How Do Symmetries Come To Children, and Vice Versa?

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Abstract

This paper showcases mathematics taught during art lessons at lower secondary school, where children selected mathematics as the best instrument to reach their selected artistic objectives. Children connected basic geometry transformations with artistic expression, then described and (on certain level) formalized them, and finally, applied their discoveries to create personal monograms. This approach has tangential connections with contextual learning, and operates by fostering algorithmical thinking and strong emotional motivation. It can be also seen as a complementary example of Papert´s MediaLab studies.

Introduction

There are various ways to introduce mathematics to children. Among these we can look for objectives or goals to be learnt in the relation to instruments to be used in the given learning environment. Such analysis brings us to our first Figure (Fig. 1.)

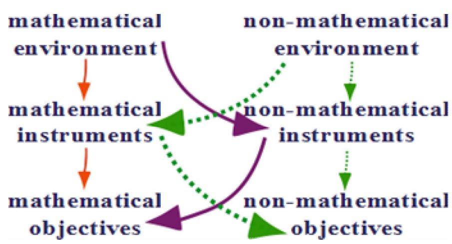

Figure 1: *Four paths of school teaching/learning*

There are four paths available to visualise fundamental learning situations. Two of the paths shown above feature small arrows represent a typical approach. In these we can perfectly distinguish mathematical and non-mathematical academic subjects simply by labelling them as such. In this case, there is no trace of interdisciplinarity or any other contextual connection between the mathematical and non-mathematical environment, instruments or objectives. We are teaching/learning mathematical objectives using mathematical instruments in the mathematical environment – during subject matter lessons. On the other side, working in a non-mathematical environment means teaching/learning non-mathematical objectives with the help of non-mathematical instruments.

These distinct paths have already been criticised since the time of Comenius [1] who considered them "not too efficient" regarding leared content and their possible application. Yet they are still largely employed at all educational levels, in almost any corner of the world. My personal experiences show various examples of this split, one of them repeated almost in any group questioned: Children often don't see any relationship between graphs of linear function, learnt during their mathematics lessons, and graphs tracing the trajectory/time dependence of uniform rectilinear movement, taught during physics lessons, or lines expressing the linear growth of prices following stable inflation rates; and served during their social studies lessons.

Almost any study investing in interdisciplinary or in contextual teaching/learning – be it some particular experience [2], learning theory [3] or curricular project [4] – attempts to follow the other path from Figure 1. The path with long dark arrows shows the “classical” path followed by inventive teachers, who during mathematics lessons use some non-mathematical (contextual or cross-disciplinary) instruments to allow children to arrive at the intended mathematical objective. This is valid for numerous studies exploring more efficient teaching/learning approaches to mathematics. The “other” context takes on the role of providing a motivational environment or emotionally engaging topic.

Experiencing mathematics in arts

For our purposes we focus on the last path, marked with long dashed arrows. Working in a non-mathematical environment with non-mathematical objectives means, in this case, that over a five year period, I had the possibility to teach arts and music in the first three years at two secondary schools (K5-K7 level). With all 11 classes ranging from 16 pupils at a private school, to 36 in the public lyceum, I had been working on the understanding of basic concepts of arts and music. The Slovak educational system allows arts and music teachers to work without emphasizing any “comprehension” or interest in developing the competences of non-gifted children, simply distributing marks, and therefore evaluating not the efficiency of their improvement, but simply the status quo of their knowledges and skills.

While the previous statement sounds similar to the “staus quo” for math lessons, the situation joined to my mathematics background convinced me to try to familiarising children with some “other” mathematics and, at the same time, with some “serious” artistic concepts.

Following Seymour Papert´s maxim [5] I wanted not “*to make children do math they hate*” but “*to make math they will love*”. Papert, with his MediaLab group, attempted to elicit the motivation to do math via interesting tasks, essentially non-mathematical – e.g. dancing and working with a robotic turtle. Finally, children develop their algorithmical thinking as well as their spacial imagination and mental representation. There was a strong emphasis on an emotional engagement of the teacher and pupils in the same task – desire to achieve something they like.

I had a chance to choose my own topics and instruments as well as pedagogical approach. This liberty allowed me to work with all children, having explained to them the necessity of a well developed ear, voice and manual skills, as well as sight for the benefit of their adult lives. After our discussions about the effects of a good job in arts, all concerned classes (age 10-12) accepted to work much harder than they were used to. One of the first artistic concepts to be developed was ornament. This topic emerged from discussions with children who hated “obvious” topics like “my favourite sport”, “my summer holiday”, “autumn leaves collages” etc. Thus, I started with the history of art. Children noticed beautiful ornaments in renaissance draperies and fabric designs as well as those in illuminated manuscripts.

The beginning of the children´s work was research on the qualities of basic elements of ornament. We started by analyzing letters (Figure 2).

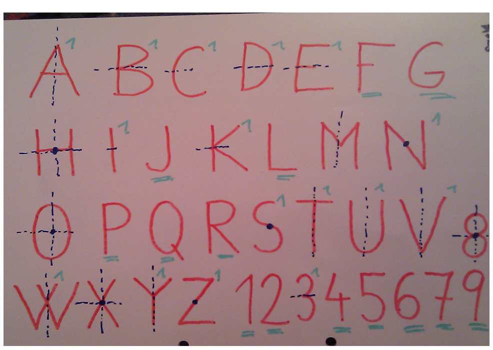

Figure 2 *Research on symmetry*

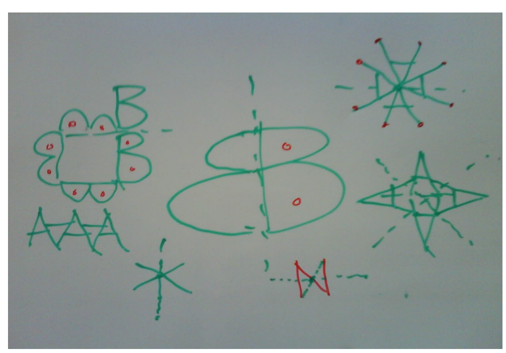

Figure 3 *Usage of the three transformations*

Children discovered very quickly that some letters appear "like in a mirror", some looking "turned around" some unchanged and some completely different. One girl remarked that for the circular "O" there are infinite symmetry axes.

Three years prior the official curriculum, I had explained the word "symmetry" and had also asked if they could tell me what the differences were between "mirror-like" and "turn-around" symmetries. The discovery of reflection and rotational principles followed immediately, and then (Figure 3) they added translation, to create the "bordering ornament" – periodically repeated pattern or element.

The interesting moment came when I asked if there should eventually be some "other transformation" necessary to get the other letters, to which we could not find symmetry (e.g. F, J). At that moment, the murmuring of a strong discussion occupied the classroom and the children decided that the three discovered transformations sufficed. To prove their statement, two girls showed how we could compose an algorithm to create a figure or shape. At the end of their "dissertation" the first girl showed that for the circular "O" letter it was sufficient to rotate a single point around the centre. After a little reflection, another boy added that a single point would be enough for ANY shape. He came to the whiteboard and explained that with the three transformations it was possible to generate any segment, line or shape. Some children added that they could describe the "recipe" to create any shape. Other children then demonstrated how to create various patterns from basic elements or how to use letters instead of abstract geometric shapes as shown in Figure 4.

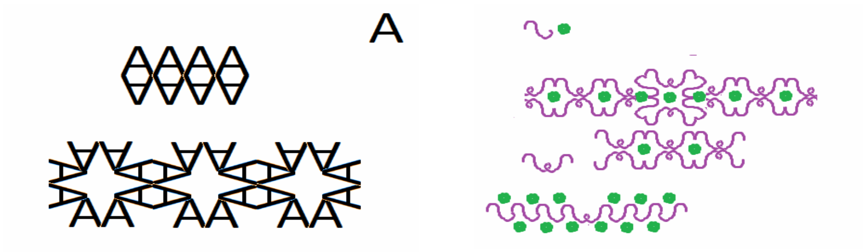

Figure 4: *Pattern generation*

I saw an opportunity to develop their algorithmical thinking, so unexpectedly discovered, in a school computer room to generate patterns using geometric transformations and describing them to their peers. This activity delivered very interesting results when used by children –with a lot of pleasure- for family greeting cards. Concentrated reading of art books and books about the symmetries [6] seen in Figure 5 gave children an idea about the hidden beauty of generative algorithms and formalized symmetries.

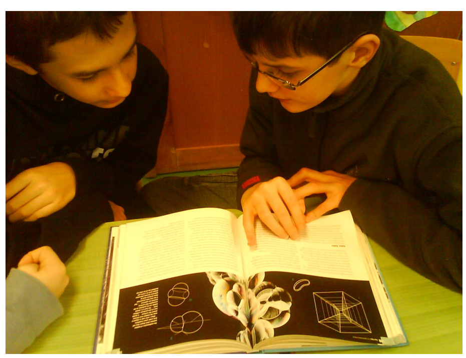

Figure 5: *Reading books about symmetries*

After a long discussion there were some classes which chose tapestry and some classes that chose plain initial or monogram. For the "monogram" group we discussed the expression of personal, individual features, concentrated in one or two letters with some characteristic objects and colours to define the personality to be expressed in the monogram. As we discovered, the selection of framing shapes had to be connected with the content, and to the shape of the letters to be included in the monogram placed on the A4 sheet.

The study of ornaments, tapestry and illuminated manuscripts added to our discussions about symmetry and brought children to a request for accurately formulated rules for a “good” monogram. As shown in Figure 6a and 6b, the newly learnt concepts on geometrical transformations were applied with ease and with a lot of precision and accuracy in very inventive compositions.

The atmosphere of “serious work” was characteristic for each class working on the “matemathized art lesson”, be it the aforementioned monogram, tapestry or any other “unusual” topic. In spite of the low age and therefore typical absence of formal operational ability, the major part of classes manifested deep insight and comprehension for such non-trivial concepts as geometrical transformations, with an immediate recognition of its appearances in the world around.

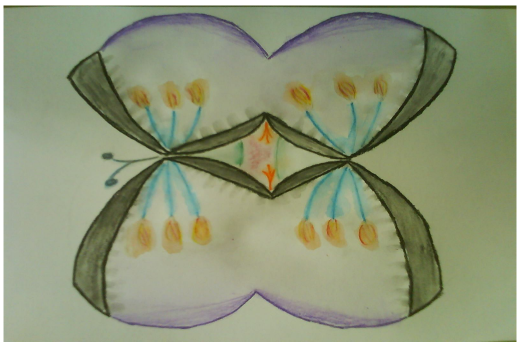

Figure 6a: *Double reflexion/rotation*

Figure 6b: *Rotation*

I also appreciated the effort children made to discover more than the immediate application of symmetries. They found them in architecture, fashion and other domains. I would like also to emphasize the impact on their algorithmical thinking with the development of spacial reasoning and mental rotation as well as the mental representation of shapes. Additionally – from the motivational point of view – who wouldn’t like to have such beautiful monograms at home?

Acknowledgements

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