The Shape Snacker: a Bite of Origami and Math

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Abstract

Origami, the Japanese art of paper folding, seems to naturally fall at the intersection of art and mathematics. This workshop shares a traditional origami model which was repurposed for mathematics instruction in the elementary classroom. Participants will learn to fold the traditional candy dish and become familiar with the Shape Snacker which is a candy dish modified for classroom implementation to study shape and size with students in the second grade. We will then get an overview of other modifications for several other classrooms. Aside from printable materials, participants in the workshop will get to brainstorm further uses of these models for their own classrooms and goals.

Introduction

The Grant. I received a grant from the **Japan Foundation's Center for Global Partnership** in early 2011. "*Origami as a Gateway to the Study of Japanese Culture and Society*" allowed me to lead two after-school sessions of professional development with thirty K-12 teachers and follow up a week later helping them implement new lessons with their students. The grant allowed us to collaboratively modify six traditional origami models for use throughout the K-12 curriculum and among disciplines from language arts, science, social studies, and mathematics.

I have spent much of the time since the grant revising and implementing the activities with several groups of school-age students. This workshop shares a single model that we used for elementary mathematics. Participants and those who read this work are strongly encouraged to modify and revise this model to help them meet their own goals for their discipline and age-specific classroom.

The Workshop. Those planning to lead a workshop on origami should fold early and often. The folding offers its own motivation and there is plenty of discussion time while the folding is taking place. With this in mind, the article shows the folding sequence for the traditional model followed by the modifications we made for classroom implementation. One successful lesson is shared and insights are given. Once we understand the model better, I will discuss several other ways it has been used successfully with school-aged students. Finally, the workshop participants and those reading the article are strongly encouraged to brainstorm how the model might help them meet their own classroom goals with their own age-specific classroom.

The Candy Dish Becomes a Shape Snacker

Origami. The *candy dish,* or salt cellar in origami, has now become infamous in elementary classrooms as the fortune teller or cootie catcher. Traditional origami favors the four compartments which happen to be useful for storage in the elementary classroom. Students use the back-and-forth movement and then partially unfold the model to reveal a fortune. Very few manipulatives use the partial unfolding successfully so the Shape Snacker relies heavily on the back-and-forth movement with only a brief mention of the partial unfolding.

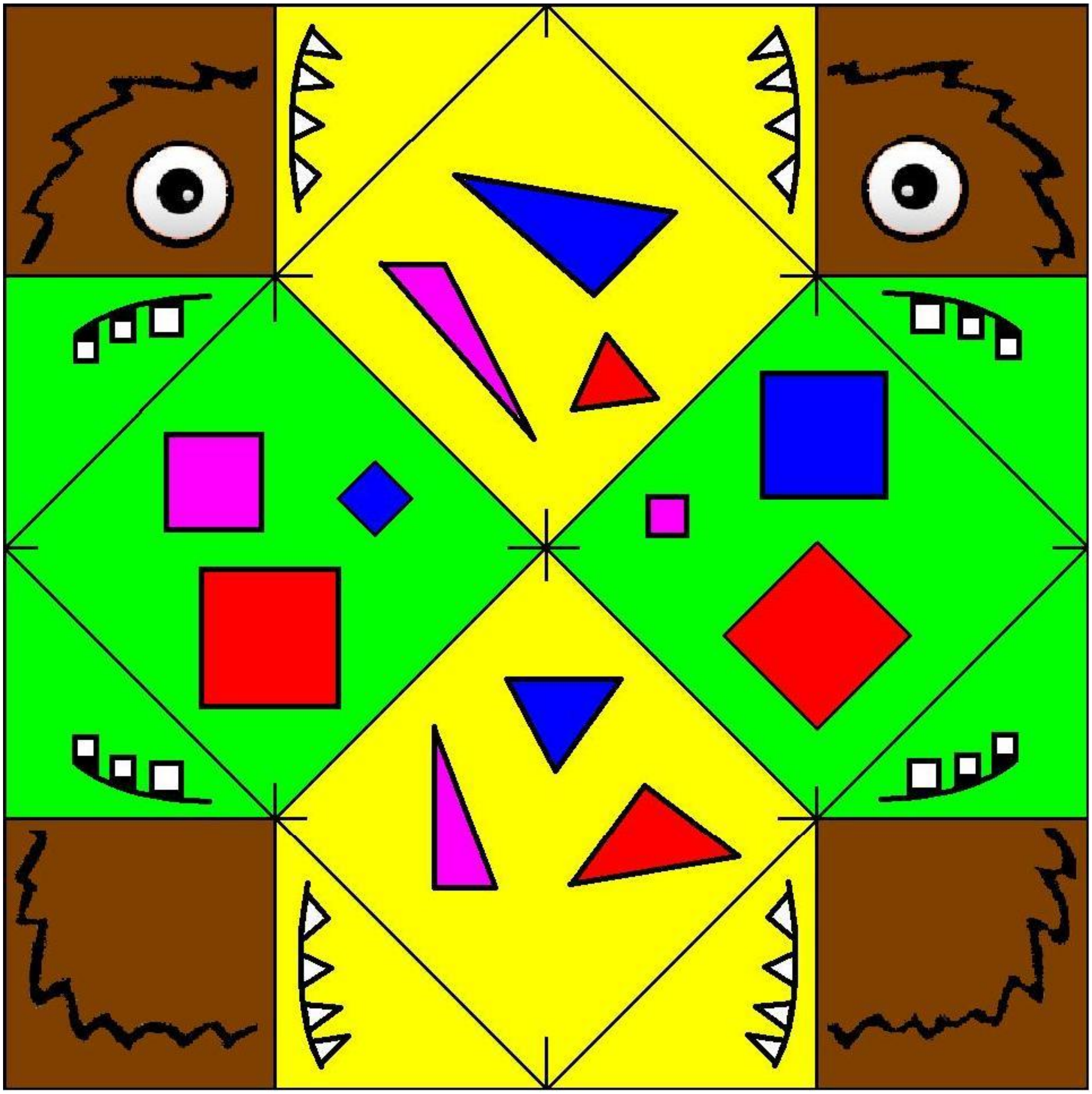


Figure 1: *The Shape Snacker starting square.*

The Shape Snacker. A reliable method for introducing new manipulatives gives them the starting square shown in Figure 1 and asks them to generate Good Questions and Good Math Questions. Good Questions often include: “Is this a monster?”, “Why does he have different shaped teeth?”, and “How does his mouth work?” Good Math Questions often include: “Why do all of the squares live in the green?”, “Why are there so many different types of triangles, but the squares all look the same?”, and “Why does each square (region) have three shapes (pink, red, and blue)?”

After taking a few moments to explore the piece, we process our ideas by putting them on the board. Elham Kazemi's Discourse That Promotes Conceptual Understanding [2] is a wonderful resource for considering and answering questions through discussion. I try to get students to answer questions collaboratively during the discussion. For example, students will help each other quickly understand that the variety of squares can only vary by size. The rules for squares require them all to look similar while triangles can take on quite a variety of shapes and sizes. Some of the questions can be answered by folding the Shape Snacker, so that's our next stop.

Folding the Shape Snacker. I have included the five-step folding method for the Shape Snacker.

1. With the picture face down, fold both diagonals and unfold.
2. Fold all four corners to the center.
3. Turn the paper over and fold all four corners to the center again.
4. Fold the model in half edge-to-edge both ways and unfold.
5. Place a finger in each pocket and the Shape Snacker appears.

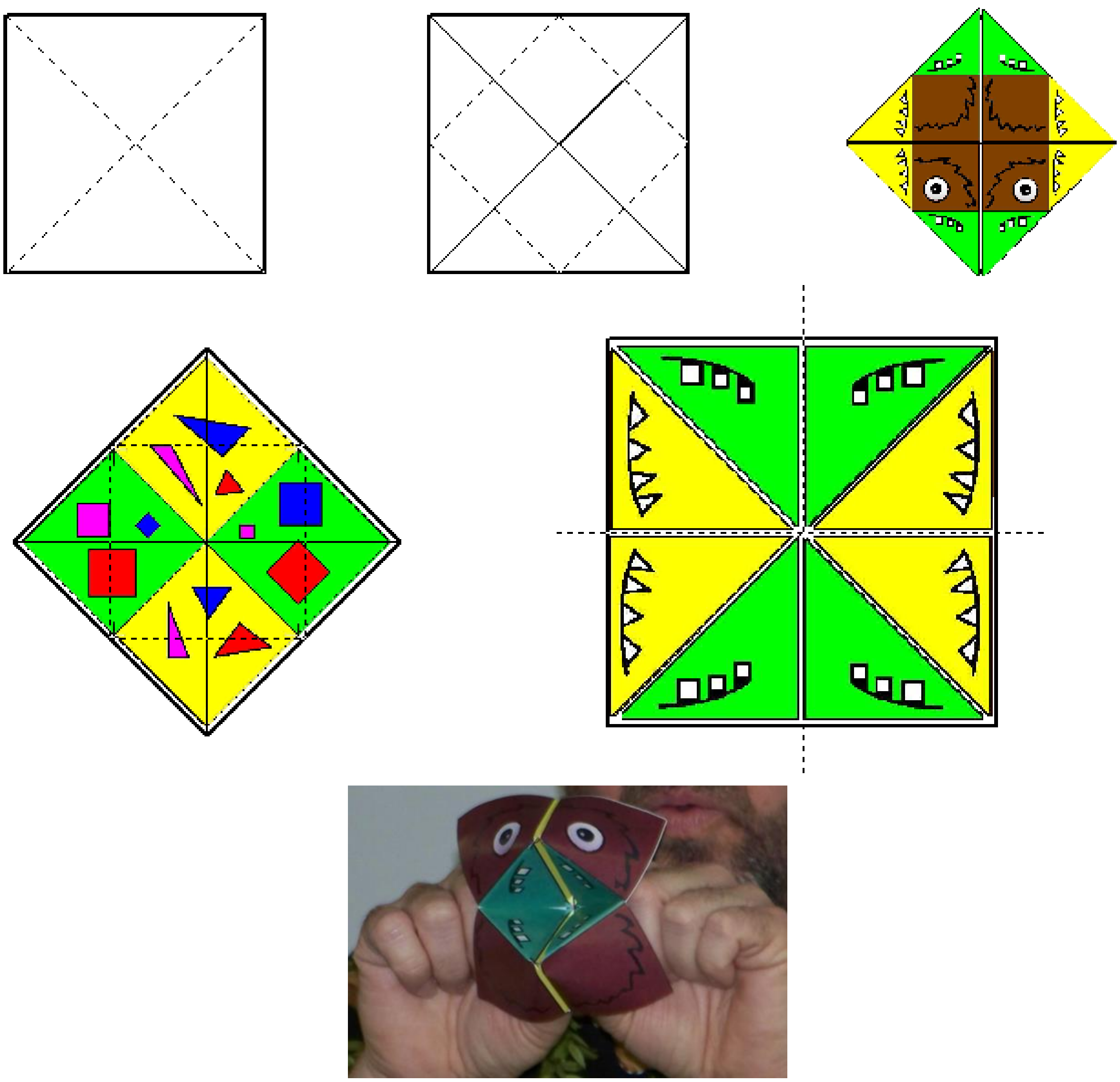


Figure 2: *Folding the Shape Snacker.*

Shape Snacker Eating Habits. Now that the students have their own Shape Snacker, they have to know how to feed it. There are a few clear rules for feeding a Shape Snacker:

1. Shape Snackers eat squares with their green mouth
2. Shape Snackers eat triangles with their yellow mouth.
3. Shape Snackers eat no other shapes.
4. If given a choice between squares and triangles, a Shape Snacker will always choose the one with the largest area.

Here is where the partial unfolding of a fortune teller comes in handy. The students see that the Shape Snacker has a green mouth with square teeth for eating squares. If they unfold slightly, they will see some leftover squares. The yellow mouth has triangular teeth for eating triangles. Again, a slight unfolding can show some of the triangles this Shape Snacker has eaten recently. It clearly needs to brush its teeth.

The Activity

Common Core. The result of these explorations is an activity which can be completed in a single 50-minute session. The activity addresses Common Core State Standards [1] by focusing on standard 2.G.A.1 and beginning to explore 3.G.A.2. From 2.G.A.1, students engaged in this activity have several chances to show they can “recognize shapes having specified attributes” as they identify triangles, squares, and those shapes which are neither triangles or squares. From 3.G.A.2, this activity promotes the ability to “partition shapes into parts with equal area”. The activity also encourages the Standards of Mathematical Practice as students “make sense of problems and persevere in solving them”. Through discussions of students reasoning, we reinforce the student’s ability to “construct viable arguments and critique the reasoning of others”.

Snacker Food. The final pieces required for the activity are two large squares of paper which will be used to create Snacker Food. These squares should be pre-creased as shown in Figure 3.

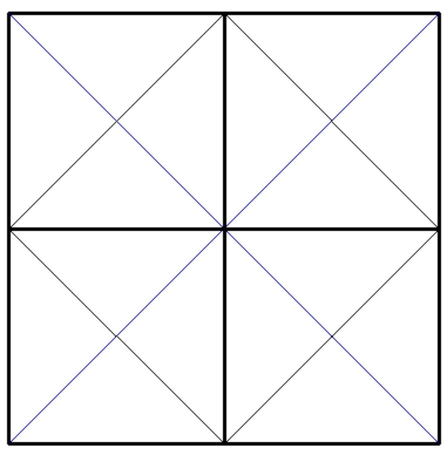


Figure 3: *Snacker Food.*

The classroom teacher and I made two shapes at the front of the room, without adding any new creases, and asked the students to choose which shape the Snacker would eat by opening the Snacker’s mouth to either the square-eating mouth or the triangle-eating mouth.

Choosing. We began with Choice 1 in Figure 4. Students immediately chose the square and opened the Snacker's green mouth wide. It was easy for both the teacher and me to see that the answers were correct because we didn't observe any yellow mouths.

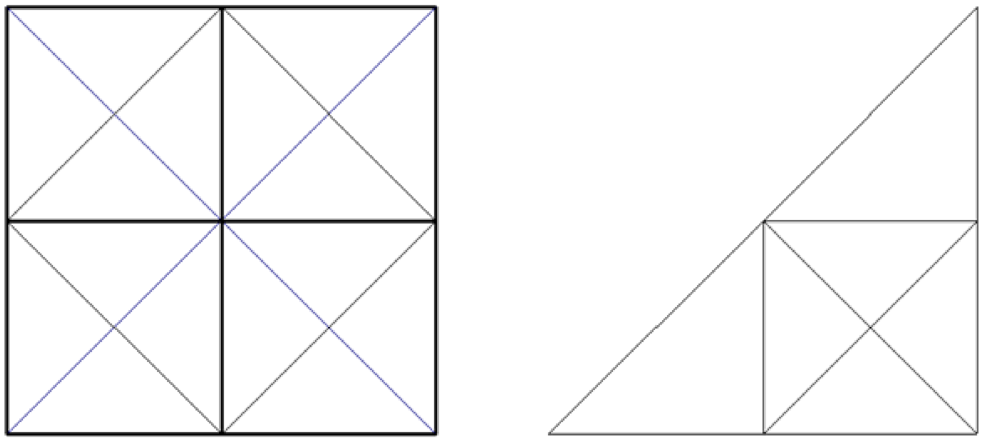


Figure 4: *Choice 1.*

Choices 2 through 5 are shown in Figure 5 for sake of space. Please note that these were the important choices in our investigation, but certainly not the only choices. I had one class who took a couple moments to get their bearings so we included more examples at the front of the room before we introduced challenging choices.

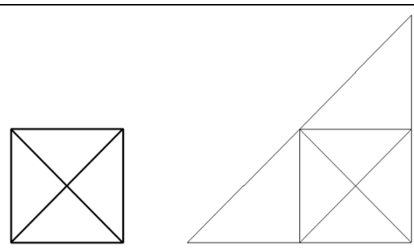
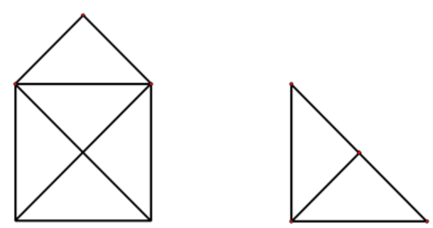
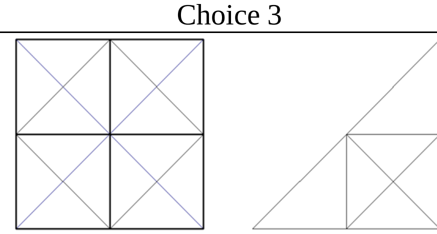
	
Choice 2	Choice 3
	
Choice 4	Choice 5

Figure 5: *Choice 2 through 5.*

Choice 2 resulted in every student opening the yellow mouth of their Shape Snacker. Choice 3 offered a challenge. Many students instinctively opened the square mouth and then quickly realized that the shape's outline actually had five sides and was inedible. For those who didn't see their mistake, we asked a student who chose to open the yellow mouth why they chose the smaller figure. Proudly, they will proclaim that they made a correct choice because "the Snacker cannot eat that other shape!" Pentagons are not on the menu for a Shape Snacker.

Choice 4 is the best for discussion. Some students open the yellow mouth, some the green, while still others sit there and struggle. A few may open the entire Snacker showing both the green and yellow mouths simultaneously. This was the students' way of saying that the choices were equal. Resolving this dispute usually generates about five minutes of discussion as the students try to construct arguments to convince one another. Before long, a student will emerge who chooses a dissection argument. "Looking at the folds, the square has two triangles in it. The triangle also has two triangles in it. They have the same area." At this point, in sharing this idea, students begin to understand that area can be measured in "little triangles". This is still challenging for many students because they understand the figures are made of identical pieces, but the triangle "is wider and taller than the square". This conservation of area may be difficult for many of the students depending on geometric maturity.

The first time we did this activity, my wife and I were the presenters at the front of the room. I was pleased with the results and about to conclude the activity when my wife asked me to "make sure the students have it". So, we gave them Choice 5 which looks suspiciously like Choice 1. I wanted to see a green mouth and pack up. Instead, I saw a sea of students counting little triangles. The same students who knew the square was bigger ten minutes prior were now counting component pieces to get the area. For one, counting to sixteen without missing one or double-counting is difficult, but more importantly they had given up on their intuition. With Choice 4, they had learned that intuition might be a faulty system and that only through counting could they reach true clarity in their answer.

In subsequent attempts at this activity, I have repeated Choice 5 and without fail, the students begin counting small component triangles. So, we use the end of the activity to talk about how we choose problem solving strategies. This is one of the keys to promoting conceptual understanding [2]. Now that they have two strategies, it is time to learn how to implement them appropriately. If they can tell quickly by visual inspection, they should use that strategy. If there is doubt, they can then move to the counting strategy. I find quite a few college students who have lost the ability to estimate a reasonable answer because they have pitched out large-scale approximation in favor of this type of fine-grained approach. In fairness, I had "taught" these second-graders the counting strategy. It then became my job to teach them when to use visual inspection and when to use counting.

Extensions. There are many things that we have added to the activity if time allows. Students, in pairs, have been given Snack squares and asked to create their own problems to reinforce the strategies. Another teacher asked me to explore the snacks with her students as its own manipulative. "Without adding any new creases, what are the largest and smallest snacks that can be folded? How many different shapes can be folded that the Snacker can eat? That is, how many different sizes of squares and triangles are possible? Starting with pentagons, how many non-edible shapes can be folded without adding any new creases?" A third teacher used the Shape Snacker as a personal response device for the class. He gave them a choice with two outcomes and asked them to open the green mouth for option 1 and the yellow mouth for option 2. This quickly reveals consensus in responses because you will easily see all green if that is the option they chose.

Brainstorming

General Uses. Once teachers have tried the candy dish and the Shape Snacker, we begin to explore the key uses of this model. First, when used as the candy dish, the model can hold items in four separate compartments. Second, when used with the back-and-forth movement, the Shape Snacker could reveal two different colored mouths. I will now share two alternate versions of the Shape Snacker which have been used across the curriculum: Classroom Response Systems and Four Containers.

Classroom Response System (CRS). The other common variation is a take on the Shape Snacker which yields three choices. Thoughtful teachers had noticed that the Snacker with a closed mouth was brown. So, they determined they could ask students questions with three responses assigning brown, yellow, and green to the three responses. Figure 7 shows the blank for this model. We see that this new model is yellow when closed and can be opened to red or green. Once this model has been created, teachers have typically used it throughout the entire year.

There are numerous resources for the use of CRS in the classroom. My favorite resource discusses both elementary and secondary classrooms [3]. The basic premise has the teacher posing a challenging question with three possible solutions. The class is polled and the results are shown to the entire class. Students are then asked to discuss reasons for and against the three responses. This ties back to Kazemi [2]. Finally, the class is polled a second time and the results are shared with the class again. The teacher can then use the information to reteach a concept or to breakdown misconceptions. Originally developed for physics classrooms, the pedagogy seems quite sound. Thankfully an origami manipulative can replace the high-priced technology usually associated with the CRS.

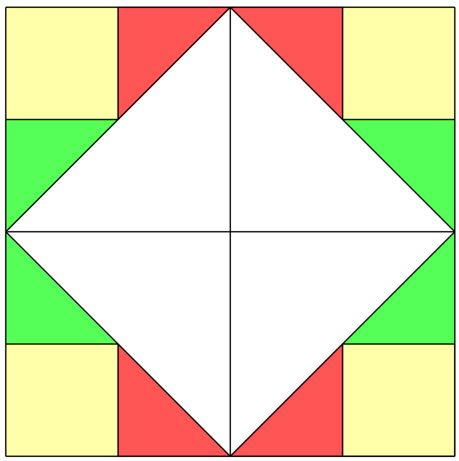


Figure 6: *PRS Blank.*

Four Containers. We use one variation of the candy dish across the curriculum and across grade levels as a method for gathering data. Figure 6 shows the blank for this model. The blank features a place to write the question and a place for each of the responses in each of the four corners.

In small groups, students take a question blank, fill in the question with four possible answers, and fold it into a candy dish. A typical class may have from five to ten questions to be answered. So each student is given five to ten beans or counters and asked to place the counter in the appropriate answer for each question. Allow approximately five minutes for all students to move around the room weighing in on all the questions. Once the questions have been answered, groups return to their candy dish question and construct a graph showing the classroom results as a bar graph using the counters.

Some popular questions in the past have included: Favorite pet – Dog, Cat, Fish, Bird; Favorite Pizza Topping –Cheese, Pepperoni, Other, Don't eat pizza; Favorite Ice Cream Flavor – Vanilla, Chocolate, Strawberry, Other; Favorite Color – Red, Yellow, Blue, Green; Favorite Season – Winter, Spring, Summer, Fall.

Student groups are then encouraged to create two statements based on their bar graphs. They double-check each other's work before the results are posted. Finally the results can be explored by the entire class looking for any interesting patterns.

Four Containers Extensions. If boys and girls are given different counters, the results can be broken down by gender. This results in stacked bar graphs. Create two yes/no questions to place on either side of the candy dish. In this way, two sets of data can be created and two graphs can be generated. A group may create two candy dishes if they can generate a question with 8 responses. For example: Favorite Crayon Color – Brown, Red, Orange, Yellow, Green, Blue, Purple, and Black.

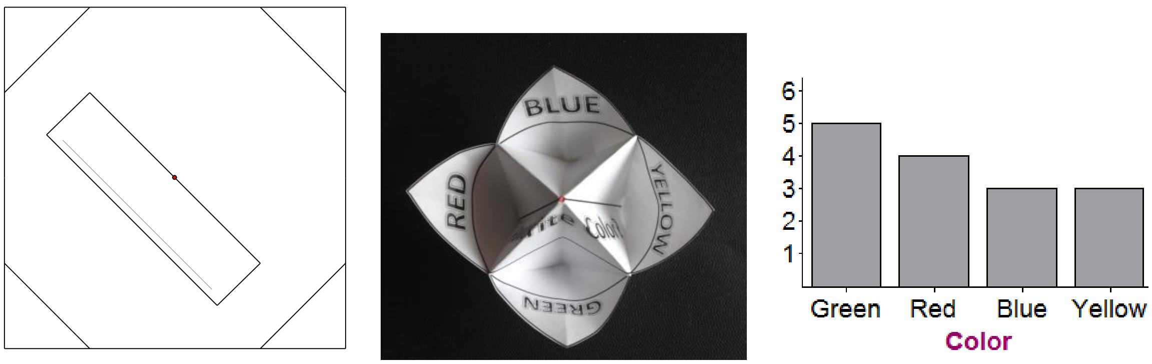


Figure 7: *Question Blank, Folded Container, and Graph*.

References

[1] Common Core State Standards Initiative (CCSSI), *Common Core State Standards for Mathematics*, National Governors Association Center for Best Practices and the Council of Chief State School Officers and Standards for School Mathematics. 2010.

[2] E. Kazemi, *Discourse That Promotes Conceptual Understanding*, Teaching Children Mathematics, Vol. 4, No. 7, pp. 410-414. 1998

[3] W. Penuel, C.K. Boscardin, K. Masyn, and V.M. Crawford, *Teaching with Student Response Systems in Elementary and Secondary Education*, Educational Technology Research and Development, Vol. 55, No. 4, pp. 315-346. 2007.