A 7-Fold System for Creating Islamic Geometric Patterns Part 2: Contemporary Expression

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Abstract

This paper is an abbreviated report on some two decades of research by the authors in the field of 7-fold Islamic geometric patterns. We have expanded the application of this design methodology beyond the historical 7-fold systematic practices identified in Part 1: finding a wide assortment of new polygonal sub-grid elements upon which pattern lines are pre-arranged. What is more, by applying the same basic methodologies used in the *5/10 System*, the inherent proportionality of the *7/14 System* allows for the creation of 7-fold quasi-crystalline patterns with inflation symmetry, 7-fold non-periodic designs, and 7-fold self-similar designs. This material demonstrates the remarkable flexibility of the 7/14 System of pattern generation, and is intended to open the doors of creative pursuit for others captivated by the beauty of these 7-fold patterns.

The *7/14 System* of geometric pattern generation has a significantly greater number of sub-grid elements than the *5/10 System* [**Figure 1**]. This is a direct result of the greater number of heptagonal and tetradecagonal sides over those of the pentagon and decagon. Edge-to-edge arrangements of these sub-grid elements will create an infinite number of

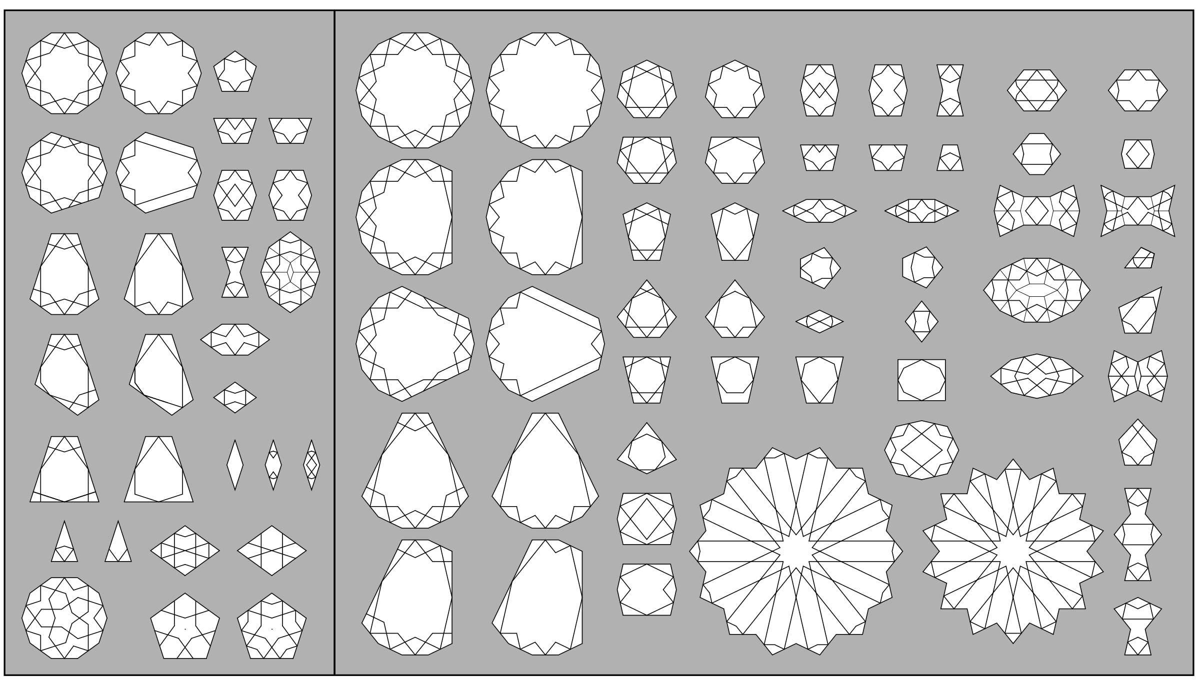

Figure 1: *On the left is shown some representative elements of the* 5/10 System (median *pattern family*). *On the right are some representative elements of the* 7/14 System. *There are strong analogies between elements within these respective systems: in both the inherent qualities of sub-grid elements, and in terms of how they are used.*

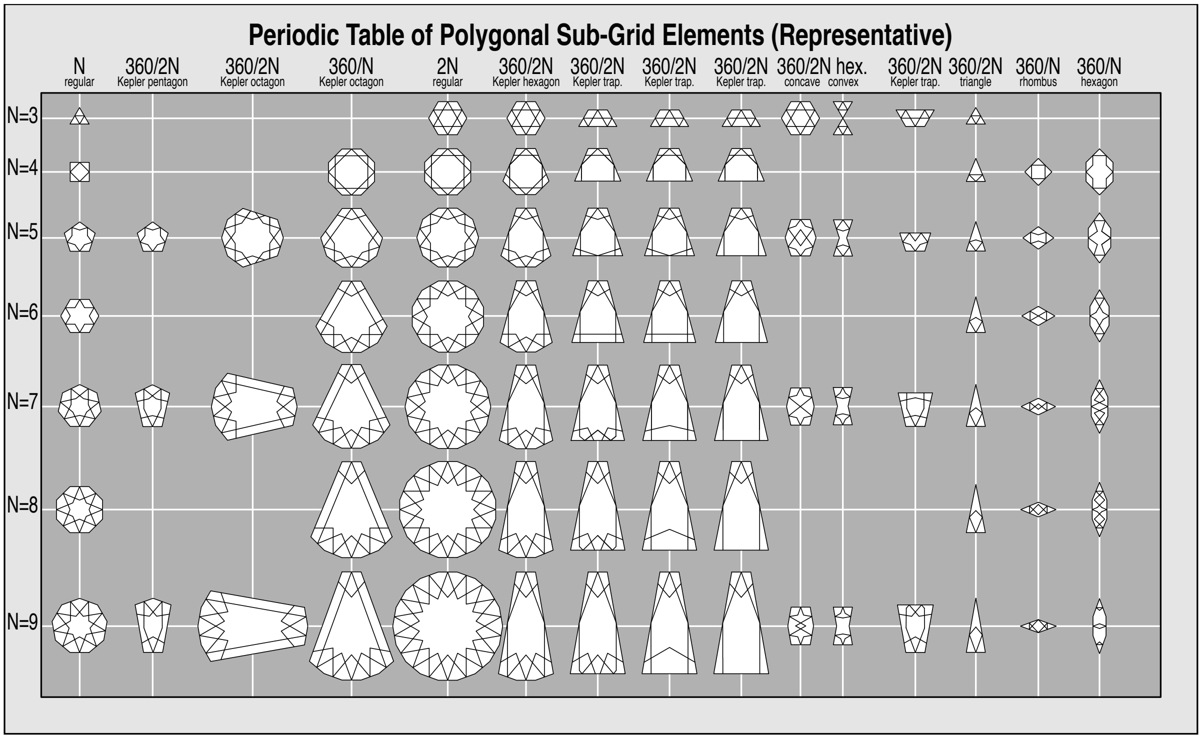

Figure 2: *The consecutive rows are arranged in order of increasing symmetry systems (N). The columns include elements with analogous properties and affinities. This has been especially helpful in exploring less common systems such as the 7/14 and 9/18.*

distinct tessellations. Depending on the contact angle of the applied crossing pattern lines at each polygonal edge, each tessellation is capable of producing a range of different geometric patterns (see Part 1: Figure 1); and each of these individual patterns is open to stylistic variation through additive and subtractive design conventions [1]. Truly, the relatively few patterns from the historical record only touch upon the generative potential of this 7-fold system. In seeking a more complete set of 7-fold sub-grid elements, it has proven immensely helpful to find analogs from the 5/10 System, and indeed all the other systematic generative design systems used historically [2]. Generalization based on analogy can be a very powerful tool for understanding what is known, and what remains unknown. Important examples include the mathematical theory of higher dimensional spaces (see Coxeter), and the Periodic Table of the Chemical Elements. This table arranges the elements into families according to atomic weight or number. When the Periodic Table was discovered, Mendeleev noticed three gaps in it. He predicted that these gaps must be undiscovered elements, and predicted with great accuracy their atomic weights and chemical properties. Studying the diversity of underlying generative polygonal systems used for creating Islamic geometric patterns has led us to create a "periodic table" of polygonal sub-grid elements [**Figure 2**]. As one moves to higher symmetry sub-grid systems, there are clear analogs between specific elements within these systems. What is more, these analogs anticipate new polygonal sub-grid elements within higher-order systems. One can thereby make use of analogous tessellating arrangements that are employed within a more familiar system, such as the 4/8 or 5/10, helping to reveal the potential of less widely known, or well-explored, systems. In this way, designing geometric patterns with the 7/14 and 9/18 systems becomes immediately familiar and expedient.

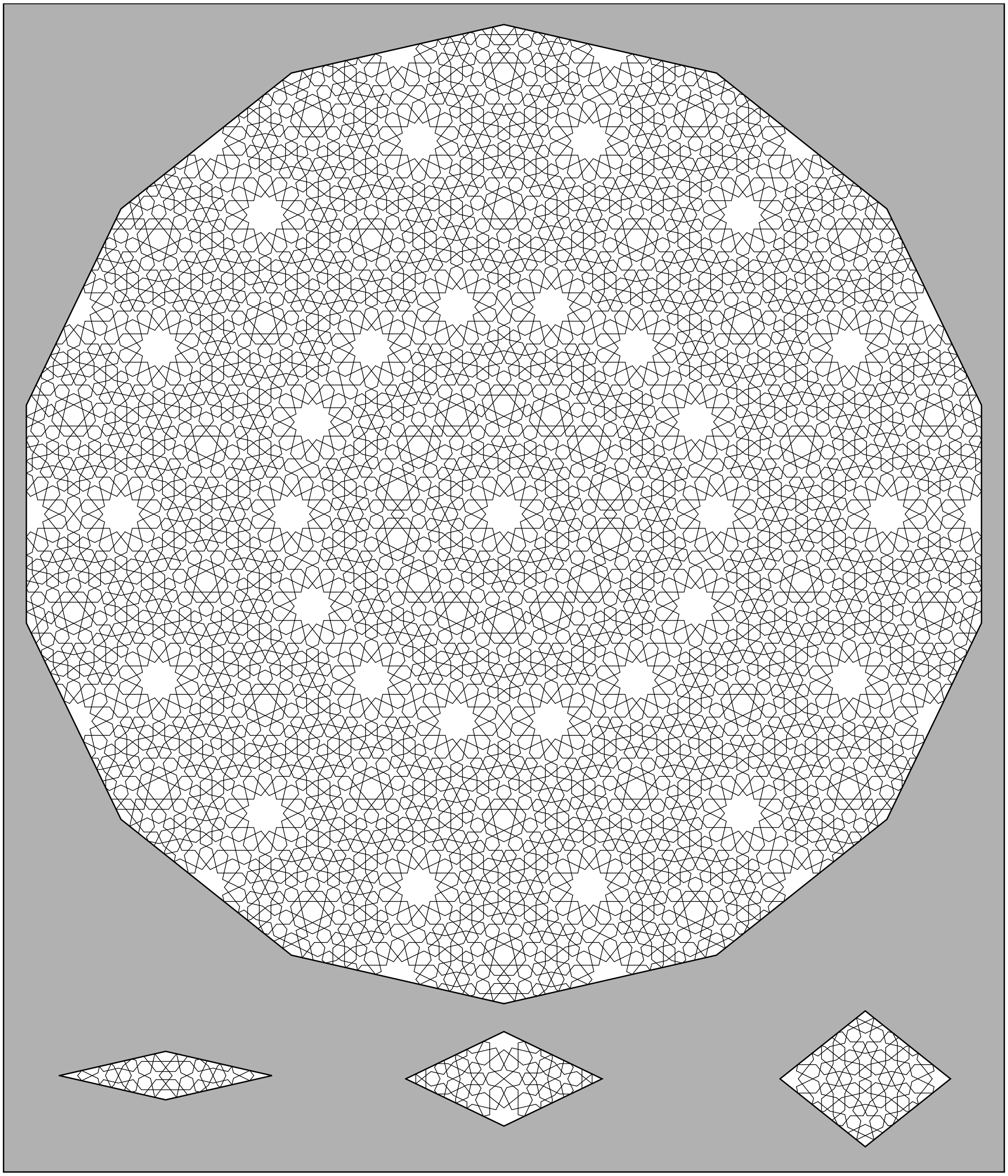

Figure 3: *A radial design made from decorating the three rhombic tiles associated with the 7/14 System. The tiles are decorated in a binary fashion (even, odd, even, odd) such that every odd vertex has a 7-pointed star and every even vertex has a 14-pointed star.* (© Marc Pelletier, 2012.)

The three rhombi associated with 7-fold symmetry can tessellate periodically, radially, and non-periodically (see Part 1: Figure 11). The application of sub-grid elements into the rhombi requires matching edge configurations, and generally places either heptagons or tetradecagons at the vertices [**Figure 3**]. Each sub-grid tessellation always has a dual [**Figure 4**], and both the initial sub-grid and its dual are equally capable of generating

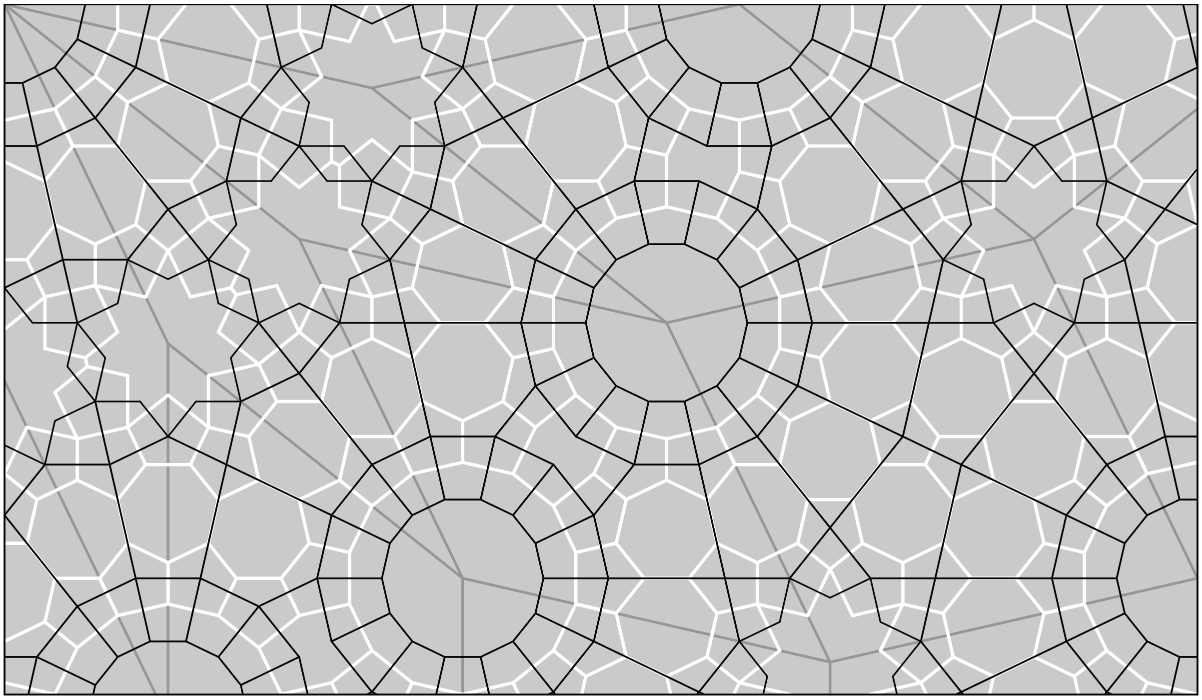

Figure 4: *Sub-grid detail from the design in Figure 3 showing the underlying generative sub-grid in black lines, and the associated* dual *sub-grid tiling in white. Dual sub-grids will create identical geometric patterns, and are a fundamental aspect of this design methodology.* (© Marc Pelletier, 2012.)

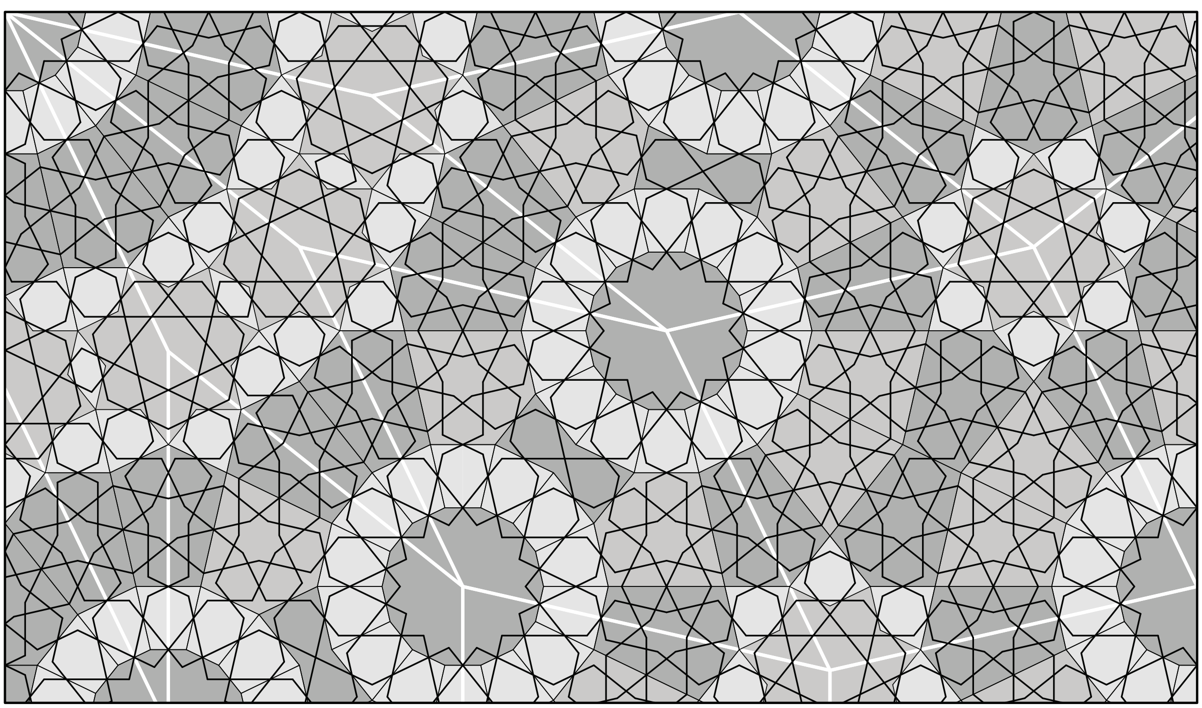

Figure 5: *This illustration shows the 14-*s2 obtuse *pattern that is generated from the primary sub-grid elements as seen in Figure 4*. (© Marc Pelletier, 2012.)

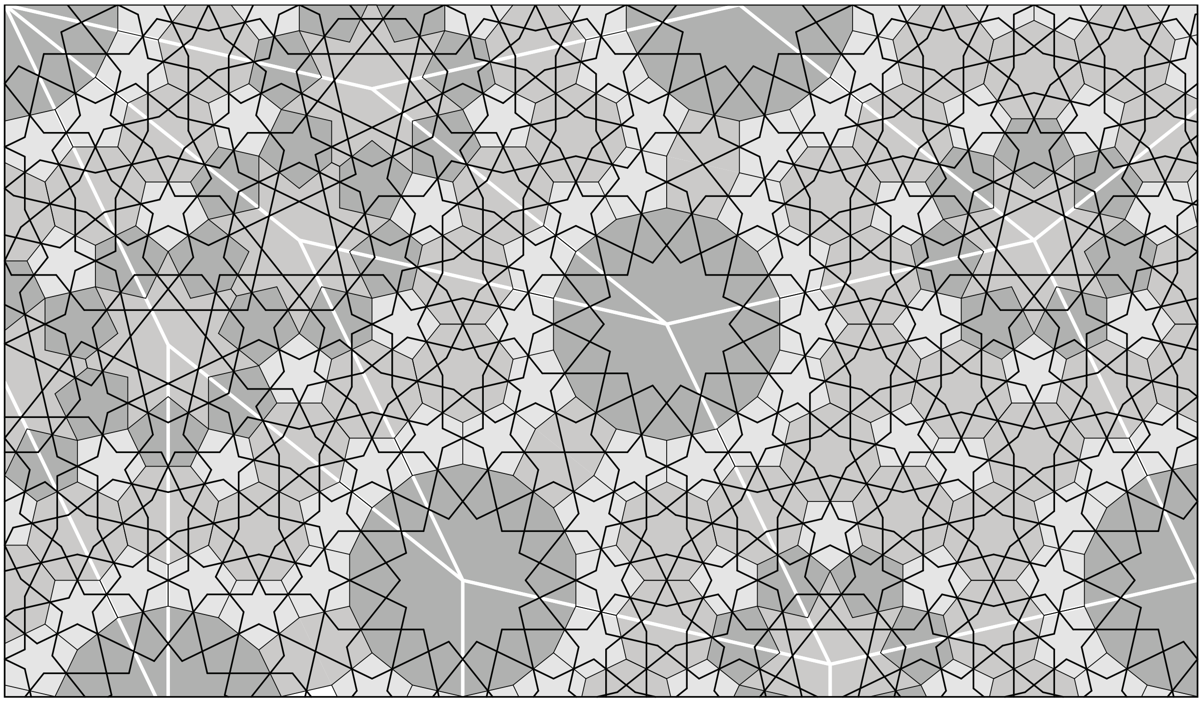

Figure 6: *This illustration shows the 14-*s*5* acute *pattern that is generated from the dual sub-grid elements shown in Figure 4. This geometric pattern is identical to the pattern in Figure 5.* (© Marc Pelletier, 2012.)

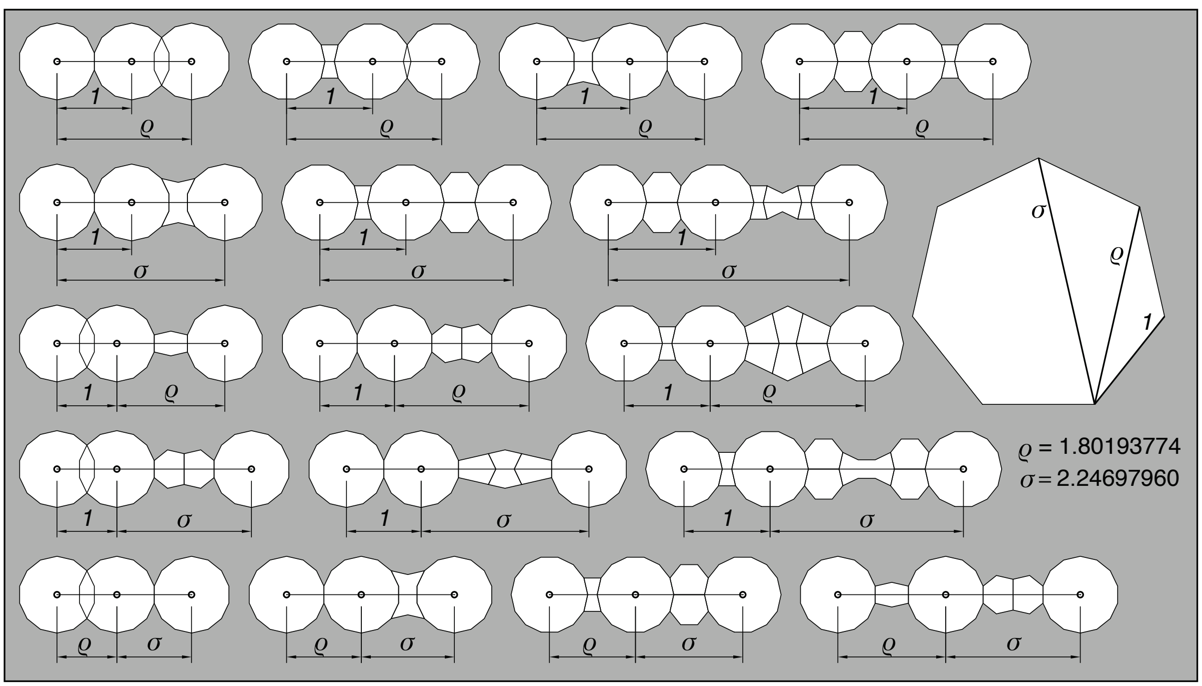

Figure 7: *Sample edge configurations for the* 7/14 System. *The lengths of these edges correspond to powers in a geometric series that have similar properties to the Golden Section. In the* 5/10 System, *the Golden Section = τ such that $\tau^2 = \tau + 1$. Analogously, the* 7/14 System *has two ratios, ϱ and σ, such that $\varrho^2 = 1 + \sigma$ and $\varrho\sigma = \varrho + \sigma$ and $\sigma^2 = 1 + \varrho + \sigma$. These proportional ratios ensure that scaling between recursive levels of a pattern have many beautiful and surprising pattern alignments.*

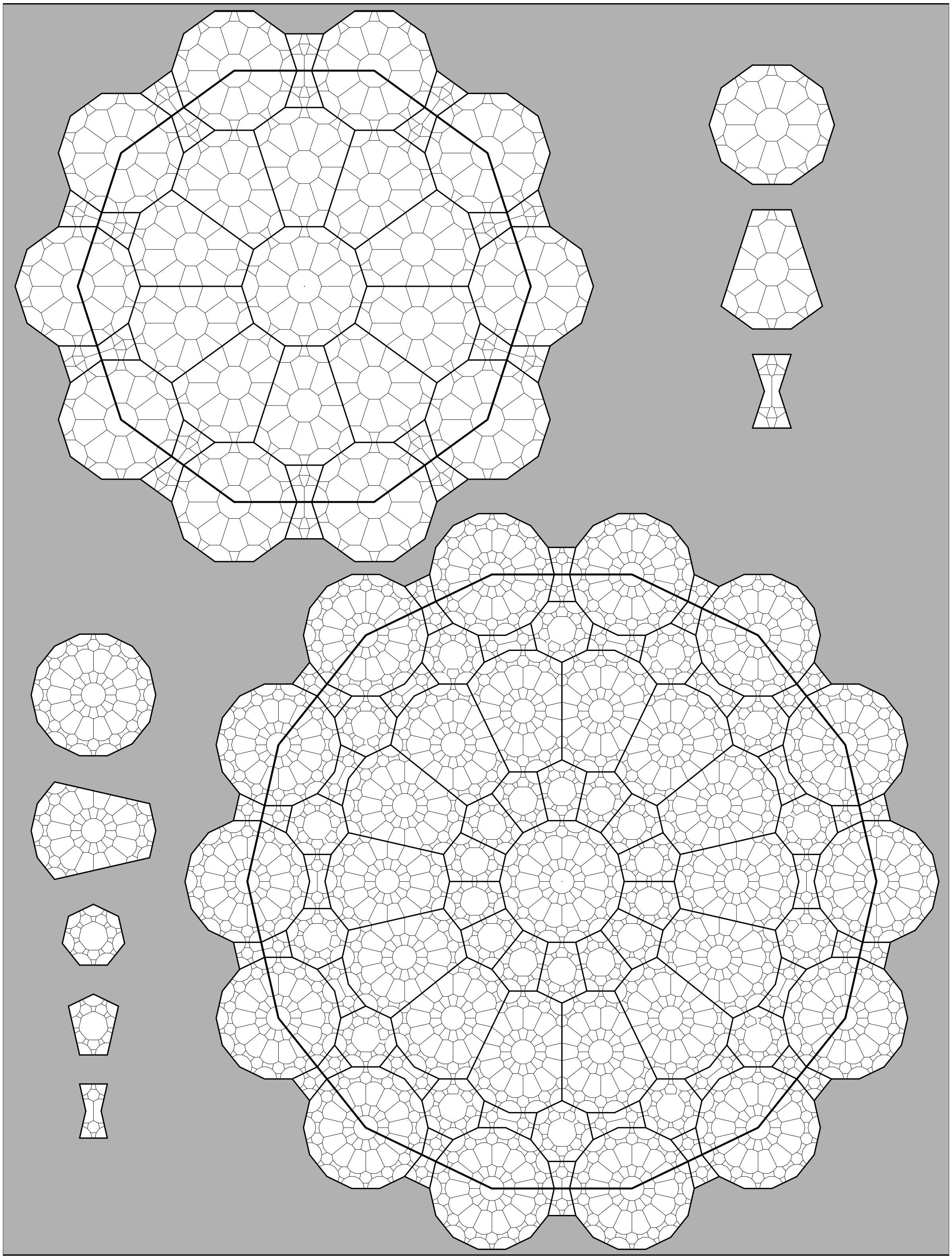

Figure 8: *Comparison between 3-level sub-grid recursions in both the 5/10 System and the 7/14 System. The secondary and tertiary sub-grids are both using an analogous edge configuration: two n-gons separated by a "bowtie" element. Details of the three-level recursive geometric patterns created from these quasi-crystalline sub-grids are shown in Figures 9 and 10*. (© Marc Pelletier, 2012.)

geometric patterns [1] (see Part 1: Figure 6). As in the 5/10 System, a specific 14-*s*2 *obtuse* design made from a given sub-grid [**Figure 5**] can equally be regarded as a 14-*s*5 *acute* design generated from the dual sub-grid [**Figure 6**]. The established conventions for creating multi-level recursive designs within the more familiar 4/8 and 5/10 systems [2] are directly transferable to the creation of quasi-periodic designs with inflation symmetry [3] created from the 7/14 System. A key feature of this methodology involves the placement of scaled-down sub-grid elements into the primary design [2]. Generally, this is accomplished by placing scaled-down tetradecagons at the vertices of the primary sub-grid and/or the vertices of the primary geometric pattern. These scaled-down elements will connect edge-to-edge with one another along the lines between the vertices, creating an *edge configuration* [**Figure 7**]. After producing the edge configurations within the primary design, one then fills in the remaining space with additional scaled-down sub-grid elements. Just as the 5/10 System is ubiquitously governed by the golden mean, the polygonal sub-grid elements that make up the 7/14 System are imbued with proportions specific to heptagonal symmetry [4], and the different edge lengths that require populating with scaled-down sub-grid elements always have a proportional relationship specific to the heptagon. And as with the 5/10 System, this inherent proportionality enables the sub-grid elements in the 7/14 System to manifest recursively through substitution tiling [**Figure 8**]. Despite recent interest in identifying quasi-crystallinity within the Islamic ornamental tradition [5], true quasi-crystalline designs created from substitution rules have not been found within the historic record. However, the polygonal modules of both the 5/10 and 7/14 systems are ideally suited to producing recursive quasi-crystalline designs with substitution rules, and the quasi-crystalline patterns that can be created from the 5/10 System [**Figure 9**] are equally elegant with analogous designs created from the 7/14 System [**Figure 10**]. Although never employed historically, the inherent scaling properties of the *7/14 System* also allow for the construction of designs in

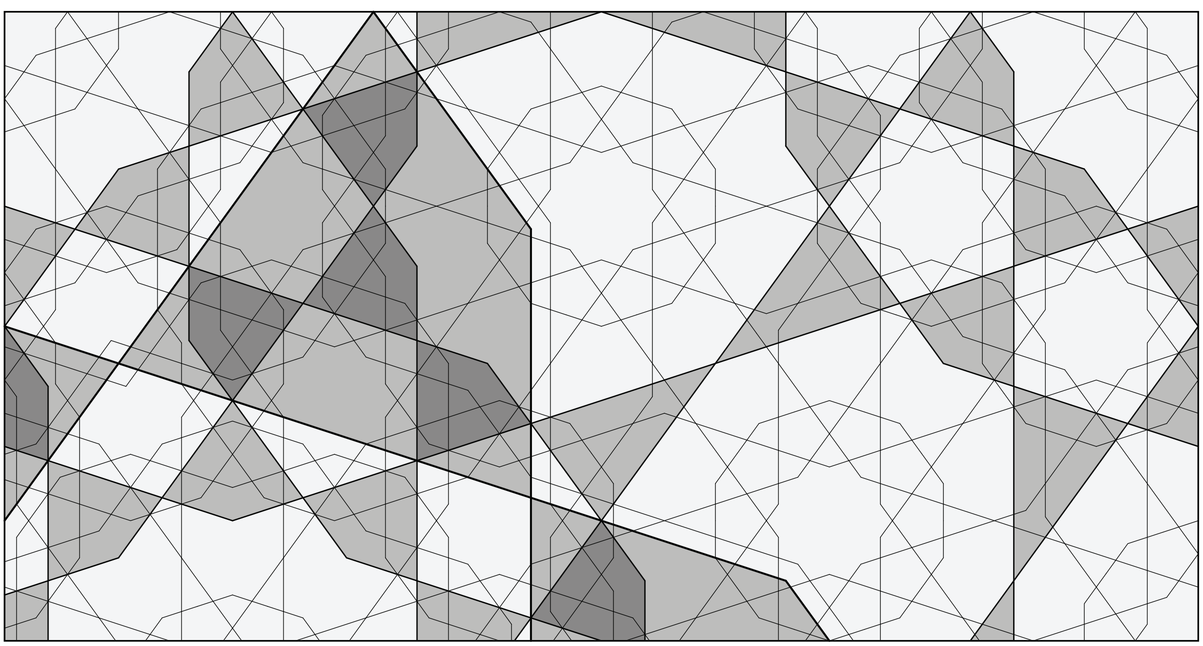

Figure 9: *Detail of the three levels of recursion for the 10-*s*3* median *pattern created from the* 5/10 System *sub-grid in Figure 8. Note the remarkable alignments of the patterns of all levels*. (© Marc Pelletier, 2012.)

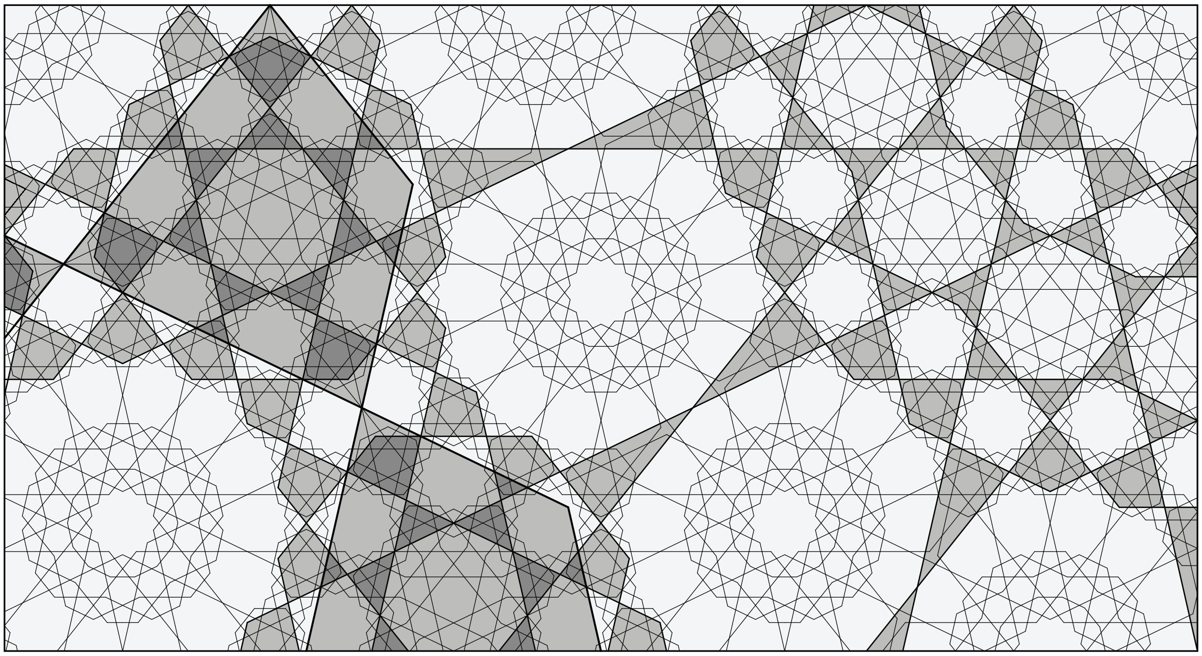

Figure 10: *Detail of the three levels of recursion for the 14-*s4 median *pattern created from the* 7/14 System *sub-grid in Figure 8. Note the remarkable alignments of the pattern lines of all levels*. (© Marc Pelletier, 2012.)

each of the three traditional Islamic self-similar design conventions developed during the fourteenth and fifteenth centuries (for example: the Darb-i Imam in Isfahan [3]).

The design potential of the 7/14 System has been largely overlooked. As of this date, very few designers other than the authors are known to have also worked with this system*. Our sincere wish is that others will join in these creative endeavors and help to expand the still narrow boundaries of this very beautiful 7-fold system of geometric design.

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* Using many of the same 7/14 sub-grid elements, Nathan Voirol has independently produced a number of remarkable 7-fold self-similar designs in the Persian style. Joe Bartholomew has also working independently with many of these same ornamented modules and has produced a variety of very lovely single-level 7-fold patterns.