Meta-Vermeer: A Topological Reinterpretation of a Masterpiece

Silvia De Toffoli*
Dept. of Mathematics • Technical University of Berlin
Straße des 17. Juni 136 • 10623 Berlin • Germany
toffoli@math.tu-berlin.de

Yasuhiro Sakamoto
Institute for Art and Visual Studies • Humboldt University of Berlin
Georgenstraße 47 • 10117 Berlin • Germany
yasuhiro.sakamoto@culture.hu-berlin.de

Abstract

The global shape of space is investigated in topology, a mathematical theory that focuses on the 'primordial' properties of spatial objects, the ones that do not depend on their geometric shape. Topology gives the possibility of presenting different models for our universe, different configurations which are compatible with our usual 3-dimensional Euclidean geometry. We present a way to directly experience these different topological configurations through a masterpiece of the past: *The Girl with a Glass of Wine* by Johannes Vermeer. *Meta-Vermeer* consists of a 3-dimensional model of the room depicted in Vermeer's painting and a light simulation. The room is then visualized (though a 3d portal) as different topological spaces.

Introduction

As the title *Meta-Vermeer* suggests, our analysis is about a painting of Vermeer from an outside perspective. We focus on light constraints and geometry. In the first stage we make the assumption that Vermeer used a camera obscura to create many of his paintings, and we study the consequences of this. This device served as a third eye for the painter: it connected him with the real space of the room and with the geometry of the surrounding space. With a space simulation, new possibilities emerge to imagine other instruments (not necessary optical), which create different spaces and which can also serve as third eye of the painter. These new devices are a connection between the artist and the space of the painting, a "conceptual lens" which plays a role in the perception of the space of the room. The instrument we use consists of a mathematical simulation and the transformations we focus on are topological, instead of geometrical. This is a reinterpretation of the same process Vermeer used, after the mathematical development of topology of the twentieth century. The aim is to share the experience, which normally only mathematicians have, of being in a different topological space, through visualization and the simulation of light.

In his work *Meta-Kandinsky* [4], the Swiss kinetic artist Jean Tinguery discloses hidden movements in Kandinsky's constructivist paintings; in a similar fashion, *Meta-Vermeer* shows from a painting a variety of new images created through topological transformations: "movements" of a different nature become visible.

The painting that we consider here is *The Girl with a Glass of Wine* by Vermeer. The topological transformation in which we are interested consists of interpreting the space of the room depicted in the painting, not as the ordinary 3-dimensional space, but as a different space, a 3-manifold (for example the 3-dimensional torus), which can be obtained by different gluing patterns of the boundary of a cube.

In the next section we explain how we created a 3-dimensional model of the room of the painting using information gathered from 11 different works of Vermeer and how the light simulation was implemented. After that we show which topological interpretations are acceptable and carry out some concrete examples. To conclude, we discuss the theoretical value of this analysis and propose practical implementation.

Reconstructing the Room and Simulating Light

Vermeer's paintings have often been analysed by means of geometric theories of perspective [5, 3]. The starting point for our work is the analysis by Philip Steadman [8]. He claims that eleven paintings by Vermeer depict different scenes set in the same room and that a camera obscura was used. Each painting adopts a slightly different observation point for the same room. Thus, it is possible to extract from them different facts about the shape of the room. Using them, Steadman was able to build a physical diorama of the room. Following his method, we also created a diorama of the room, and a 3D computer model of it (Figure 3). Figure 2 depicts the room with the elements of the various paintings fitting in the same space[1].

The diorama and the computer model have been presented by the second author, in cooperation with the first author, in the exhibition *The Structure of Light - Light in the time of Rembrandt and Vermeer* in Kassel, Germany [9]. The structure of light in Vermeer's atelier was simulated through a physical interface, a digital camera obscura (placed in front of the physical diorama) that worked as a light sensor capturing the light coming from the window of the diorama. This was located in front of a real window and thus illuminated by sunlight. The data from the sensor were then sent to a computer, which in real time simulated the light behaviour in the digital model of the room. The variation of light in the space of the room of the exhibition thus directly affected the virtual space (Figure 3) [6]. In this work, the geometry and light constraints which interested Vermeer so much could be clearly visualized: with our digital camera obscura we were able to present the hidden information of the painting. In the next section we present a further step: Instead of using an *optical lens* we use a *conceptual lens*, which instead of connecting the viewer with the geometry of the painting, connects him with its topology. This means to go beyond the limits of mathematics of the seventeenth century (the path traced by Vermeer), and encompass the "revolution" of Poincaré: that is, not only to focus on the geometry of space but on its topology too.

Figure 1 : *The Girl with the Wine Glass*

Figure 2 : *The reconstruction of the room*

Vermeer's Room as a New Topological Space

As we have seen in the previous section, the room represented in Vermeer's painting is modelled as a 3-dimensional mathematical cube. The idea is now to imagine the cube not as a object *in* space, but *as the* space itself. While for Vermeer the room was the geometrical space of the painting, for us it becomes a topological space. The cube is now our "universe". Topologically, it is a closed compact 3-manifold, which means that it is locally indistinguishable from ordinary Euclidean space. Moreover it has no boundary –

[1]This model was created using the software: *Max/Msp* and *Blender*.

Figure 3 : *The computer model*

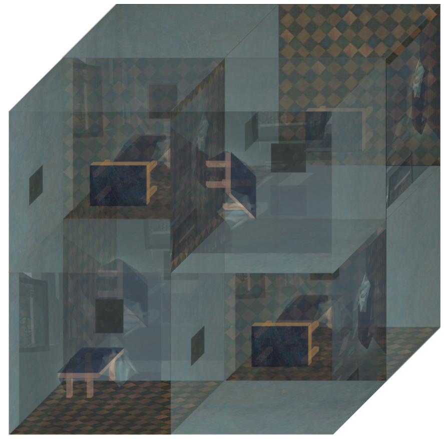


Figure 4 : *Eight copies of the room glued according to the first row in Table 1*

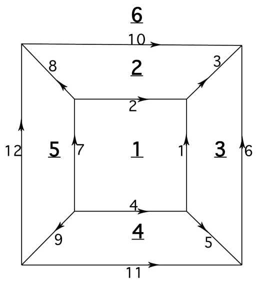
indeed what would be the space outside when all our universe is the room? We have to imagine that each time that we cross a wall (that is, a face of the cube) we enter the room/cube again from a different place.

To create these manifolds, we paired the faces of a polyhedron and then glued them. One problem was that not all gluings are allowed: some constraints must be followed in order for the space we obtain to be a closed manifold. To find out whether the gluing is compatible or not we have to check the orientations and then to calculate the Euler characteristic of the resulting space [7, p. 221]. Moreover we are not satisfied with any result: we want to obtain a manifold that can be equipped with an Euclidean metric. That is, the new space has to be locally compatible with our ordinary experience of Euclidean geometry. To visualize the gluings we can imagine making infinite many copies of the cube/room and placing them together in such a way that they fill all the space.We imagine each of these copies to be identified. The fact that we can obtain different manifolds from a cube is reflected by the fact that we can fill space in different ways with copies of the same cube: we can glue them not only in the most natural way, so that the orientations are always the same, but we can also perform rotations of the cube prior to gluing, as in Figure 4.

A list of such gluings was computed by Everitt [2, Table 4], and reported here[2]. The schema in Table 1 assigns an orientation and a numeric label to each edge and a numeric label to each face (the underlined numbers). The correspondence between the cube and the room is the following: The face labeled $\underline{1}$ is the back wall of the room, the outer face (number $\underline{6}$) is the front wall, right and left walls are numbered $\underline{3}$ and $\underline{5}$ and floor and ceiling, $\underline{4}$ and $\underline{2}$. The codes are to be read in this way: The *Face* column codes the identifications of the faces. The position number of the letter (a, b or c) in the code correspond to the face with that number (from $\underline{1}$ to $\underline{6}$), and when two letters are equal, then the faces corresponding to their position must be identified. For example in the first code we glue the faces $\underline{1}$–$\underline{3}$ together (because both correspond to the letter a), $\underline{2}$–$\underline{5}$ (letter b) and $\underline{4}$–$\underline{6}$ (letter c). The *Edges* column codes the identifications of the edges. First we read the word ignoring the $+$ and $-$ signs and use the same logic as before. In the first examples we would glue edges with number 1–3–4–12 together (all corresponding to the letter a) and so on. The signs indicate in which way we have to glue the edges, with the normal orientation or the reversed one. In the first code all edges are glued with the same orientation.

The 4th identification is the most natural one and corresponds to the 3-torus. It consist in gluing the cube always in the same direction. So if we think in terms of the room, the front wall is identified with the back wall, the ceiling with the floor wall and the right with the left wall. Moreover all orientations of

[2] There is a closed orientable Euclidean manifold ($c6$ [1, Table 6]) that is not listed in our table because it is not arising from gluings on a cube. The list present in [2, Table 4] has an extra row because two gluings (8 and 12 in his notation, 2 in ours) correspond to the same manifold: $c2$ in [1].



n	$Faces$	$Edges$
1	$abacbc$	$a(+++)b(+++)aac(+++)bccbcba$
2	$abbcca$	$a(-+-)ab(--+)c(-+-)bacbbacc$
3	$abccba$	$a(-+-)ab(--+)c(+--)bccbbcaa$
4	$abcbca$	$a(+++)b(+++)c(+++)bcaaccbba$
5	$abcbca$	$a(+++)b(+++)c(-+-)cbaacbbca$

Table 1 : *Possible identifications on the cube giving rise to different closed orientable Euclidean manifolds*

the edges are positive. When visualizing this space in a virtual reality theatre we would look right and see repetition from the left and so on. The space could also be revealed by a peculiar behaviour of a light beam (or of a solid object like a billiard ball). In this case the light would enter from the window (upper part of the left wall) and then touch the floor, but, instead of reflecting would continue through and appear from the ceiling and so on. If instead, we chose the first identification, then the back wall would be identified with the right one (a, in the code): In this way the light will travel in yet another way. We can visualize this gluing in Figure 4, where 8 copies of the room are glued together.

Conclusion

By examining Vermeer's room and considering it in different ways, we challenge our standard geometric perception and oblige the spectator to experience a different space, to be in another world. Even if this process is done mathematically, it is only using the tools of algebraic topology that we can calculate which spaces can be obtained, the result can be experienced by any audience.

In practice, the way we visualize such different spaces is through virtual reality or with the simulation of light beams travelling around a model of the room. The virtual reality is particularly appropriate to experience repetitions and reflection: looking right it would be possible to see the same room from the left or another point.

References

[1] J. Conway and J. Rossetti. Describing the platycosms. *arXiv:math.DG/0311476*, 2003.

[2] B. Everitt. 3-manifolds from Platonic solids. *Topology and its Applications*, 138(1-3):253–263, 2004.

[3] T. García-Salgado. Modular perspective and Vermeer's room. *Bridges Conference Proceedings*, page 379, 2006.

[4] F. Gerber and P. Hulten. *Museum Jean Tinguely Basel: The Collection*. Art Data, 1996.

[5] G. Gutruf and H. Stachel. The hidden geometry in Vermeer's the art of painting. *Journal for Geometry and Graphics*, 14(2):187–202, 2010.

[6] Y. Sakamoto and S. De Toffoli. Einfürung in *Meta-Vermeer*: Experimentelle Studien über originale und manipulierte Bilder. In *Das Licht im Zeitalter von Rembrandt und Vermeer, Ein Handbuch der Forschungsgruppe Historische Lichtgefüge*, pages 50–53. Jovis, 2012.

[7] H. Seifert and W. Threlfall. *Seifert and Threlfall, A textbook of topology*, volume 89. Academic Pr, 1980.

[8] P. Steadman. Vermeer's camera: uncovering the truth behind the masterpieces. 2001.

[9] N. Windmöller. MetaVermeer, 2011. In *Exhibition Catalogue (Lichtgefüge - Museum Schloss Wilhelmshöhe , 18 Nov. 2011 - 4 Mar. 2012)*, pages 172–175. Michael Imhof Verlag, 2011.