Edge Color Patterns in the Bead Truncated Icosahedron

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Abstract

I explore methods of deriving edge color patterns in truncated icosahedra constructed with beads and thread. I discuss methods that include mathematical progressions with differing parameters, artistic intuition, and experimentation. I examine methods of orienting the bead polyhedra and construction perspectives.

Introduction

In 1997, I made my first truncated icosahedron with beads and thread following a line drawing from a book. I created an open framework polyhedron where the beads sit at the edges of the form. The planar faces are open or negative space and there is a small void at each vertex where the beads/edges meet. I later drew a diagram of the net pattern where each bead was numbered and wrote step-by-step instructions. I have taught the form nationally and internationally for over twelve years. This paper discusses my process of creating a repertoire of color patterns. The bead polyhedra generally are made with beads of the same size (4mm) and shape (bicone). The thread used is a clear monofilament.

Edge Counts

In the bead truncated icosahedron, a bead stands in for each edge of the 32 faces of the structure. These 32 open faces consist of 12 pentagonal faces and 20 hexagonal faces. The 12 pentagonal faces do not touch each other and are surrounded by 20 hexagonal faces which touch both the pentagonal faces as well as other hexagonal faces. These open face pentagons and hexagons have a combined total of 90 edges. Sixty of the 90 edges form the pentagonal open faces which are also shared with the adjacent hexagonal faces. The remaining 30 edges are shared between the hexagonal faces. The 20 hexagonal faces have a potential number of 120 edges. Subtract the 60 edges that are shared with pentagons and 60 potential edges remain. Of the remaining 60 possible hexagonal edges 30 are shared between hexagons leaving a final edge count of 90. An open framework truncated icosahedron affords nearly triple the color pattern permutations—90 possible positions that grow exponentially versus the possible 32 colors of 32 planar faces (assuming the faces are each one color without additional graphics).

Figure 1: Diagram Net  Figure 2: Monochromatic  Figure 3: “Ten Points”  Figure 4: “Meander”

The step-by-step instructions for the bead truncated icosahedron are 18 pages long (see Figure 1). Please contact the author if you would like a copy.
Progressive Edge Patterns

The first color pattern is monochromatic (see Figure 2). All of the beads/edges are the same color. This pattern is the most difficult to execute as there are no differing color points to refer to that will help mark the beader’s place as she creates the bead truncated icosahedron. I am using the term “bead truncated icosahedron” rather than “beaded truncated icosahedron” as “beaded” implies covered with beads and a bead polyhedron is a “bead” (something containing a hole) in its own right.

The next logical color pattern adds a second color (preferably of high contrast to the first color). The first pattern of a bi-color format results from changing the color of 1 bead (color A is 89 beads and color B is 1 bead). Because we have two different polygonal faces and not all the edges are shared between the pentagons and hexagons there are 2 different possible locations within the pattern net for this single bead of the second color. I can locate this bead at the edge of any pentagonal face or as the edge between any 2 hexagons that share an edge.

Consider how many color patterns each of these 2 edge colors will generate. Examining first the new color placed on a pentagonal edge, I find I have the potential for 60 patterns. However if I do not fix the polyhedron in space, 60 separate patterns are not necessary as there will be only one reference point to differentiate one pattern (or edge location) from another. I can partially fix the polyhedron in space by running a rod vertically through two symmetrically opposed pentagons. This partial fix allows individual patterns to be distinguishable as they move in relative distance to the position of the rod. The partial fix still permits the bead polyhedron to rotate. To fully fix the polyhedra in space requires a second rod passing through a second set of 2 symmetrically opposed pentagons. The new bead color now has fixed reference points. And I have 60 more patterns. When I move the location of the single new color bead on the spatially fixed polyhedron to a position on an edge shared by 2 hexagons I add 30 more possible patterns. I can also fix the truncated icosahedron in space by passing the rod through two pairs of symmetrically opposed hexagons. (The intersection of the rods through the pentagonal faces will have angles of 60° and 120°. The intersection of the rods through the hexagonal faces will be 90° as well as combinations of 60° and 120°.) This alters the perspective in space from which I can view the polyhedron but does not change the 90 new bi-color single bead patterns. Because these patterns are singular and don’t yet have mirror images they are located progressively if asymmetrically in relation to the fixed poles.

![Figure 5: Basic AB “Soccer Ball”](image)

![Figure 6: “Zig Zag”](image)

![Figure 7: Sections of Stripes](image)

The logical progression of pattern generation continues by adding a bead of the same color B. The first step in this instance would be to add 1 bead in an adjacent position relative to another bead so that each shares a vertex. Again I have 2 choices: to add another pentagonal edge to the first pentagonal edge or to add another pentagonal edge to an exclusively hexagonal edge. Each edge addition to a hexagonal edge will always be an edge that is both a pentagonal edge as well as a hexagonal edge. (Remember that each pentagonal edge also doubles as a hexagonal edge but that 30 edges are exclusively hexagonal and none of the 30 exclusively hexagonal edges shares a vertex with another strictly hexagonal edge.)
A second step would be to add another bead of the same color B, but to place this new color at a non-contiguous edge to the first. The additional possible variants become 1) the two separated pentagonal color edges, 2) two separated pentagonal and exclusively hexagonal edges, 3) two separated exclusively hexagonal edges. The number of possible patterns begins to grow while the possible positions for the new patterns on the polyhedron begin to diminish.

A third step would be to add another bead so that the pattern cluster is now 3 beads of the same color B. The beads/edges options are 1) connected to each other, 2) two beads connected and one bead separate, 3) three beads not connected to each other. Each of these 3 options would also have variations where beads in each option are pentagonal edge locations or exclusively hexagonal locations. The pattern possibilities become exponential.

**Aesthetic Considerations**

I do not think the singular patterns of one or more beads that I have explored up to this point are particularly interesting. I believe that the aggregate of the 90 patterns of the first group of second colors could be artistically satisfactory if they were strung together, hung as a group mobile, or attached to each other at shared faces with other bead polyhedra. An early pattern that I developed has ten separated second color beads placed equidistantly on the truncated icosahedron. I call it “Ten Points” (see Figure 3).

![Figure 3: “Ten Points”](image)

My first pattern explorations were intuitive, experimental or just plain hit or miss. Before I attempted a monochromatic truncated icosahedron I constructed (diagramming came later) the soccer ball pattern of 12 circular loops of 5 in one color with a second color at the remaining 30 hexagonal edges. I called it the “Basic AB” color pattern or “Soccer Ball” (see Figure 5). Color A denotes the 12 pentagonal bead loops and color B the 30 hexagonal edges.

My next intuitive pattern experiments revolved around distinguishing circular loops that run horizontally around an imaginary pole or stripes that give the impression of running vertically along an imaginary pole (see Figures 4, 6 and 7). The diagram net that I use to teach students is oriented to begin with a pentagonal circular loop. The next “row” out then consists of 5 connecting hexagonal circular loops. Rows 3 and 4 have alternating pentagonal and hexagonal circular loops. The last row is 5 hexagonal circular loops that also include the beads of the last row which is a pentagonal circular loop. I also work from a second diagram net that begins with a central hexagon. I find that this change in perspective enables me to see new pattern combinations.
The Color Frontier

At this writing the largest number of colors I have worked with on any single bead truncated icosahedron is 13—one color for each circular loop of 5 (all pentagonal edges) and the thirteenth color for the 30 exclusively hexagonal edges. Theoretically I could work with as many as 90 colors, but managing patterns or other aesthetic considerations gets more complex. I tend to work primarily with 2 or 3 colors. Because I work most often with crystal beads I have to consider the properties of transitive light (through translucent beads) and reflected light (that bounces off the surface of opaquely colored and coated beads) when I choose a palette to work from. To date I have diagrammed 40 different edge color patterns in single, double, triple, and larger multiples of color (see Figures 8 and 9).

Conclusion

An exponential number of edge color patterns for the bead truncated icosahedron are possible. Aesthetic considerations influence which patterns are created. I have diagrams for over 40 color patterns at this time. That number grows when I tally all of the potential color combinations. The possible number of pleasing patterns begins to escalate again when I factor in using other types of beads. Other variations to experiment with are changing the shape of the beads and the bead material (glass, crystal, stone, plastic, organic, metal, and more). In the future I will explore changing and combining sizes of beads to create transformational changes in the shape of the truncated icosahedron without changing the pentagon to hexagon relationship. Varying the bead sizes changes the equilateral form of the open planar faces to non-equilateral polygon shapes (see Figure 10).

Figure 9: Bi-color Edge Patterns

Figure 10: Transformed Truncated Icosahedra

References