Symmetry in Mathematics, Physics and Art

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Abstract

The mathematical concept of symmetry, invariance and equivalent relation allows physical sciences to define precisely the reality of matter. Crystallographic point groups classify crystals in terms of Euclidian geometry. Art itself is often defined in terms of beauty, balance, and harmony. The following describes how the 32 crystallographic point groups diagram was used in the electronic environment to produce an artistic outcome based on scientific rigor and to evaluate art relevance to the larger debate on symmetry and the perception of beauty.

Introduction

"Symmetry, as wide or as narrow as you may define its meaning, is one idea by which man through the ages has tried to comprehend and create order, beauty and perfection."
H. Weyl [1].

This short paper is based on a series of illustration of the 32 crystallographic point groups. It describes the few practical steps I took, after briefly revisiting several fundamental scientific and artistic theories relating to the concept of symmetry to evaluate if highly conceptual information translated into visual practices can increase the viewer's appreciation of an artistic statement in terms of beauty, balance and harmony.

Applied sciences have mapped out a system of point groups built on mathematics theories and symmetries to identify and catalog most known crystals. Crystals hold a universal recognition in terms of beauty. The concept of symmetry is also very present in the vernacular of the art world. Can a concept that unites scientific disciplines be translated in an artistic format and still be relevant to the larger appreciation of our environment?

Background

Mathematics Mathematics is the study of numbers, quantity and space using abstract and logical reasoning. Euclid in "The Elements" [2] includes the element of shape and symmetry to define a relation of commensurability to create balance and rationality. Symmetry deals with the property of a system that can be applied to a structure. Mathematicians have mapped several areas of symmetry – reflection symmetry, rotational symmetries, point symmetries as well as non-isometric symmetries, scale symmetries and fractals. The larger concept of symmetry as invariance [3] opened the door to other field of scientific investigation of nature and the world as we know it.

Physics A crystal is a type of mineral that possesses an orderly repetitive internal pattern of atoms. Crystallography is the study of molecular and crystalline structure. It measures the atomic arrangement of a crystal and its properties. In crystallography, a crystallographic point group is a set of symmetry operations, like rotations or reflections, that leave a central point fixed while moving other directions and faces of the crystal to the positions of features of the same kind. "Fundamental symmetry principles dictate the basic laws of physics, control the structure of matter, and define the fundamental forces in nature," says Nobel laureate Leon M. Lederman in "Symmetry and the beautiful universe [4]". The 32-crystallographic point groups' table defines minerals that are often associated with beauty. Can the parameters of this template be applied to an artistic visualization and enhance the viewer appreciation of art?

Art

All stiff regularity (such as borders on mathematical regularity) is inherently repugnant to taste, in that the contemplation of it affords us no lasting entertainment … and we get heartily tired of it.
Immanuel Kant

Symmetry is regularity. Immanuel Kant's highly subjective statement may go against what most agree on throughout history but more fundamentally it could not have been formulated without looking first at order and symmetry. The classic notion of symmetry as defined by Greeks, Romans, and Renaissance architects or art theoreticians from Plato to Vitruvius and later Alberti, anchors the canon of beauty into balance, harmony, correspondence between right and left [5]

A different notion of symmetry emerged in the seventeenth century, grounded not on proportions but on an equality relation between elements that are opposed, such as the left and right parts of a figure. Similar efforts were made to redefine the scope and extend of symmetry and asymmetry in the field of architecture and art. Today, the concept of symmetry and invariance is still central to the larger conversation on esthetic.

Practices

Set up To evaluate in practical terms the concept of symmetry in a graphic environment, I selected a variation of the Hermann-Mauguin 32 point groups' notation grid [6]. I identified one specific crystal in each category of the table and used each mineral technical profile as a reference from which to convert the variables in digital terms.

In the following example (Figure 1) an orthorhombic crystal is identified as having a hardness of 8.5 M. and a density of 3.5 g/cm3. I converted the parameters accordingly in terms of opacity and fill in the graphic application I used for this experiment.

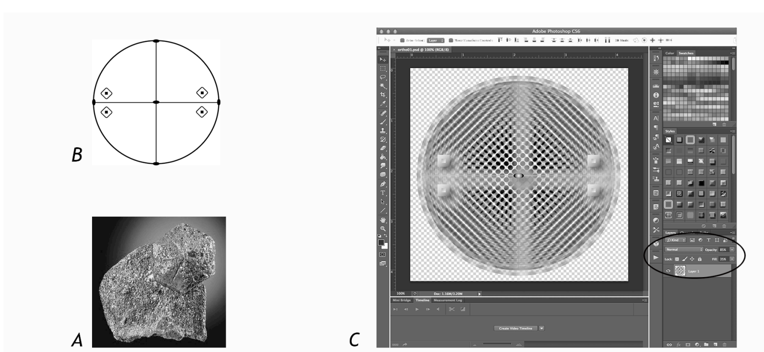

Figure 1: *From A) Chrysoberyl mineral to B) Orthorhombic symmetry to C) Black and white object.*

I defined the symmetry conversion in a grey scale format to extract volumes and shapes. The color scheme was added at the final stage, using the mineral color information I extracted from a scan of the crystal image to establish the RGB conversion (Figures 2 and 3).

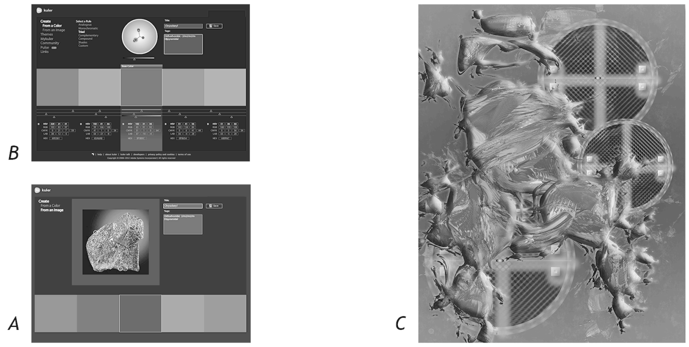

Figure 2: *A) Chrysoberyl color scan B) Triadic color scheme. C) Color scheme insertion in the composition. Notice the color spectrum variation in function of its intensity in the grayscale mode.*

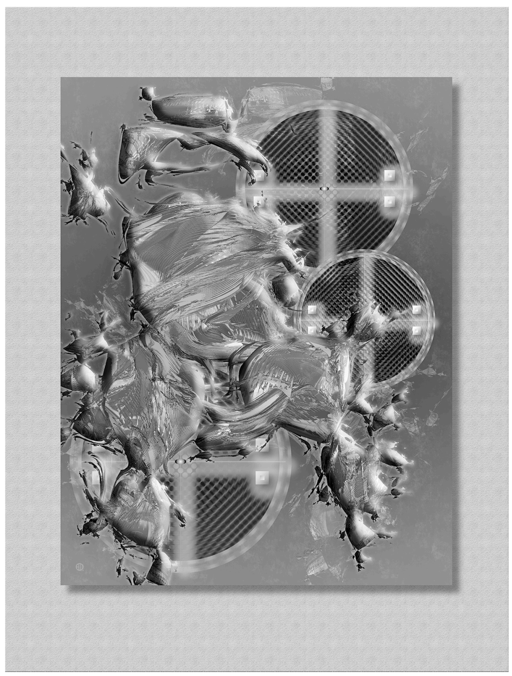

Figure 3: *Final composition. The initial symmetry object was positioned in a 2:1 grid and reduced twice proportionally at 75%. The foreground object is the original orthorhombic symmetry rendered in a 3D program's scattering filter.*

Expansion Each color scheme was selected according to the color context of the existing crystals in the category in which I was doing the rendering. 32 plates corresponding to each of the 32-point groups symmetries were made following that process [7]. The background of each image was added on a purely intuitive and creative perspective to acknowledge both the subjectivity of the art process and its contribution to a larger scheme that integrates elements of scientific disciplines (Figure 4).

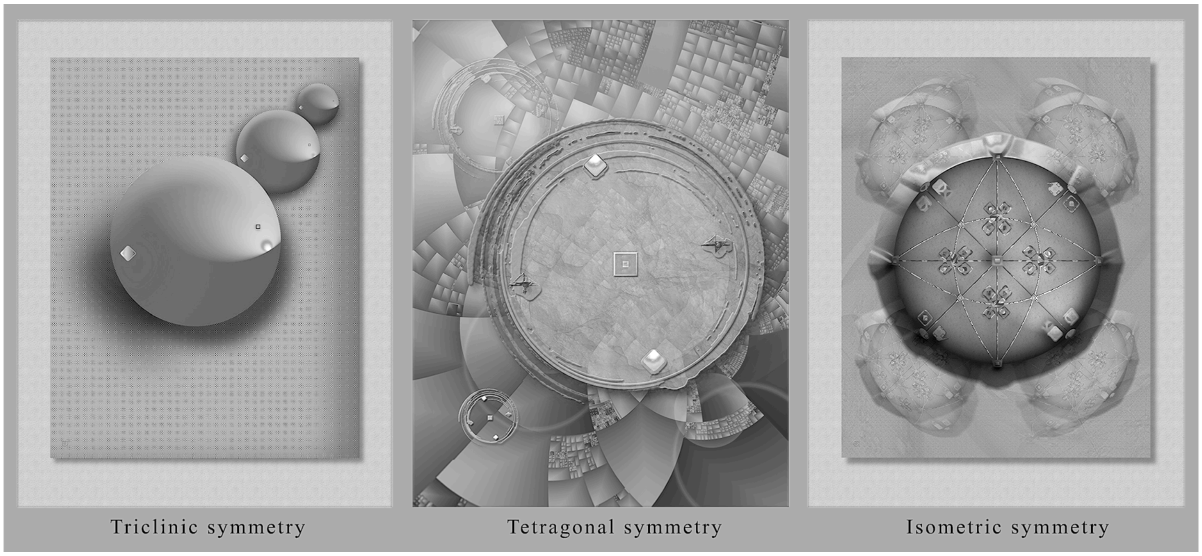

Figure 4: *Example of completed work in 3 of the 7 categories of the 32 point groups.*

Conclusion

Oscar Wilds said: "All art is quite useless", but there must be some use for it or else people wouldn't do it. Understanding the larger concept of symmetry starting from a mathematical and applied science perspective directed my effort to investigate this phenomenon in a visual context. Whether absolute or finite, symmetry permeates all fields of activities and is an effective tool by which to create and evaluate standards of balance, harmony, effectiveness and beauty.

Mathematical and applied sciences establish fundamental principles of nature. Art focuses on the perception of our environment. There is no doubt that all fields of investigation benefit from each other. Closer collaboration between all involved should encourage better appreciation of the many facets of our perception and the extent of our diversity.

References

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