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*Mathematics, Music, Art, Architecture, Culture*

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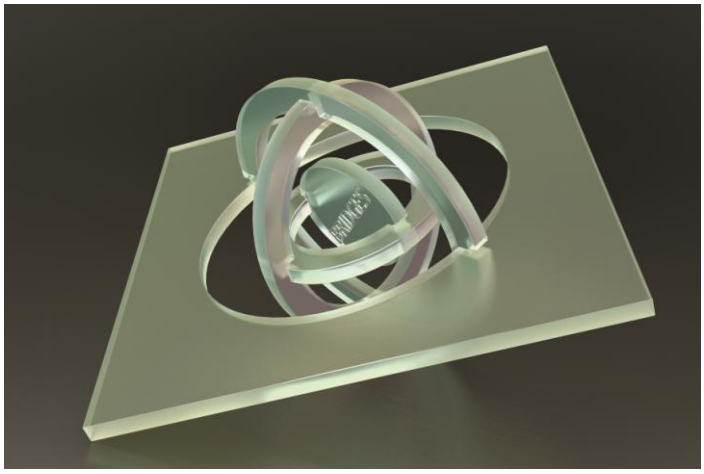

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## Preface

The International Bridges Conference, created in 1998 and running annually since, provides a model of how to integrate the seemingly diverse disciplines of mathematics and the arts. Practicing mathematicians, scientists, artists, teachers, musicians, writers, computer scientists, sculptors, dancers, weavers, and model builders come together in a lively and highly charged atmosphere of mutual exchange and encouragement.

After visiting many countries, including a flourishing and fruitful Bridges 2008 Conference in Leeuwarden, The Netherlands, the city where M.C. Escher was born, this year Bridges returns to the Netherlands, but now to the city of Enschede, one of the most attractive and cultural cities in the eastern part of the country, the province of Overjssel, and the Twente region.

The Universiteit Twente (University of Twente), a university with mostly technical studies, is located in Enschede. It's one of the three technical universities in the Netherlands (besides Delft University of Technology and Eindhoven University of Technology). The Universiteit Twente is also the only large campus university in the Netherlands.

Enschede is also home to one of the three campuses of Saxion University of Applied Sciences (Saxion Hogeschool Enschede), a polytechnical school offering internationally recognized Bachelor's degrees and Master's degrees in a wide range of fields, including engineering, economics, and health care. The other campuses are located in Deventer and Apeldoorn.

Enschede also has an academy of arts and design combined with a conservatory, named ArtEZ.

The reviewing process for the Bridges Conference Proceedings for this year, was chaired by George Hart, who led a diverse program committee of forty experts from around the world in a rigorous review of these 115 papers in three categories: regular papers, short papers, and workshop papers. The program committee in turn obtained the assistance of additional expert reviewers. Mara Alagic, Bob Bosch, Paul Gailiunas, Craig Kaplan, Douglas M. McKenna, Reza Sarhangi, and Carlo Séquin generously provided extra support. We thank all the authors and these many volunteers for their careful work, which made possible this volume you are holding.

**The Bridges Organization**
bridgesmathart.org