A Workshop on Making Klein Bottle using 4D Frame

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Abstract

In this workshop we will show how to make a Klein Bottle using 4D Frame which invented for learning Math and Science. We will bring the material for participants to experience the activity and make it without using paper and glue. This activity will help people to have better understanding regard the Mobius strip and Klein bottle. Once we decide the radius for the bottle then calculate each circumference by using π.

Introduction

When traveling by train, you may have experienced being obsessed by thinking that the train is moving backwards if you woke up from dozing. Of course, you will soon realize that it's not possible. When following along the face of triangle shape, cylinder and sphere, it will keep the same direction. The directions like front and back don't change in real world. However, there is an extraordinary surface that the direction is changed and it is called a Mobius strip.

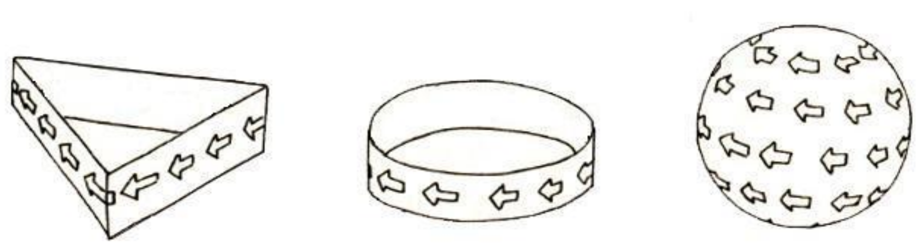


Figure 1: *Examples of orientable surface*

General Mathematical Definition of Mobius Strip and Klein Bottle

Mobius strip is made by splicing two sides of a rectangle, $\overline{AB}$ and $\overline{CD}$ by attaching A to C and B to D as below **Figure 2**. It was named after August Ferdinand Mobius (1790~1868), a mathematician from Germany.

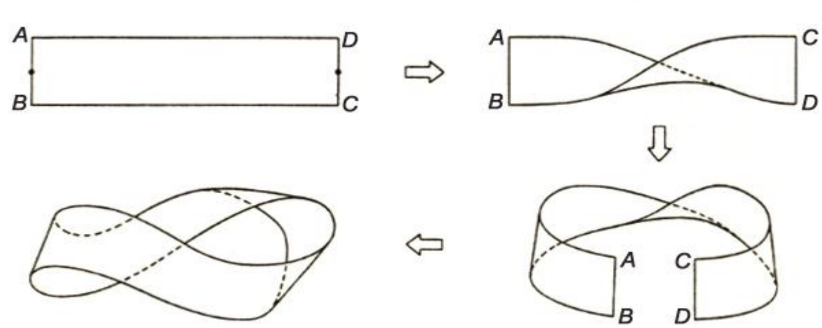


Figure 2: *Making Mobius Strip*

It is well known that this Mobius strip has only one surface while cylindrical strip having two surfaces, front and back. In other words, you can see all the curved surface of Mobius strip when you follow the strip round. Its applications in real world are easily observed around us. For examples, belts of machines in the mill or factory, cassette tapes of audio, Korean traditional knots **Figure 3**, and roller coasters you enjoy at the amusement park. Also, double helix of DNA (same as bisection of Mobius strip) can be another example. It was discovered that the genetic factors present in the form of double helix in early 1960s, however, it was very difficult to uncoil the tangled double helix to duplicate it. In 1981, Professor Walba in Colorado University discovered that the DNA has the same form of bisected Mobius strip. Other than that, Mobius strip is often used in artworks. A famous Dutch artist, Escher has made many famous paintings with the motive of Mobius strip **Figure 3**. Also, it is found in a famous Korean novel as well which is [A small ball shoot up by a dwarf] by Se-Hee Cho. It is a series of dozen stories of which first one is called, Mobius strip. A famous mobile service provider in Korea has recently adapted Mobius strip in T form as their logo as you can see from **Figure 3**. It seems they use the meaning of Mobius strip efficiently.

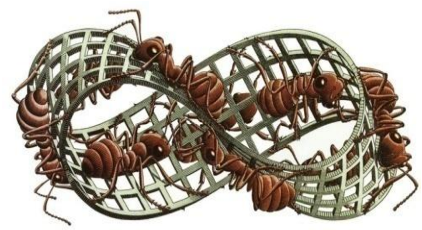
Korean traditional knot	**Mobile service provider logo**	**Mobius stripe Ⅱ by Escher**
 (a)	 (b)	 (c)

Figure 3: *Application of Mobius strip*

A German mathematician, Christian Felix Klein (1849~1925) has presented of a four dimensional solid in bottle shape of which the inside and outside is connected by gluing the ends of Mobius strip in three dimensional space. It is called the Klein bottle after his name. Since its inside and outside is basically one thing, the structure has only one entry and exit. So the water will be poured out when you pour the water into the bottle neck. Also, an explorer can go into the cave and come back through the entry he/she has already passed if there is a cave in the structure of the Klein bottle. In other words, one cannot move from one space to another and eventually come back to the same place no matter how much he/she tries to escape. It is the mathematical concept which proves that passing the wall without opening the door as shown in the movie, 'Harry potter and the philosopher's stone'.

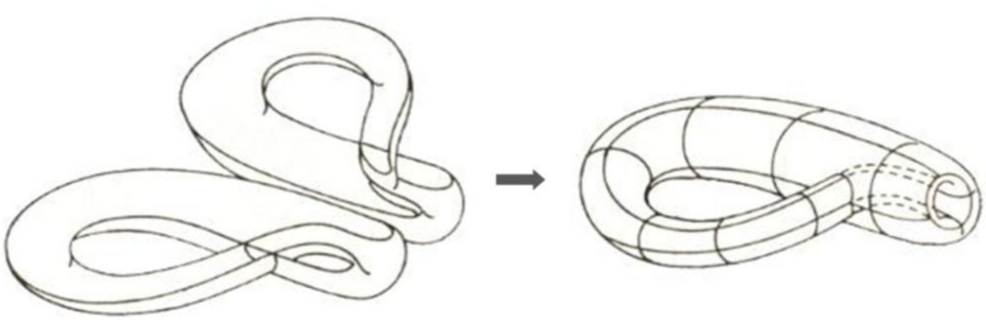
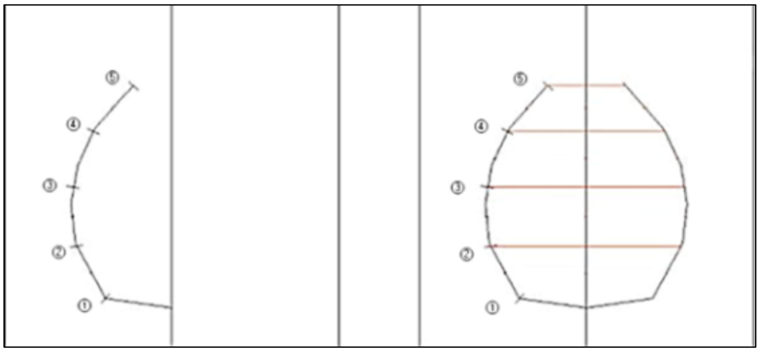


Figure 4: *Two Mobius strips and Klein bottle*

Drawing of the Klein Bottle and Application of 4D Frame

(1) Making the body of the Klein bottle – egg-shaped structure



(a)

①, ⑤ : 7.7 × 3.14 ÷ **8** ≒ 3

④ : 12.7 × 3.14 ÷ **8** ≒ 5

②, ③ : 15.3 × 3.14 ÷ **8** ≒ 6

(b)

Figure 5: *Blue print of Klein bottle*

First, draw a line on the center of a piece of paper and curved line arbitrarily based on the center. Make regular intervals on the curve and mark numbers on it. Estimate the radius and diameter based on the center point and connect the line on the opposite side symmetrically. Then calculate the circumference by multiplying diameter by 3.14.

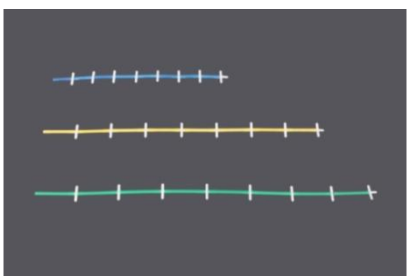
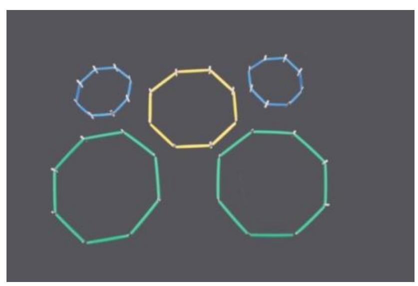
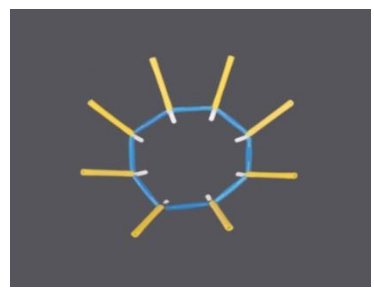
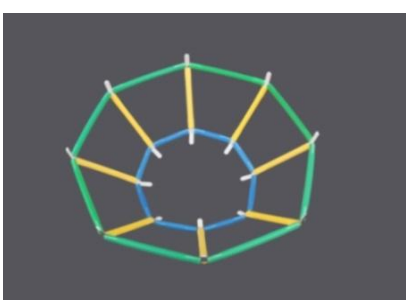
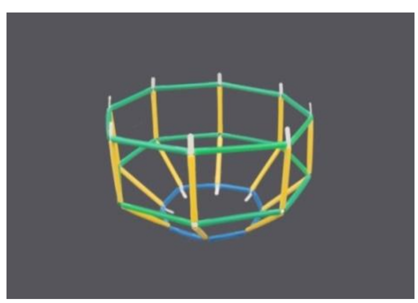
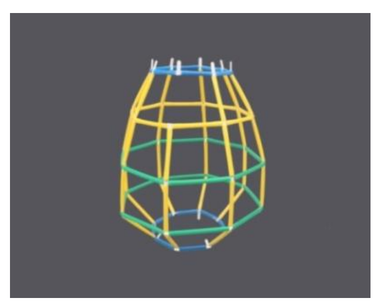
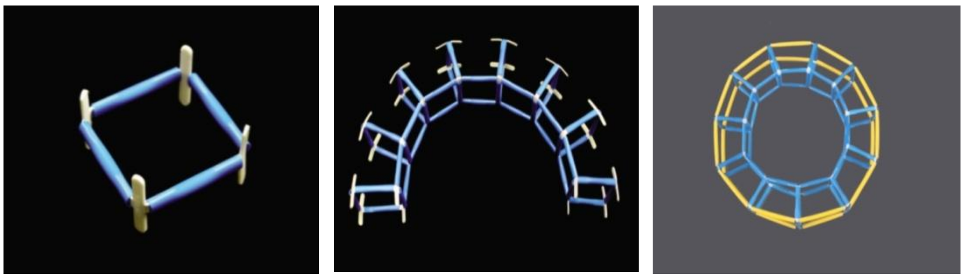
		
(a) Connect 8 of 3cm, 5cm and 6cm frames with 4pod as above picture**.**	**(b)** Connect **(a)** like above and make circles ① to ⑤ from **Figure 5-b**.	**(c)** Connect 5cm frames to one of small circle from **(b)**.
		
(d) *Connect circle ② to* **(b).**	**(e)** *Connect circle ③, ④ and ⑤ to* **(d)** *by same way***.**	**(f)** *Finished the body part of Klein bbottle***.**

Figure 6: *Making the body of the Klein bottle*

Divide the circumference by constant number to cut frames for body part like **Figure 5** *(b)*. Then you can follow the instruction from **Figure 6** to complete the body part of Klein bottle. Instead of calculating circumference using 3.14, you can simply decide to use random numbers. Separate each set of symmetrical rings by cutting the connecting line of the relevant ring number. Find the connecting line of each length of the frame and connect the curves each other, from the opposite side to make it a center point and then connecting the rings keeping the balance. Connect the frames around the central curve and then connect the rest of the rings in constant size from the upper and lower side of the central ring as long as they are making the curve.

(2) Making the neck of Klein bottle from a torus structure



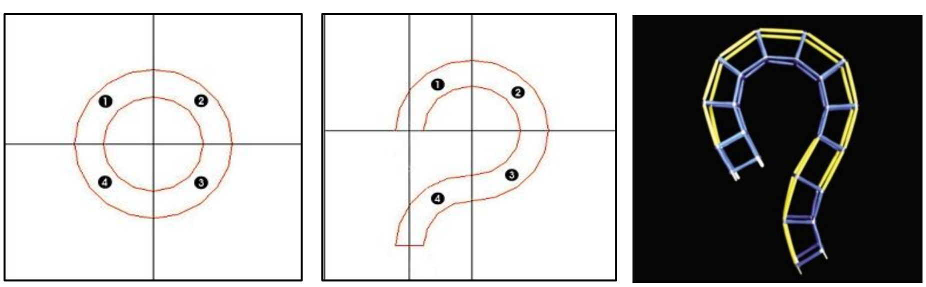
(a) (b) (c)

Figure 7: *Making a torus structure*

A torus can be defined parametrically as

$$(x, y, z) = ((R+r\cos\varphi)\cos\theta, (R+r\cos\varphi)\sin\theta, r\sin\theta)$$

where φ and θ are angles which make a full circle, starting at 0 and ending at 2π, so that their values start and end at the same point, R is the distance from the center of the tube to the center of the torus, r is the radius of the tube. Its surface area and interior volume are easily computed using the Pappus' Centroid Theorem[1] giving Area as $A = 4\pi^2 rR$ and Volume as $V = 2\pi^2 Rr^2$[1]. For making neck part of the Klein bottle, you should choose a polygon with even numbers (square, hexagon) first. Once you choose the shape then make 11 polygons with 4pod like figure 6-a. Then connect them with frames for inner circle of torus like (b) and connect outer circle with longer frames than inner circle. When you work with outer circle to complete a torus structure, connect every other pod first to have exact shape. Then connect rest of them with checking shape and balance to have a proper one.



(a) (b) (c)

Figure 8: *Change torus structure to question mark shape*

Draw two lines like (a) and disconnect number 4 then twist it to have the question mark shape from (c). If needed, you can change some frames to keep the proper shape.

(3) Connecting neck and body part together

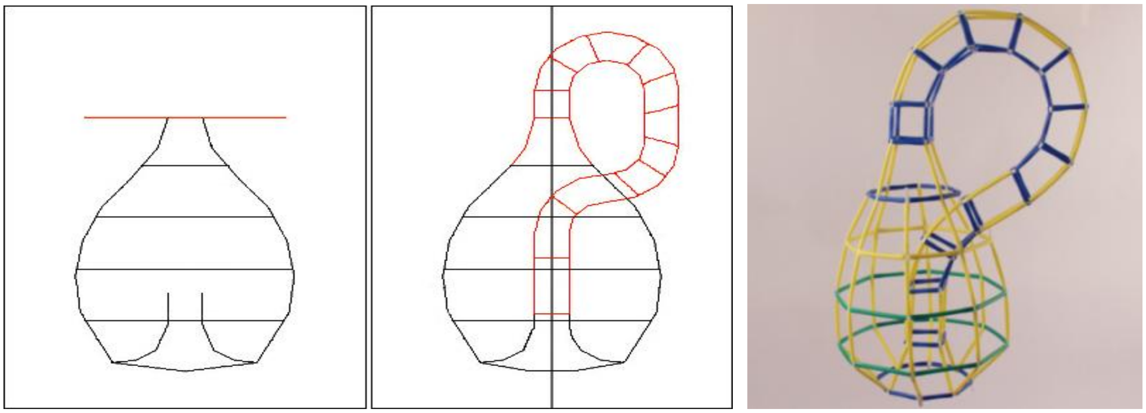


Figure 9: *Klein Bottle*

Connect the one end of the torus to the bottom part and the other to the top of the body part. When connecting the body part and question mark shape together, cut the length of the frame if needed.

(4) Finding Mobius strip from Klein Bottle

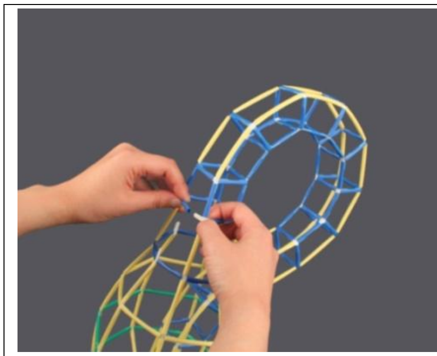
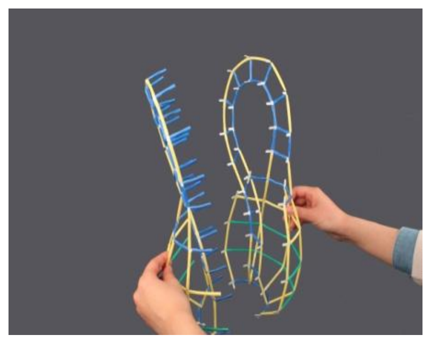
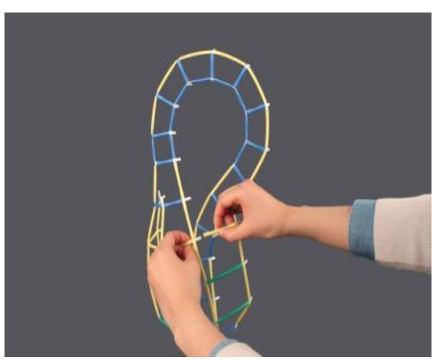
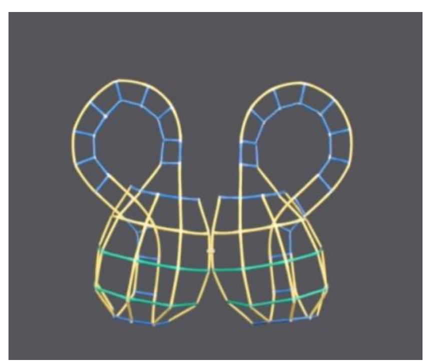
	
1. Disconnect frames at the center of Klein Bottle.	2. Separate Klein Bottle in half.
	
3. Remove frames and cut pod after to see Mobius strip clearly.	4. Use C-shape connector to connect two Mobius strip.

Figure 10: *Cutting Klein Bottle to see Mobius strip*

(4) Various forms of the Klein bottle made with 4D Frame

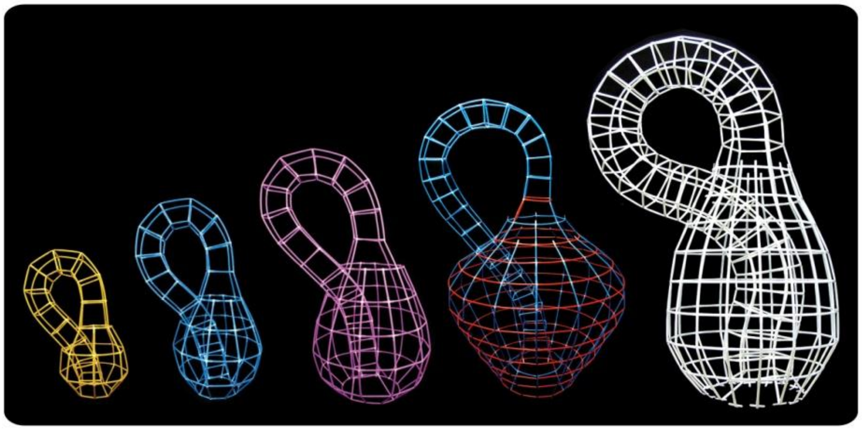


Figure 11: *Various Klein Bottle[2]*

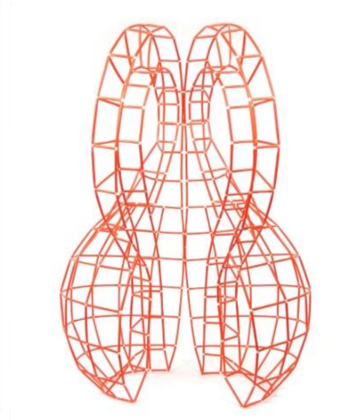


Figure 12: *Two Moubius strip in a Klein Bottle*

References

[1] A Workshop on N-regular Polygon Torus by Ho Gul Park, Bridges Enschede, Proceedings 2013
[2] The 3rd soil, Ho-Gul Park, 4D Math and science Creativity Institute
[3] Weisstein, Eric W. "Klein Bottle." From *MathWorld*--A Wolfram Web Resource, http://mathworld.wolfram.com/KleinBottle.htm

[1] In mathematics, Pappus' centroid theorem (also known as the Guldinus theorem, Pappus–Guldinus theorem or Pappus' theorem) is either of two related theorems dealing with the surface areas and volumes of surfaces and solids of revolution.