The Magical Power of Our Eye
A Student Centered Approach to Building Bridges between Mathematics and Art

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Abstract

This workshop paper provides both the necessary background knowledge for the facilitator of a workshop and a series of student-centered activities designed to enable students to explore the relationship between mathematics and art, in particular, the connection between the golden ratio and paintings. These links stimulate student interest in both disciplines as well as improve critical thinking skills.

Introductory Challenges

1. Draw a vertical line thru the rectangle below so that the distance to the right hand edge is twice the distance to the left hand edge without using any measuring device.	
2. Draw a vertical line thru the rectangle below, so that the distance to the right hand edge is about 1.61803399 the distance to the left hand edge, without using any measuring device.	

The second challenge is rather absurd – who could possibly do this? Yet in many pieces of artwork, the composition actually suggests where such a line belongs. In this workshop we will explore the fascinating link between mathematics and art as described by the golden ratio, approximately 1.61803399. How could this strange number describe anything? Where exactly does this number originate? In the workshop activities we investigate this number through the lens of mathematics, the artist's eye, and patterns of walls.

An Artistic Introduction

A basic prerequisite for most artistic composition is some form of continuity, visually connecting smaller units to larger units, and helping to weave the whole structure together. How is geometric proportion involved in the design of art? In architecture where the rectangle rules the shape of most structures, the concern for the harmony of geometric proportion is most obvious. But it is actually quite similar in painting a picture. Since the pictorial space is usually a rectangle, it invites a system of geometrical proportioning which will provide a unifying invisible grid to guide the arrangement of the subject matter. The artist develops a systematic underpinning to allow for a variety of placement possibilities in accord with the subject or content of the work.

What division of space looks attractive? Which point on a line divides the line 'perfectly' so that the relationship of one part to the other is visually satisfying? Or, if one selects a rectangle where the height to width seems 'just right', that is having perfect proportion, most people select one where the shorter length is to the longer, as the longer is to the whole. This interesting relationship is known variously as the golden ratio, the golden section, or the golden mean. The golden ratio is an irrational number, approximately equal to 1.6180, and known as Phi, (pronounced 'fee'), the twenty-first letter of the Greek alphabet. It is a canon of proportion explained in virtually every book on the fundamentals of art and design. Thought to be in use as far back as Stonehenge and utilized by artists and architects through the centuries; it occurs in Egyptian art, especially Greek art, the art of the Middle Ages, the Renaissance, and throughout much of Modern art. Past masters were all well acquainted with this system, the "divine proportion". The ratio, for some, has an almost mystical quality, and some would venture to say is a universal underpinning for the structure of the cosmos. [1]

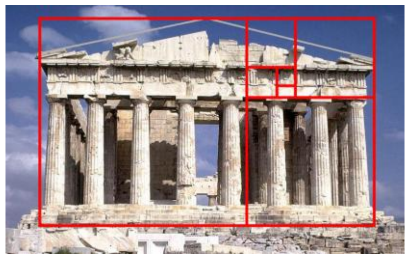


Figure 1 *The Parthenon, a canon of Western architecture designed on golden ratio proportions here defined by a golden ratio rectangle (sides in a 1:1.618 ratio).*

The Mathematical Definition

The actual definition of the golden ratio comes from dividing a rectangle into two parts, measuring a units and b units, so that the ratio of the smaller part to the larger part, $\frac{a}{b}$, is the same as the ratio of the sum of the two parts to the larger part, $\frac{a+b}{b}$, that is $\frac{a}{b}=\frac{a+b}{b}$. See Figure 2.

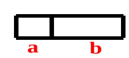


Figure 2

The Greek letter phi, ϕ, is used to denote this golden ratio, $\phi=\frac{a}{b}=\frac{a+b}{b}$. Notice that since $\phi=\frac{a}{b}$, the reciprocal of ϕ is $\frac{b}{a}$, $\frac{1}{\phi}=\frac{b}{a}$. One of the unusual properties of ϕ is that when we add the reciprocal of ϕ plus 1, we get ϕ, as shown in the following computation: $\frac{1}{\phi}+1=\frac{b}{a}+1=\frac{b}{a}+\frac{a}{a}=\frac{a+b}{a}=\phi$. Thus, $\frac{1}{\phi}+1=\phi$. Let's solve the equation $\frac{1}{\phi}+1=\phi$ to find the actual value of ϕ. Keep in mind that ϕ must be positive since it is the ratio of two distances.

Student Proof Activity: If this workshop is being used for students who have studied the quadratic formula, consider a Student Proof Activity by cutting the strips below and have pairs of students put them in the appropriate order matched with the correct reason. This inventive approach forces the students to review the mathematical ideas as they put together a proof from a puzzle of equations!

$\frac{1}{\phi}+1=\phi$	We begin with the equation satisfied by the golden ratio.

$\phi\left[\dfrac{1}{\phi}+1\right]=\phi[\phi]$	Multiply both sides of the equation by ϕ.
$\dfrac{\phi}{\phi}+\phi=\phi^2$	Simplify.
$0=\phi^2-\phi-1$	Rewrite.
$\phi=\dfrac{-(-1)\pm\sqrt{(-1)^2-4\cdot 1\cdot(-1)}}{2\cdot 1}$	This is a quadratic equation so we use the quadratic formula to solve for ϕ.
$\phi=\dfrac{1\pm\sqrt{1+4}}{2}=\dfrac{1\pm\sqrt{5}}{2}$	Since ϕ is the ratio of two lengths it must be positive, so $\phi=\dfrac{1+\sqrt{5}}{2}$.

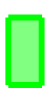
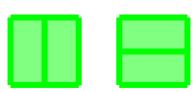
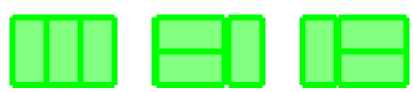
The Golden Ratio appears in the strangest ways. In the following activity, see if you can find the connection to the golden ratio as you build walls! Note: This exploration provides students with the opportunity to use critical thinking skills as they "discover" the golden ratio. [2]

Activity: Building Walls

"Mending Wall" by Robert Frost

Something there is that doesn't love a wall, . . .
Before I built a wall I'd ask to know
What I was walling in or walling out. . . .
He only says, 'Good fences make good neighbors'.
Spring is the mischief in me, and I wonder
If I could put a notion in his head:
'Why do they make good neighbors?'

Suppose we want to build a low wall out of bricks to divide our garden. The length of each brick is twice as long as the height and the wall must be two units high. Our challenge is to determine how many different patterns exist to build a wall of any particular length. If the wall is only one unit wide, it consists of one brick standing on its short edge, no choices here. For a wall that is 2 units long we can construct two wall patterns, using either two bricks on top of each other or vertically next to each other. The patterns become more interesting as the length of the wall increases, as shown below.

Brick Walls 1 Unit Long	Brick Walls 2 Units Long	Brick Walls 3 Units Long
		

1. Use the bricks to build all possible walls of length 4. Count the number of walls constructed and fill in the appropriate box in the <u>second</u> row of the table I. Repeat this step for walls of length 5 and 6.

2. Describe how to find all possible patterns for a wall of length n, if you know the entire list of patterns for walls less than n units long.

3. Use the information from 1 and 2 to complete the <u>second</u> row of table I.

4. In the third row compute the ratio of the number above to the right divided by the number above to the left; use at least 4 decimals. The first one is done for you.

Wall Length	1	2	3	4	5	6	7	8	9
# of Walls	1	2							

$\frac{2}{1}$ =2.000							

Table 1: *Counting Walls*

5. What number does the ratio in the third row approach? A bit strange, isn't it?

Focal Points In Art

A focal point in art is a center of attraction or attention, a point of interest, emphasis, or difference that captures the viewer's attention. There is usually a primary focal point with minor focal points by which a viewer may be led through the composition. A focal point may be a face, a hand, an area of bright color, contrasts in value or texture, busy areas or an object in isolation. Viewers tend to seek out information about what is happening in the picture, to discover a story or narrative to read. While most will naturally move from one element to another in their own fashion, an artist can control to some extent where the eye moves next. In Porter's *July Interior*, for example, one will naturally look at the face of Porter's wife, Anne, the lady in bed, who seems deep in introspective thought, and then look for clues about her, by moving on to the objects that surround her. (Figure 3) Her face is the obvious primary focal point, then maybe her things, and so on. The viewer of the painting gathers this information in a split second. So where should the artist place the primary focal point and minor focal points? Since the concern in composition is the relationship between the canvas rectangle and the placement of these elements, a concern for geometric harmony is warranted.

An Example: Porter's *July Interior*

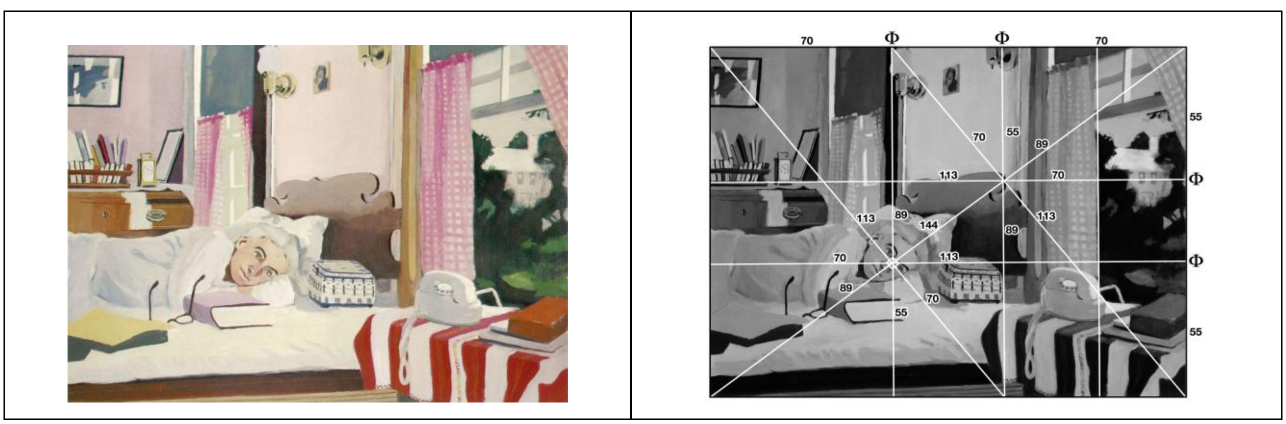


Figure 3: *Fairfield Porter*, July Interior *1964, Hirshhorn Museum and Sculpture Garden, Washington, D.C. (based on the published size of 143/4 x 183 cm)*

Although Porter's painterly brushwork and casual domestic subject matter may belie his formal division of space, his paintings demonstrate a carefully articulated arrangement of forms. Through sequential division of space, a geometric progression of parts, he achieves an analogous correlation between the rectangle of the whole picture plane and the grid of rectangles defined inside it. By this means his work has an asymmetrical harmony of parts, an architecture which houses his imagery in a unified whole. As the artist, Jacques Villon wrote, "The framework of a work of art is also its most secret and deepest poetry."

It might seem as if everything is arbitrarily placed in the painting but Porter's compositional structure is secretly hidden in plain sight. His chosen proportions of *July Interior*'s canvas sides are one to the root of the golden ratio, 1: $\sqrt{\phi}$. Because of this, successively dividing the sides by the golden ratio number (1.618) or its root (1.272) gives a series of harmonizing measures to form a spatial grid. The 1:1.272 root golden ratio rectangle has sides and diagonal that are in a geometric progression of 1: $\sqrt{\phi} : \phi$. Dividing the long side successively by 1.272 yields the short side and all the same golden ratio proportions on both long and short sides. (Figure 3)

The sides and diagonal intersect at the bridge of Anne's nose. The strongly demarcated dark vertical immediately above Anne's face (identifiable by reference to other paintings, as a post of a four-poster bed) forms the primary golden ratio vertical division of the long side. Anne's face is at the division of diagonal, vertical and horizontal each divided by 1.272, giving a numerical sequence of 183 (long side): 144 (short side): 113: 89: 70: 55: 43: 34. This center is found by using a main diagonal of the canvas, and a diagonal drawn at right angles to it from the opposite corner. Unique to the root golden ratio rectangle is that the vertical and horizontal drawn at the intersection of these diagonals are at the golden ratio divisions of the short and long sides. The position of other elements in the painting can also be determined by using a compass set at golden ratio numbers and the arcs drawn. [3]

Activity: Exploring Paintings

Part 1

Figure 4: Fairfield Porter, *View of the Barred Islands* 1970, Herbert W. Plimpton Collection.

1. Carefully examine the painting in Figure 4. Describe the place to which your eye goes first. Next, place a dot at the point in that area that seems the most important, and label the point A.

2. Repeat #1 two more times, labeling the points B and C.

3. Complete the following table for these points.

4. Does the golden ratio occur in the vertically or horizontal ratios?

Point	Distance to left edge of painting	Distance to right edge of painting	Ratio	Distance to top of painting	Distance to bottom edge of painting	Ratio
A						
B						
C						

Part II

Figure 5: Chris Bartlett, *The Golden Horn, Istanbul*, Artist's collection

1. Carefully examine the painting in Figure 5. Describe the place to which your eye goes first. Next, place a dot at the point in that area that seems the most important, and label the point A.

2. Repeat #1 two more times, labeling the points B and C.

3. Complete the following table for these points.

4. Does the golden ratio occur in the vertical or horizontal ratios?

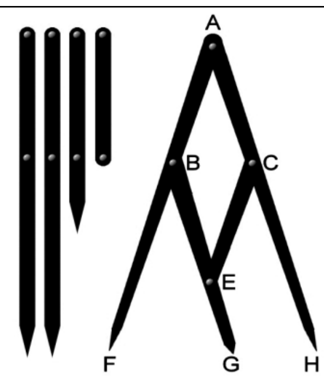
Point	Distance to left edge of painting	Distance to right edge of painting	Ratio	Distance to top of painting	Distance to bottom edge of painting	Ratio
A						
B						
C						

Golden Section Calipers

To determine golden ratio proportions in small reproductions of art works a useful tool is a gauge that opens with the same ratio regardless of size and can quickly determine if a primary focal point is a golden ratio distance from the sides.

Activity: Exploring Paintings, Extended

III. Construct a golden ratio caliper using the template provided. It may be constructed from stiff cardboard, plastic, balsa wood or even long stirring sticks. Make holes, and then place a paper fastener at each of the designated points. When the gauge is adjusted, the middle arm will always show the golden section ratio point between the two outer arms.



AF = AH =136 mm
BG = 84 mm
AB = AC = BE = CE = 52 mm
EG = 32 mm

IV. Use your golden caliper on the paintings in Part I and Part II to explore additional points in the paintings that satisfy the golden ratio and determine if the artist is directing your eye to those points.

Suggestions for the Workshop Facilitator

Since there is a significant difference between reading a paper, participating in a workshop, and leading a workshop, in this section we provide a possible agenda for a 60 – 90 minute workshop.

I. Introductory Challenges

Drawing a line to form a ratio of approximately 1 to 2 is a reasonable request, but a ratio of 1 to 1.61803399 is rather absurd, but a fun way to engage workshop participants. The facilitator might ask students to actually measure and see who got closest to the given number.

II. Mathematical Definition of Golden Ratio

NCTM, the National Council of Teachers of Mathematics, suggests that students should begin to see and do proofs as early as possible. The Student Proof Activity provides the opportunity for a student to apply their knowledge of mathematics to putting the steps and justifications in the appropriate order.

III. Building Walls Activity

Constructing fences satisfying the given conditions is an open ended problem where the students do not know how many walls of a given length exist. It illustrates how the golden ratio number appears in the strangest places.

IV. Exploring Painting Activity

A very brief explanation of the definition of focal point is all students need before beginning this activity. Be sure to emphasize that the focal point must be a specific point not a region. For example, the focal point might be the point between the eyes on a face, but not the face.

V. Exploring Paintings, Extended

The construction and use of golden calipers is fun and enables students to find potential focal points without the more traditional form of measuring with a ruler.

VI. Historical Background and Details of a Particular Painting

A brief discussion of the golden ratio in history followed by a more detailed exploration of a specific painting pulls together all of the ideas the students have now explored in a hands on manner.

Conclusion

This series of hands-on projects opens a door that both motivates and enables students to view mathematics as a key for artistic explorations and art as a means to apply mathematics. As students seek to explore the interdisciplinary connections, a new dimension enters their world.

Activity Solutions

Building Walls

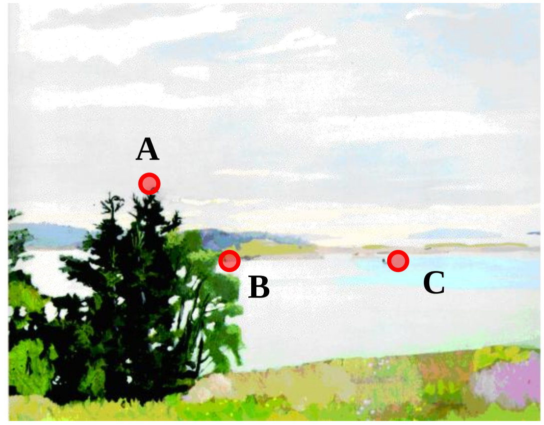
Wall Length	1	2	3	4	5	6	7	8	9
# of Walls	1	2	3	5	8	13	21	34	55

$\frac{1}{2}=$ 2.0000	$\frac{3}{2}=$ 1.5000	$\frac{5}{3}=$ 1.6667	$\frac{8}{5}=$ 1.6000	$\frac{13}{9}=$ 1.444	$\frac{21}{13}=$ 1.6153	$\frac{34}{21}=$ 1.6190	$\frac{55}{34}=$ 1.6176

The ratio approaches the golden ratio.

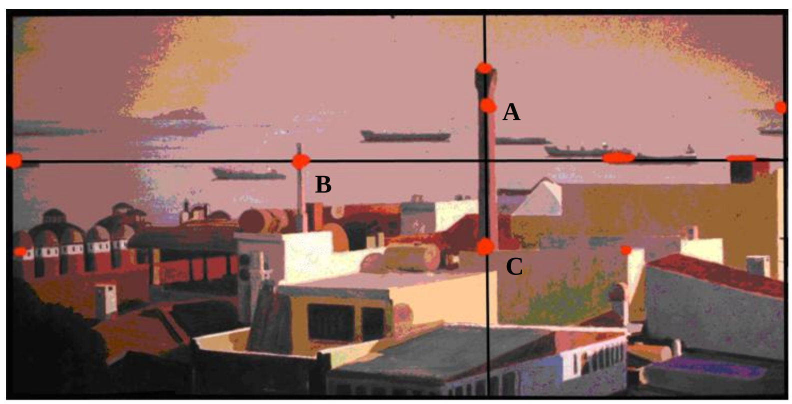
Exploring Paintings

Part I A sample response



Point	Distance to left edge of painting	Distance to right edge of painting	Ratio	Distance to top of painting	Distance to bottom edge of painting	Ratio
A	44	119	2.7	55	70	2.272
B	62	100.5	1.62	79	49	1.612
C	115	47	2.5	79	49	1.612

Part II A sample response.



Point	Distance to left edge of painting	Distance to right edge of painting	Ratio	Distance to top of painting	Distance to bottom edge of painting	Ratio
A	102	63	1.619	20	60	3.0
B	62.5	101	1.616	31	50	1.613
C	102	63	1.619	81	50	1.62

References

[1] Christopher Bartlett," Fairfield Porter's Secret Geometry", *Bridges Proceedings*, 2005,17-24
[2] http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibpuzzles.html#domino
[3] Christopher Bartlett, "Decoding Fairfield Porter's July Interior", (Chicago; University of Chicago Press/Smithsonian American Art Museum, *American Art*, Spring 2007) 98-101