Introducing the Möbius-Twisted Turk's Head Knot

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Abstract

This paper shows how the classical braided Turk's Head knot can be changed from a cylindrical configuration into a single-sided Möbius band. Realizations are shown using latigo leather and colored paper strips, respectively.

Introduction

Cowboys have a long-standing tradition of creating beautifully woven knots from strips of rawhide or latigo leather [1,2,3,4]. They often elevate the necessary leather gear used around a horse into art objects. Figure 1 shows various "bosals" (the noseband portion of the hackamore headstall) decorated with different types of woven braids.

Figure 1: *Raw-hide and latigo leather bosals showing decorative braided knots.*

(a) (b) (c)

Figure 2: *Turk's Head knots: (a) 5-part 4-bight; (b) 5-part 11-bight; (c) 6-part 7-bight.*

The Classical Turk's Head Knot

One of the basic patterns that shows up in many different contexts is a flat braid that is formed into a wreath or ring by means of a single "thong" (a narrow strip of leather) [1]. This braided ring has two parameters: *p parts* and *b bights*. The number of *parts* measures the width of the braid by the number of thongs that would be severed, if the braid were to be cut straight across. The length of the braid is measured in *bights*, i.e., the number of repeating patterns in the longitudinal direction, which can most easily be counted by the number of "bays" (or, alternatively, the number of scallops) along one side of the braid (Fig. 2). To realize this ring-braid with a single thong, these two measures have to be mutually prime. If they have a common divisor, then, when the braid is being closed into a loop, there will be more than one individual thong that forms a loop closing back onto itself.

The Challenge: Developing a Möbius Pattern

Latigo leather strands have two different-looking surfaces: the smoother, shinier outer side of the hide and the coarser inner side. In the Turk's Head knot, and almost all cowboy braids, normally the shiny side is pointing outwards consistently. In other words, practically all of the leather weaves are "orientable" surfaces, and the visible side is the shiny, outer side of the hide.

Now, if one plans to weave any kind of single-sided surface, like a Möbius band, both sides of the finished weave need to look exactly the same, because there must be no pattern discontinuities as one moves along the surface around a Möbius loop with a 180° twist. This implies that *p* must be even; both sides of the woven surface need to show the same amounts of the outer and inner surfaces of the leather – in exactly the same pattern. The most obvious way to achieve this is to alternate the surface orientations of adjacent strands that run "side-by-side" around the braid. This leads to a basic pattern as shown in Figure 3a. There are also some basic constraints on the edges of the braid. A Möbius band has only a single border; so the left and right edges of a segment of the woven ribbon must be two views of the same physical pattern – but seen from opposite sides. Therefore, the symmetrical pattern shown in Figure 3b will not work. What looks like an over-pass of the shiny side amounts to an under-pass for the coarser back side. Figures 3c through 3e show patterns that have the required properties.

There is also a change in the rules that guarantee that the whole Möbius band can be woven with a single thong: Since the Möbius flip reverses the left-to-right pattern of the strands in the braid, we must compensate for this by adding or subtracting a half-period (*p/*2 bights) to the chosen (relatively prime) length of the loop; in a plain woven braid, a segment *p/*2 bights in length achieves the same reversal.

(a) (b) (c) (d) (e)

Figure 3: *Patterns for a Möbius braid: (a) area coverage; (b) bad "symmetrical" edges; (c) an elementary pattern that works; (d) enlarging the pattern; (e) a proper Möbius pattern.*

Weaving a Möbius Knot

Now that we have designs that meet all our criteria, there is still the key problem of how to actually realize such a knot in leather. To construct a physical Turk's Head knot, one can use a cylindrical pegboard (Fig. 4a) and gradually weave the leather strand around the pegs. In the first round this is just a zig-zag path wrapping around the cylinder. On subsequent rounds one must be careful to observe proper over- and under-crossings. Once the whole knot is completed in a loose configuration, the pegs can be pulled out and the knot can be removed from the cylindrical core. The knot is then gradually tightened, until the strands lie tightly packed against one another.

(a) (b) (c) (d)

Figure 4: *Knot construction: (a) using a cylindrical 'peg board'; (b) using the substitution method; (c) a resulting 4-part, 15-bight Möbius knot; (d) 6-part, 34-bight 540°-twist Möbius knot.*

When weaving a non-orientable surface, it is not possible to have a supporting peg-board surface on one side of it; it would tightly encroach the emerging woven surface from both sides, making access to it impossible. We first considered using some kind of a gridded guide surface through which we could weave the leather strand as required by the needed sequence of over- and under-crossings. In the end this supporting guide-surface has to be removed in some way. This implies that the guide surface would have to be constructed from individual row-and column-struts or that it is made of some dissolvable material. This would be a huge departure from the way cowboys weave their knots.

A more practical solution starts out by weaving a p-part braid from p separate strands to whatever length (measured in bights) desired. Then the two open ends of the braid, each showing p strands, are joined with one another while imparting a twist of 180° to the whole band (Fig. 4b). The strands are labeled individually, so that one can make sure that the various strands join into one single filament forming the whole knot. The individual strands can be hooked together temporarily to form a single overall filament; but we are not considering fusing any strands together – this would be a breach of knot-weaving etiquette. Instead, we now take a long contiguous leather filament and gradually replace one of the temporary strands after the other by threading the new filament along the path of its temporary place-holder, removing the latter one in the process. When all the temporary filaments have been replaced, we then have a Möbius-twisted Turk's Head knot realized with a single filament in the style of many other woven knots (Fig. 4c). Figure 4d shows a larger, 6-part, 34-bight Möbius knot with a twist of 540°.

An "Origami" Version of the Möbius-twisted Turk's Head Knot

Latigo leather or rawhide strips may not be readily available to many participants of the Bridges conferences. Thus we now present some models made from paper strips that allow experimentation with this novel type of knot. Long, two-colored paper bands may be another item that is not readily available, but paper strips can readily be glued together. Thus for the proposed construction we just form a suitably woven rectangular braid and then twist it through an odd multiple of 180° before joining the ends of the paper strips sticking out from the two opposite narrow edges.

Because thin paper strips fold easily, but do not readily submit to tight lateral turns in the plane of the paper, we introduce another modification. Rather than forming a rounded scallop when a paper strip reaches the edge of the Möbius band, we simply fold it over as shown in Figure 5d. This means that all strips will flip front-to-back at these locations. Figure 5a is a proof of concept that such a knot with folded ribbons can be woven with a single thong. The single thong has identical front and back sides and rainbow coloring in the longitudinal direction; it passes through the full 360° hue (H) range of the HSB color space, with saturation (S) and brightness (B) set to 100%. This is a 4-part, 21-bight Möbius-twisted Turk's Head knot. Next, we use a thong that is colored only on one side. This seems like a minor change, but it turns out that the patterns proposed in Figure 3 can no longer be achieved. Instead, a pattern emerges in which all front-facing segments of the thong run diagonally in one direction across the Möbius band, while all the back-facing segments run diagonally in the other direction (Fig. 5b). It seems that an odd number of parts is now admissible, since the thong reverses front-to-back orientation every time that it touches the edge; but this leads to a discontinuity in the over–under sequence around the loop. Figure 5c shows a knot actually woven from paper strips colored on one side; this is an 8-part, 31-bight Möbius-twisted Turk's Head knot. Figure 5d presents an intermediate phase in the construction of this knot.

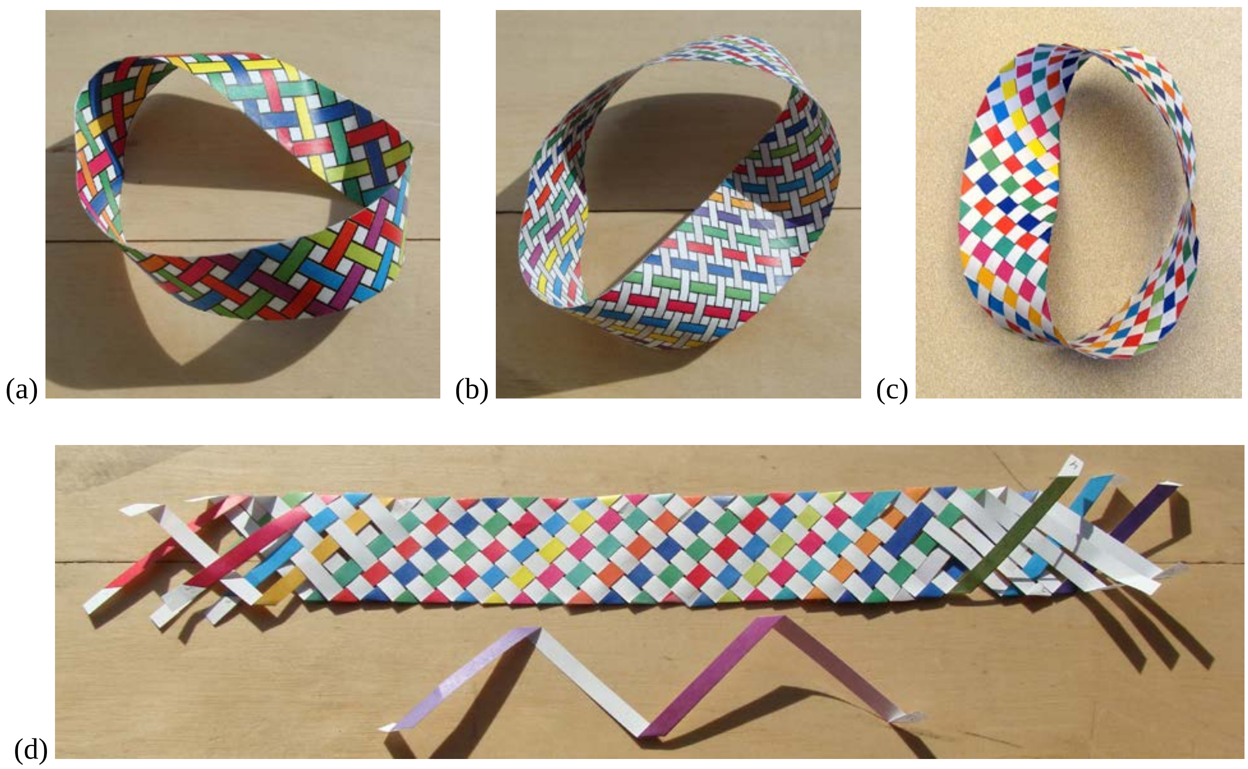

Figure 5: *Paper knots: (a) model with a double-sided thong, p=4, b=21; (b) using a single-sided thong, p=8, b=31; (c, d) a woven Möbius-twisted Turk's Head knot made from selectively colored paper strips.*

Discussion and Conclusions

We believe that this paper describes the first realization of a Möbius-twisted Turk's Head knot. This novel knot may not have any immediate use in the gear that one can find around a horse's body; but it certainly is an intriguing and aesthetically pleasing object, and it was a delightful challenge to develop a practical technique for its construction after the conceptual idea had first come up. Strange and wonderful things can happen when one brings together a mathematician and a cowboy!

References

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