Hexagons and Their Inner World

Dániel Erdély
Option.hu Ltd.
Hársfa St. 8/b, 2011 Budakalász, Hungary
E-mail: edan@option.hu

Abstract

The hexagons I am going to present some findings about below are regular or semi-regular. Some are spatial, some are planar. The angles of their plane projections are either all equal or alternate between two values. They are also distinguished by their inner worlds. The spatial ones may be possible to flatten into the plane, mobile or rigid. In the table below, I attempted to classify the group of hexagons I am interested in by their properties so as to make it easier to compare Spidrons and other hexagonal surfaces.

Dimension	2D		3D				
Kind	Regular (RH)	Semiregular (SH)	2-fold	6-fold	Escher's	Semiregular Sp	Spidron
Angles	$\alpha = 120°$	$(\alpha + \beta) = 240°$	$g_1 = 120°$ $g_2 < 120°$	$\alpha, \beta < 120°$	$g_1 = g_2 = f$ $f < 180°$	$g_1 \neq g_2 < 180°$	$g_1 = g_2 = f$ $f < 180°$
Tessellates	2D	–	3D	–	3D	–	3D
Rotation	6-fold	3-fold	2-fold	3-fold	3-fold	3-fold	3-fold
Symmetry	Mirror	Mirror	Chiral	Chiral	Enantiomorph	–	Enantiomorph
Origin	–	–	RH	RH	–	SH	RH
Center	360°	360°	360°	360°	424,8°	Hole going flat	Hole going flat
Bound in	–	–	Hemisphere	Hemisphere	Sphere	Semiellipsoid	Ellipsoid
Foldability	–	–	Yes	Yes	Limited	Limited	Limited
Movable relief	–	–	Yes	With 2-fold	–	–	Yes

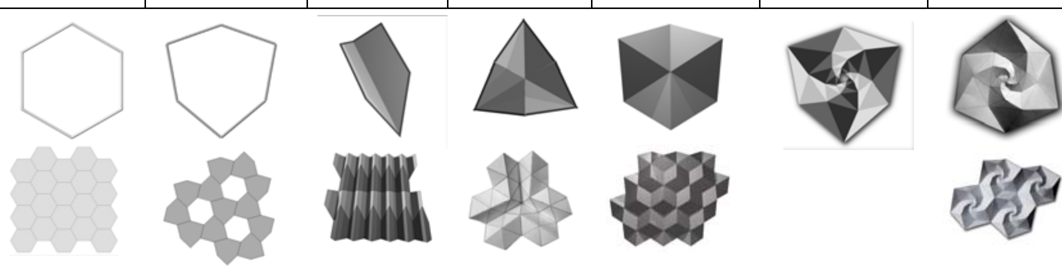

Table 1 Hexagons with equal edges & rotational axis
α , β – internal angles in the plane, g_1, g_2, f – internal angles in space, RH – Regular hexagon, SH – Semi-regular hexagon

An additional point about Spidrons

We can make an interesting observation about Spidron plates. For that, let's remind ourselves of some of the known properties of the classic Spidron nest. Spidrons are composed of rings, that is to say one way to form them is to keep adding rings consisting of 12 triangles (6 equilateral ones and 6 120° isosceles ones) inside and outside each other [1]. As in the general case, the angles between the rings and the base plane keep increasing as the size of the rings increases, the process will only continue indefinitely if the rings remain in the plane. Otherwise, we soon reach the physical limit of adding further rings on the outside, because the triangles forming the ring would cross each other in space. In the direction of smaller rings, molecular size is the limit of physical viability. So is the Spidron a plane or a physical, relief-like shape? Let's look at the question a little more closely. Let's take a finite set of folded Spidron rings consisting of adjacent rings. Let's say, arbitrarily, that the largest ring we are observing is infinitely large and at an infinite distance from the center. *(If we were to measure distance in the number of rings inside it rather than in length, we would indeed be at an infinite distance from the center.)* It is known that the triangles composing that furthest ring would touch, they would be in the same plane. [2] In that position, the outer edges of the isosceles triangles are at a 60°, while the inner edges of the equilateral ones are at a 30° angle with the base plane. This, in theory, characterizes the infinitely distant ring. Although we know that all the rings are similar to each other as long as the entire Spidron is laid out in the plane, in the deformed (but not distorted!) state the Spidron is not a plane figure, so only the smallest, last ring can be completely plane. But as, similarly to the largest one, the smallest ring is also "inaccessible", the only thing we can do is to say, arbitrarily, of one of the plane rings that it is the last one in the center and its inner edges have zero length. [3] But if we have some statements, facts about the smallest and the largest ring, and we also know how the angles of intermediate rings vary with the base plane, we must reach some rather hairy conclusions: the Spidron plate and the so-called "Euclidian plane" are, so to speak, "identical". This also means that something assumed, or defined, or "axiomatically" defined as smooth "doesn't suffer any damage" if it is replaced by a Spidron nest that can be spread out in the plane. (This also shows that a "real" (i.e. not ideal) Euclidian plane is only "able" to be realized in the middle of the Spidron (where it necessarily has a hole in it). If we consider an increasing number of rings to be flat, we sort of expand the space, increase the size of the Spidron plate, but its character doesn't change at all. We could also say that despite defining the plane as flat, it could also have a Spidron characteristic, which is not necessarily plane. But the "ideal" plane can never have a Spidron characteristic. That's not how we imagine it, that's not how we are used to thinking of it. It doesn't come into the picture. The same way we never think of an ideal geometric point or straight line, or even a circle of sphere as rotating, while we could still be performing operations on them just as we do on their static versions.

Time-fractal

Let's pay attention to the following: let's observe a ring that is not too close to the center. It has quite a big *f* angle. Let's say it is 20° and the next ring inside it has an *f* angle is 16° If we pull the plate out and make it flatter and flatter, after a short period of time this ring will reach a state with an angle of exactly 16°, the state its inner neighbor had at the previous stage and all of the smaller rings will have the same *f* as the bigger neighbor rings had before. And the next bigger ring will have a 20° *f* angle. Is that not interesting? It is like a miracle, which made our object 3 times smaller than it was previously. *(Because the ratio between the rings' outer and inner edges is $\sqrt{3}$.)* [4] We can call it a time-fractal, because the whole figure turns into itself periodically, in a smaller or bigger size.

References

[1] **Erdély, D :** *Spidron System*: *A flexible Space-Filling Structure*, POLYHEDRA- *Symmetry: Culture and Science,* International Symmetry Foundation, 2004.

[2] **Erdély, D**: *Some Surprising New Properties of Spidrons*> Renaissance Bridges proceedings, Banff, Canada p.179 –186. 2005

[3] **Szilassi, Lajos**: *The Right for Doubting - and the Necessity of Doubt - Thoughts Concerning the Analysis of Erdély's Spidron System;* Procceding of the "Sprout-Selecting" Conference: Computer Algebra Systems ans Dynamic Geometry Systems in Mathematics Teaching; Pécs, Hungary, 2004 Edited by Csaba Sárvári, p.78–96.

[4] **Erdély, D**: *Concept of Spidron System*; Proceedings of the "Sprout-Selecting" Conference: Computer Algebra Systems and Dynamic Geometry Systems in Mathematics Teaching; Pécs, Hungary, 2004 Edited by Csaba Sárvári, p. 68–77.

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