave ocarina requires only four holes, but considerations such as aesthetic appearance, ergonomics (including hole size and location), and the player’s finger size and fingering facility may motivate adding more holes. The author uses a five-hole system for her bird-shaped “chickarinas,” which can play 12-tone chromatic scales.

Making Clay Ocarinas

The author will demonstrate how to make a clay ocarina and will provide clay, all necessary tools and supplies, and clay/ocarina instruction for workshop participants to build ocarinas of their own. To demonstrate the design approaches discussed above, and to demonstrate that a single hole can produce sharper or flatter intervals depending on whether other holes are already open, we will work on a small scale (literally and figuratively). Thinking first like mathematicians, our goal will be to produce instruments with two holes that yield the first four tones in an ascending diatonic scale. Participants who succeed in producing pleasing tone quality with a two-hole ocarina will be encouraged to attempt additional holes and to experiment with tuning designs (thinking like mathematicians, pianists, potter-designers, or any
thinking like a pianist/mathematician/potter-designer: strategies for tuning ocarinas

participants will have the option of arranging to have their work bisque fired. (because dry clay poses an inhalation hazard, finished ocarinas should be fired if they will be played).

figure 1 illustrates steps for making a basic ocarina; a similar method is described by baird [4]. ocarinas have two essential parts: a resonating body and an airduct assembly. the resonating body is constructed by pinching a ball of clay into a hollowed spheroid having a round, smooth-walled interior cavity (a-d). the spheroid is tapped against a table or other flat surface to create a flat face (e-f). a clay rectangular prism is attached to extend the flattened side; this will become the windway (g-h).

push a beveled wood craft stick into the rectangular prism, just skirting the flat inside of the resonating cavity (i). with the stick held in place, use a second stick to map out the aperture (j). the aperture should begin at the inner edge of the resonating cavity, just past the end of the windway, with the same width as the windway stick. with the first stick providing counter-pressure, use the beveled end of the second stick to cut open the aperture and chisel a ~45° bevel (k-l).

pull the first stick out of the windway. it will usually drag a plug of clay along with it, blocking the end of the windway; the plug can be pushed out of the way or removed with one of the wooden sticks.

blowing into the windway should now produce a tone. if not, or if the tone quality is breathy, refine the airduct assembly. common causes of a poor or absent tone include obstructions in the windway; a rough, blunt, or poorly angled bevel; a poorly directed windway; an aperture cut too far from the edge of the resonating cavity; or an overly small or overly large aperture. the airduct assembly is remarkably sensitive to small changes.

once the ocarina successfully produces a tone, finger holes can be added to produce additional tones (m-n). smaller/larger holes generate smaller/larger intervals. position each new hole so that it is comfortable to finger, and tune the hole size by ear. to generate the first four steps in an ascending diatonic scale, tune the first two holes to a major second (^2) and major third (^3) above ^1, respectively; when both are open, they will add up to approximately a perfect fourth (^4). tone quality often becomes audibly breathier with each additional hole, but can be improved by further refining the airduct assembly and checking that holes are free of debris. when all else fails, a particularly troublesome hole can be plugged back up and reattempted.

note that blowing into the ocarina is necessary to check the tuning, but humidifies and thus softens the clay. excessive blowing can negatively impact both tuning and tone quality. if the airduct assembly becomes noticeably moist, wait for the windway and bevel to firm up a bit before making further adjustments.

references

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<tbody>
<tr>
<td>a.</td>
<td>Begin with a small ball of clay.</td>
<td>b. Pinch to create a round cavity.</td>
</tr>
<tr>
<td>d.</td>
<td>The hollowed closed form.</td>
<td>e. Tap form on a flat surface...</td>
</tr>
<tr>
<td>g.</td>
<td>Attach a rectangular prism....</td>
<td>h. ...extending the flattened face.</td>
</tr>
<tr>
<td>j.</td>
<td>Map out the aperture position.</td>
<td>k. Cut aperture and chisel bevel.</td>
</tr>
<tr>
<td>m.</td>
<td>Cut holes as desired.</td>
<td>n. A 2-hole/4-tone ocarina.</td>
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**Figure 1(a-n):** Steps for making a basic ocarina. The instrument shown here has two holes of unequal size and can therefore produce four discrete tones.