On Colouring Sequences of Digital Roots

Gabriele Gelatti
Italy
gabrigelatti@gmail.com
www.gabrigelatti.it

Abstract
In this paper we introduce a logic of transforming integers from 1 to 9 into colours, and present a visual method of seeing the periods of digital roots in Fibonacci-like sequences.

Transforming Numbers into Colours

The *RYB* (*Red-Yellow-Blue*) *colours model* [1] is formed by a triad of *primary* colours (*yellow*, *red*, *blue*), and a triad of *secondary* colours (*orange*, *green*, *purple*) deriving from the mix of *primary* colours. We apply it to describe some properties of positive integers from 1 to 9. The logical model is completed with the addition of a third triad of *achromatic* colours (*black, white* and *grey*). We match colours to numbers as: 1 = *yellow;* 2 = *orange;* 3 = *white;* 4 = *red;* 5 = *green;* 6 = *black;* 7 = *blue;* 8 = *purple;* 9 = *grey.*

Figure 1: *Numbers and colours.*

Digital Root

We use the operation of *digital root* [2] to reduce any positive integer bigger than 9 to a one cypher number. For example: 123 = 1 + 2 + 3 = 6. This can also be done by dividing any positive integer by 9 and considering its remainder (e.g.: 123 / 9 = 13,*666*...). Multiples of 9 leave no remainder. All positive integers receive a given colour according to the above *chromatic* rules. Assigning colours to *digital roots* is equivalent to assigning colours to the values *modulo 9* [3], considering *grey* = 0, as 9 ≡ 0 (mod *9*).

Complementarity and Sequences of Integers

The above Figure 1 shows the triads of colours (1, 4, 7 = *primary* colours; 2, 5, 8 = *secondary* colours; 3, 6, 9 = *achromatic* colours) combined in couples of symmetric *complementary* colours, except number 9.

The mix of each *complementary* couple, according to the *RYB colours model*, produces the neutral colour *grey*; in the same way the addition of the couples of numbers *1+8*, *2+7*, *3+6, 4+5* makes always 9.

The logical rules of *RYB colours model* and the *digital root* operation are here combined to create a "chromatic arithmetic". The pattern in the Figure 2 displays the symmetries of colours *complementarity* in the *digital root* reduction of the *multiplication table.*

Chromatic Analysis of Fibonacci-like Sequences

The rules of "chromatic arithmetic" applied to Fibonacci [4] and Lucas [5] sequences display:
Fibonacci = ... 2, *3*, 5, 8... (*secondary* colours + *3*); Lucas =... 1, *3*, 4, 7... (*primary* colours + *3*).

We align the *digital roots* of Fibonacci and Lucas sequences on the common term "3":
1, 2, *3*, 5, 8, 4, 3, 7, 1, 8, 9, 8, 8, 7, 6, 4, 1, 5, 6, 2, 8, 1, 9, 1, 1... ;
2, 1, *3*, 4, 7, 2, 9, 2, 2, 4, 6, 1, 7, 8, 6, 5, 2, 7, 9, 7, 7, 5, 3, 8, 2... .

The sum in column of the terms of the two sequences creates a third sequence of *digital roots*: 3, 3, 6, 9, 6, 6, 3, 9, 3, 3, 6, 9, 6, 6, 3, 9, 3, 3, 6, 9, 6, 6, 3, 9, 3... entirely made of *achromatic* colours.

The three sequences could also be simplified as "adding 3 to numbers 1, 2 and 3".

1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
4	8	3	7	2	6	1	5	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
7	5	3	1	8	6	4	2	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9

Figure 2: *Chromatic multiplication table.*

1	2	3	4	5	6	7	8	9
3	3	3	3	3	3	3	3	3
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
2	4	6	8	1	3	5	7	9
9	3	6	9	3	6	9	3	6
2	7	3	8	4	9	5	1	6
2	1	9	8	7	6	5	4	3
4	8	3	7	2	6	1	5	9
6	9	3	6	9	3	6	9	3
1	8	6	4	2	9	7	5	3
7	8	9	1	2	3	4	5	6
8	7	6	5	4	3	2	1	9
6	6	6	6	6	6	6	6	6
5	4	3	2	1	9	8	7	6
2	1	9	8	7	6	5	4	3
7	5	3	1	8	6	4	2	9
9	6	3	9	6	3	9	6	3
7	2	6	1	5	9	4	8	3
7	8	9	1	2	3	4	5	6
5	1	6	2	7	3	8	4	9
3	9	6	3	9	6	3	9	6
8	1	3	5	7	9	2	4	6
2	1	9	8	7	6	5	4	3

Figure 3: *Tables of Fibonacci-like sequences, starting by adding 3 to numbers from 1 to 9.*

1	2	3	4	5	6	7	8	9
9	9	9	9	9	9	9	9	9
1	2	3	4	5	6	7	8	9
1	2	3	4	5	6	7	8	9
2	4	6	8	1	3	5	7	9
3	6	9	3	6	9	3	6	9
5	1	6	2	7	3	8	4	9
8	7	6	5	4	3	2	1	9
4	8	3	7	2	6	1	5	9
3	6	9	3	6	9	3	6	9
7	5	3	1	8	6	4	2	9
1	2	3	4	5	6	7	8	9
8	7	6	5	4	3	2	1	9
9	9	9	9	9	9	9	9	9
8	7	6	5	4	3	2	1	9
8	7	6	5	4	3	2	1	9
7	5	3	1	8	6	4	2	9
6	3	9	6	3	9	6	3	9
4	8	3	7	2	6	1	5	9
1	2	3	4	5	6	7	8	9
5	1	6	2	7	3	8	4	9
6	3	9	6	3	9	6	3	9
2	4	6	8	1	3	5	7	9
8	7	6	5	4	3	2	1	9

Figure 4: *Tables of Fibonacci-like sequences, starting by adding 9 to numbers from 1 to 9.*

We then start any sequence by adding 3 to all numbers from 1 to 9 (Figure 3): a new *chromatic* sequence appears only in column 7 and 8. Three *chromatic* columns repeat twice (1 and 5, 2 and 4, 7 and 8 are equivalent). They are made of all colours and have a *Pisano period* [6] of 24, divided into two *complementary* periods of 12. The *achromatic* column repeats with a *Pisano period* of 8, divided into two *complementary* periods of 4. For the given equivalence of *digital roots* with *modulo 9,* only by adding 9 (Figure 4) to all numbers from 1 to 9, we produce a new column made of *grey* and *Pisano period* of 1.

From these observations we can derive a *theorem* of "chromatic arithmetic".

Theorem. All the possible Fibonacci-like sequences of *digital roots* are described by only five sequences of colours: three are made of all colours, one of *achromatic* colours and one of only *grey.*

Proof of theorem. The patterns of the *chromatic tables* of Figures 3 and 4, along with general rules of *recursive addition*, *digital root* reduction, and *modulo 9* arithmetic, provide a visual mathematical proof.

References

[1] Goethe, *Zur Farbenlehre.* Cotta, 1810; also Itten, *Kunst der Farbe*, Verlag, 1961.

[2] http://mathworld.wolfram.com/DigitalRoot.htmla, (as of Feb. 2, 2014).

[3] http://en.wikipedia.org/wiki/Modular_arithmetic, (as of Apr. 13, 2014).

[4] Fibonacci, *Liber Abaci*, Springer, 2003.

[5] http://oeis.org/A000032, (as of Feb. 2, 2014).

[6] http://en.wikipedia.org/wiki/Pisano_period, (as of Apr. 13, 2014).