Non-periodic Tiles Based on Ammann Set A2 Tiles

Chirag Mehta
chimehta@gmail.com

Abstract

I present here three different non-periodic tile sets TS1, TS2 and TS3. Tile set TS1 is deduced from the Ammann set A2, and is a triangular version of it. TS2 is a variant of tile set TS1, while TS3and TS2 are MLD (Mutually Locally Derivable). These tiles are based on Golden Ratios Ø = 0.618034 and φ =1.618034.

Introduction

Periodic tiling is one in which you can outline a region that tiles the plane by translation, that is, shifting the position of the region without rotating or reflecting it. [1] Non-periodic tiling is one that does not repeat and cannot be constructed from a single primitive cell by pure translation. The tile sets presented here are obtained by tiles that group together to form larger replicas of themselves.

Derivation from Ammann Set A2

The prototiles of Ammann set A2 consist of two tiles A-tile which is primarily a unit square, with a square of side 1-*Ø* deducted form its corner; while B-tile is a unit square with an additional square of side *Ø* attached to one of its side[2]. When similar configuration is done to a unit equilateral triangle we get tiles 'a' and 'b' respectively which constitute the tile set TS1 (Figure 1)**.**

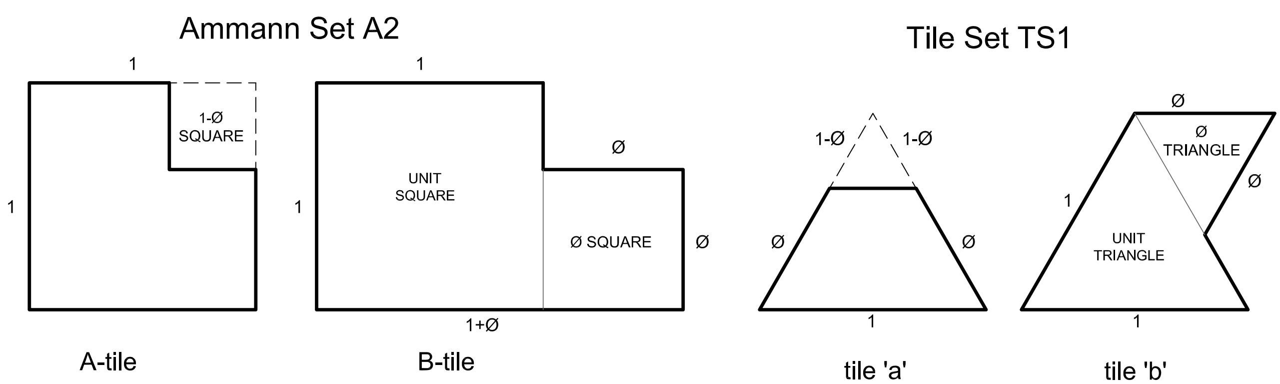


Figure 1: *Derivation of TS1 from Ammann Set A2.*

Tile Set TS1

Tile set TS1 comprises of three tiles ‘a’, ‘b’and ‘b-’. Tile‘b-’ is mirror image of ‘b’. (Tile names with a negative sign suffix shall indicate that they are mirror tiles). The dimensions of these tiles are indicated in Figure 2. Tiles ‘a’,’b’ and ‘b-’ can be combined to form larger self-similar super-tiles. Tile ‘A’=‘a’ + ‘b-’ ;‘B’=’a’ + ’b’ + ’b-’ (Figure 3). The ratio of areas of individual tile to its super-tile i.e. a/A= $Ø^2$ while the ratio of smaller tile to larger tile i.e a/b =*Ø*.

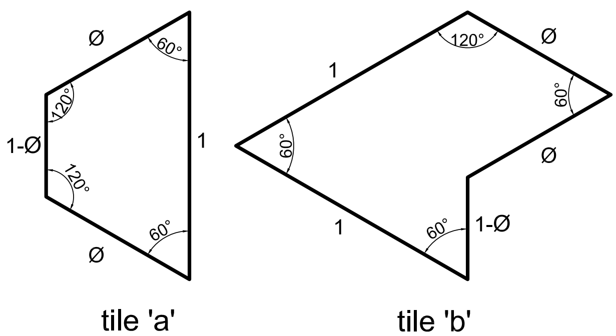


Figure 2: *Dimensions of tile set TS1.*

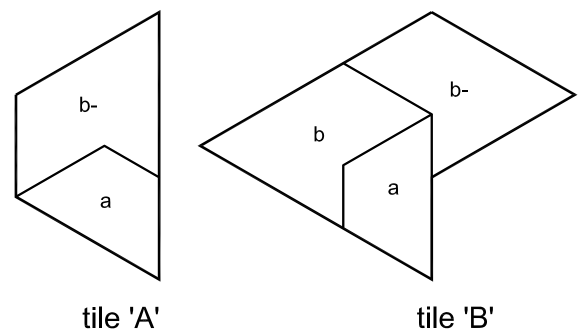


Figure 3: *Formation of supertiles in set TS1*

Tile Set TS2

Tile set TS2 comprise of three tiles ‘c’,‘d’,’e’, and their mirror tiles ‘c-’, ‘d-’ and ‘e-’. It is a simple variant of tile set TS1, constructed by adapting it to a 45° grid rather than 60°. The dimensions and geometry of these tiles are elaborated in Figure 4, while Figure 5, shows the arrangement for formation of its supertiles.

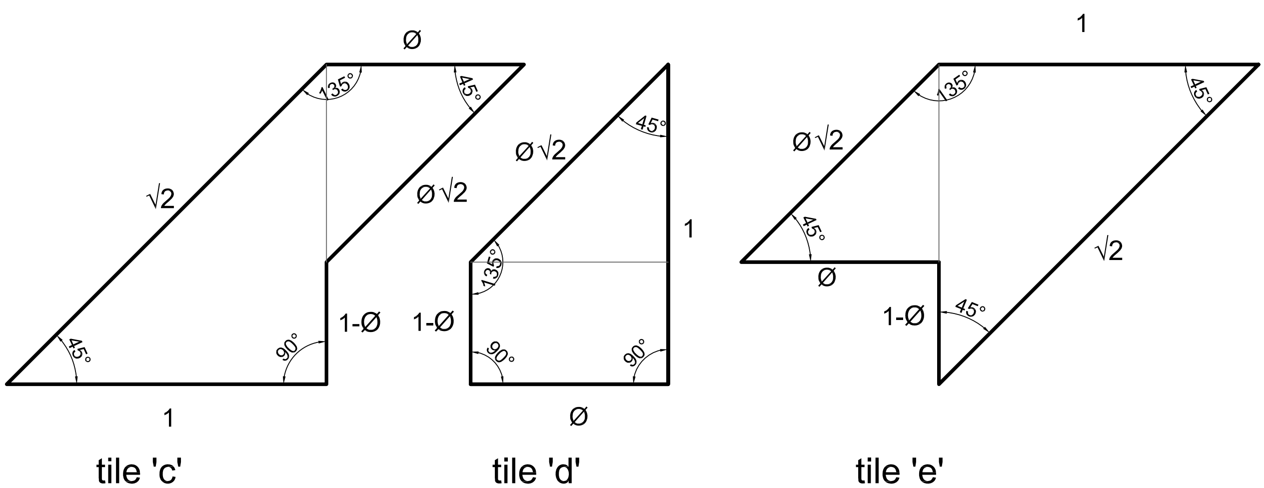


Figure 4: *Dimensions of tile set TS2.*

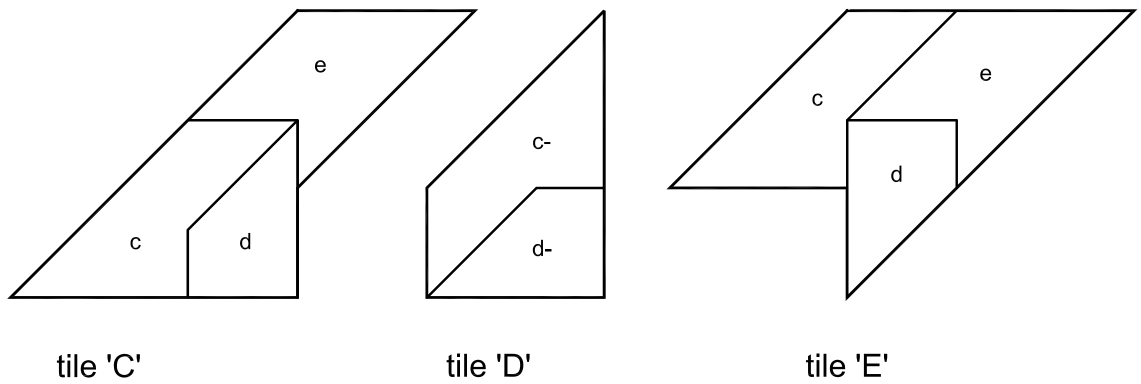


Figure 5: *Formation of supertiles in tile set TS2.*

MLD tile sets TS2 & TS3

Tile set TS2 and TS3 are MLD [3]**.** The tile 'c' of TS2 is modified into a triangle by deleting two of its vertexes to form tile 'g'; tile 'e' is transformed into 'g-' and 'd' looses its one vertex to become tile 'f'. Thus a new tile set TS3(as shown by dashed line in Figure 6) is formed which comprised of the tiles 'g', 'f', and 'g-' .

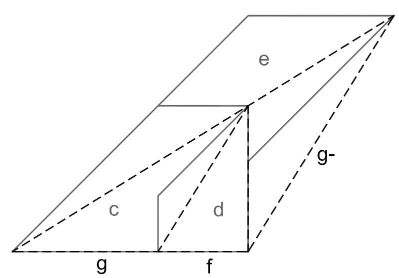


Figure 6: *Mutually Locally Derivable Tile Set TS2 and TS3.*

Tile Set TS3

Tile set TS3 comprises of three tiles 'f','g', and their mirror images 'f-', and 'g-'. These set of tiles and tile set TS2 are Mutually Locally Derivable (refer Figure 6). The dimensions of these tiles are explained in Figure 7, while Figure 8, shows the arrangement for formation of its supertiles.

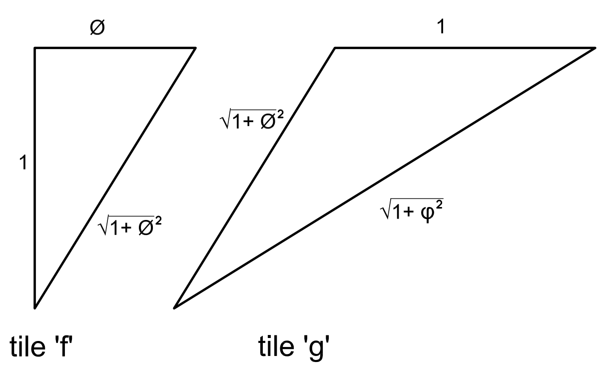


Figure 7: *Dimensions of tile set TS3.*

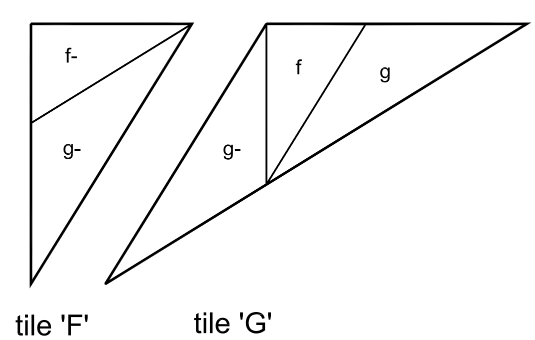
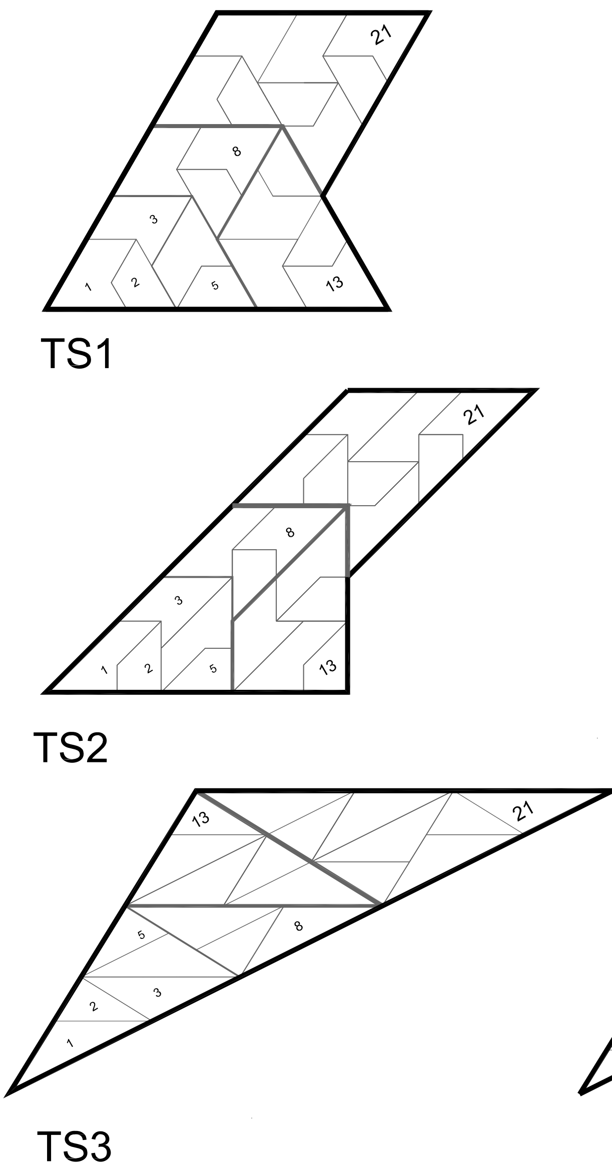


Figure 8: *Formation of supertiles in tile set* TS3

Occurrence of Fibonacci series in Tile Sets.

Since the geometry of tiles embeds golden ratio, the occurrence of Fibonacci series is natural here. The sequence can be traces as follows in TS1:- Starting from base tile ‘b’ at bottom left corner we count the number of total tiles required to form the next super-tile ‘B’, ‘A’, or ‘B-’. We observe that we require 2 tiles to form super tile ‘A’. 3 tiles to form ‘B’. 5 tiles to form ‘A’, 8 tiles to make tile B, so on and so forth.



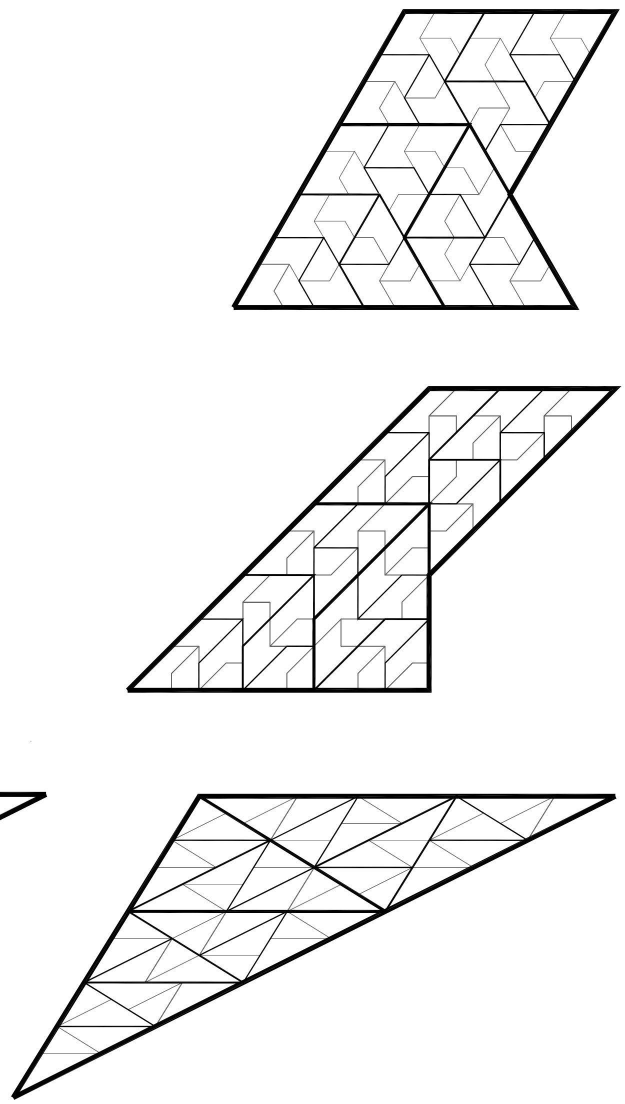


Figure 9: *Fibonacci Sequence in tiling.* **Figure 10**: *Sample set for Tiling.*

References

[1] Penrose tiling, http://math.uchicago.edu/~mann/penrose.pdf (as of March 16, 2014)
[2] Ammann R., Grünbaum B., Shephard G. C., Aperiodic tiles, Discrete & Computational Geometry, 8:1 (1992), 1-25.
[3] Baake M. et al., Quasiperiodic tiling with tenfold symmetry and equivalence with respect to local derivability, J. Phys. A: Math. Gen. 24, 4637 (1991).