Stigmetry Prints from Patterns of Circles

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Abstract
Stigmergy is a form of self-organization that is brought about by indirect coordination of agents or actions. Urbano proposed a model based on stigmergy for simulating the nest formation of the species T. albipennis that occurs from collecting virtual grains of sand. By making different colonies of the species sensitive to different colors of virtual sand grains, Urbano produced what he called “sand paintings”. We exploit this technique by carefully assigning centers, radii, and colors to colonies in such a way that the stigmergy model self-organizes a uniform density distribution of virtual sand grains into sand paintings that exhibit various types of color preserving, and color reversing, symmetry. We call these algorithmic, computer-drawn compositions “stigmetry prints”.

1 Background
So-called “swarm paintings” trace their origins to ant colony simulation experiments of Ramos [14] [15] who investigated them for image processing purposes. This early work of Ramos eventually led to the physically embodied collective robotics paintings of Moura [12][13]. Image processing and non-photorealistic rendering continue to be application domains for ant colony simulation research (see, for example, Ramos [16] or O’Reilly et al. [17]).

A research group led by Monmarché [2] appears to be the first to have used the term “ant painting” to describe abstract images made by virtual ants that roam over a toroidal grid. In their model, a small number (4-6) of virtual ants paint by depositing “scent” of one color while searching for “scent” of a different color. In fact, their image generation scheme was interactive. They used the technique known as user-guided aesthetics so that the user could evolve ant paintings. Greenfield [5] considered non-interactive methods for evolving ant paintings using a similar model, also using a small number (8-12) of virtual ants.

Urbano [19] considered an ant painting model where individual cells in the environment exuded virtual scent — the attractant — until they were visited (harvested?) by an ant. As cells were visited they were painted according to the color assigned to the species that initially reached them. By using two ant species with large numbers (25-250) of ants, and by diffusing and evaporating the exuded scent, Urbano’s technique yielded ant paintings that were “finished” once there were no more unvisited cells. Greenfield [4][5] advanced this model by also making the ants themselves exude virtual scents, and by having ant behaviors further controlled in the absence of environmental scent by treating these ant scents as attractants and repellents.

Jacob et al. explored 2D swarm painting models based on simulating large colonies of bacteria [9] as well as 3D swarm paintings based on simulating flocks of birds [10]. Annunziato et al. [1] have exhibited installation artworks inspired by artificial life that use large numbers of virtual agents behaving similar to the agents in swarm paintings.

A gray scale version of ant painting was used by Greenfield and Machado as part of an artists and critics simulation [8]. Each “artist” could develop a unique style according to the various parameter settings adopted over time for their ant paintings. Greenfield continued to explore the artistic merits of this version of ant painting in [7]. Jones modeled the evolution and formation of plasma transport networks of the slime
mold *Physarum* by using virtual ants that possessed remote sensing capabilities [11]. In turn, Greenfield further refined this method and combined it with image compositing techniques to develop ant paintings called “network transport overlays” [3].

Previously, both Urbano and Moura have pointed to the concept of stigmergy to help explain why their respective physical and virtual software-controlled entities are able to exhibit creative or artistic tendencies. Stigmergy [18] refers to the situation where the behavior of agents in swarms is controlled wholly by external, environmental factors.

## 2 Introduction

Recently, Urbano proposed using a model for nest construction by the ant species *T. albipennis* as the basis for ant paintings [20]. The idea is to have ants use a simple stochastic sand grain foraging, collecting, and depositing algorithm in order to construct circular walls defining their nest’s outermost boundary. The problem of leaving open passageways in the boundary for the ants to enter and leave the nest is not addressed in this model. The paintings themselves result from uniformly distributing sand grains in the toroidal environment, making different colonies of ants sensitive to different colors of sand grains, and using randomly assigned nest radii and centers. Figure 1 shows results from our implementation of this model illustrating the basic idea and providing an example of an ant painting of this type.

![Figure 1: Left: White sand grains on a black background are collected and deposited according to the *T. albipennis* behavioral strategy suggested by Urbano for nest formation. Right: An ant painting using multiple colonies each sensitive to one of the two sand grain colors available and having randomly assigned nest centers and radii.](image)

In this paper, rather than randomly assign nest radii, centers, and sand grain color preferences, we carefully set these parameters. In this way starting from a uniform density random distribution of sand grains, thanks to stigmergy, at the macroscopic level a pattern of circles emerges for which there are color preserving or color reversing symmetries in the patterns. However, at the microscopic level there is no symmetry to the distribution of the grains. We refer to these algorithmic compositions as “stigmmetry prints”.

## 3 The Ant Behavior Algorithm

Let \((C_x, C_y)\) be the nest center and let \(R\) be the nest radius. The stochastic ant behavior algorithm is modulated by three parameters: a constant drop probability \(K_d\), a constant pick-up probability \(K_u\), and a constant \(\tau > 0\) which helps control the speed of nest boundary formation and (indirectly) the thickness of the nest wall. If
\((a_x, a_y)\) is the ant’s current location and \(r\) is the ant’s current distance to the nest center, define the current drop probability to be

\[
P_d = \frac{K_d}{1 + \tau(r - R)^2},
\]

and the current pick-up probability to be

\[
P_u = K_u \left(1 - \frac{1}{1 + \tau(r - R)^2}\right).
\]

Note that some care must be exercised when calculating \(r\) due to the assumption that the underlying canvas grid is toroidal. With this set-up, the ant foraging algorithm is:

if ant is carrying grain
  if current pixel does not have grain
    with probability \(P_d\) drop grain
  else
    if current pixel has grain
      with probability \(P_u\) pick-up grain

Note how nest wall formation occurs indirectly due to the nature of the way the current pick-up and drop probabilities spike with respect to the radial distance to the nest boundary. After completing this action, the ant must also move. The ant movement algorithm is:

if ant is carrying grain and \(r > R\)
  move toward nest center
else
  move in random direction

The subtlety here is that if an ant is within the nest and carrying a grain then it moves in a random direction in order to help find an empty spot to drop the grain.

4 Stigmmetry Print Examples

The stigmmetry prints, so named because they combine stygmergy with circle pattern symmetry, shown here were executed on 500 \(\times\) 500 pixel toroidal canvases. The parameter settings were \(K_u = 0.99, K_d = 0.99,\) and \(\tau = 0.5.\) For the initial uniform distribution of sand grains, the density was 0.19. We always used 400 ants and set the number of time steps to 599000. Ants were assigned to nests in such a way that the number of nests required for the design each had approximately the same number of ants. For convenience, ants were initially distributed at random over the canvas. Given the number of time steps this has no impact on the final print.

A collection of circle patterns to work with was obtained by entering “circle pattern” into the search engine of our browser. Once an interesting pattern was identified, some elementary analytic geometry and trigonometry were required in order to write short program fragments to specify the centers and radii for the circles needed to execute the design. Our final color schemes were chosen from a library of randomly generated color schemes that was built up during the course of testing to verify the placement of the circles in a pattern was correct.
For computational purposes, when executing the ant behavioral algorithms, where necessary the canvas is treated as being real valued by realizing it as the rectangle in the plane whose points $(x, y)$ satisfy $0 \leq x, y < 500$. In other words, the canvas is simultaneously a $500 \times 500$ grid of pixels where each pixel either does or does not contain a sand grain, and the rectangle in the plane defined by $[0, 500) \times [0, 500)$. Since ants move one unit each time step, this implies that depending on location and direction, it may take an ant more than one time step to traverse a pixel. The fact that the grid is toroidal comes in to play when an ant attempts to cross a grid boundary during a random walk or must return to its nest using the shortest route.

Figure 2 shows a circle pattern that has the symmetry group of the square i.e. the dihedral group of order eight, $D_4 = \langle \alpha, \beta \rangle$ where $\alpha$ is the 90 degree (counterclockwise) rotation and $\beta$ is reflection across the line $y = x$. However, the two versions are colored so that the one on the left is color reversing for both $\alpha$ and $\beta$, while the one on the right is color preserving for $\beta$ but neither color preserving or reversing for $\alpha$. In fact, if for convenience we denote the reflections across the central vertical and horizontal lines by $V$ and $H$, those across the central diagonals $y = -x$ and $y = x$ by $A$ and $D$ respectively, and the rotations by $R$, $R^2$, $R^3$ and $R^4 = I$ then it is clear that the version on the left of Figure 2 is color preserving for $I$, $R^2$, $V$, and $H$ and color reversing for $R$, $R^3$, $A$ and $D$; while the version on the right of Figure 2 is color preserving for $I$, $A$ color reversing for $R^2$, $D$ and neither for $R$, $R^3$, $V$, and $H$. Of course it is not strictly true that there are any color preserving or color reversing symmetries in these prints. Up close, due to random sand grain distribution, it can be discerned that true symmetry is not possible. It is only from a distance, where the brain coalesces the nest boundaries and identifies the unused sand grains with the background that the symmetry becomes apparent.

Figure 2: Two ant paintings with the same underlying circular pattern but different color preserving and color reversing symmetries.

Because figure reproduction here is gray scale, henceforth we will only show one example of each circle pattern. For reasons of space, we will leave as exercises for the reader the determination of the symmetry groups as well as the exhaustive classification of which group elements are color reversing, color preserving, or neither. Figure 3 continues the theme of Figure 2 (left) with an even number of alternatingly colored circles evenly spaced about the circumference of a given circle.

Figure 4 shows a new configuration of circles on the left and an embellishment of it on the right. Figure 5 (left) shows a configuration of nested circles colored to give a “telescoping” effect and such that the print is color preserving with respect to reflection across the line $y = x$. Figure 5 (right) shows paired slightly overlapping circles colored so that vertical symmetry is color reversing. We feel this image has an “op art”
Figure 3: Two more alternating circle within circle stigmetry prints.

Finally, Figure 6 shows the upper left quadrant of the low resolution image of Figure 3 (left) enlarged to better reveal the grain pattern, while Figure 7 shows a high resolution 1048 × 1048 paired and overlapping circles image that is color preserving with respect to horizontal symmetry and color reversing with respect to vertical symmetry. The grain density and the number of ants used was increased for this print.

5 Summary and Conclusions

We have explained how we were able to implement an algorithm for modeling nest formation by the ant species *T. albipennis* based on previous work by Urbano. Continuing within the established history and framework of ant paintings, and inspired by the model’s ability to self-organize uniform density backgrounds of virtual particles into patterns of circles, we combined coloring algorithms with well known circle patterns.
Figure 5: Left: Nested circles arranged so that symmetry about the line $y = x$ is preserved. Right: Paired overlapping circles colored so that vertical symmetry is color reversing.

to create a series of “stigmmetry prints”. The possibilities for this art form are not exhausted, and we believe the work here continues to document its potential.

References


Figure 6: High magnification of an image fragment to reveal the grain pattern.


Figure 7: A high resolution paired and overlapping circles stigmmetry print that is color preserving under horizontal symmetry and color reversing under vertical symmetry.


