Inside and Outside the Rhombic Hexecontahedron

A Study of Possible Structures with Rhombic Hexecontahedron with the Help of Physical Models and *Wolfram Mathematica*

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Abstract

The rhombic hexecontahedron is a twenty-pointed star polyhedron that is bounded by sixty golden rhombi. It can be understood as a stellation of its relative, the Keplerian rhombic triacontahedron, or as the assembly as twenty prolate golden rhombohedra. This study investigates these relationships and the possible structures that can be created by dissecting and recombining parts of these forms. Many possible structures are introduced with the help of Wolfram Mathematica graphics, the Zometool construction set, and the author's own StyroBlock modeling system, see www.kabai.hu).

History of Rhombic Hexecontahedron (RH)

RH as a geometrical object was probably first noted and illustrated by Unkelbach in 1940 [1]. The date is rather surprising, because the closely related rhombic triacontahedron (RT), which is generally attributed to Kepler, was first discussed and analyzed in his 1619 book *Harmonies of the World* [2]. In an 1987 article, Guyot reported about the appearance of the RH shape in quasicrystals [3] (Fig. 1). The images probably inspired extended research all over the world. In particular, the author's interest was aroused, and he started to make models of the structure with StyroBlock system (Fig. 2), as well as graphical images with the help of the software Wolfram Mathematica (Fig. 3).

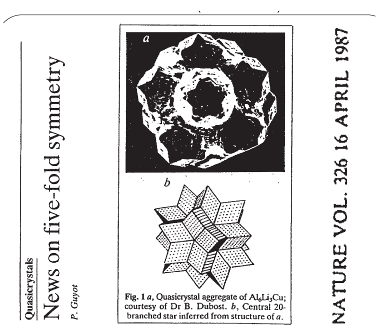


***Fig. 1**: Image from the article of Guyot*

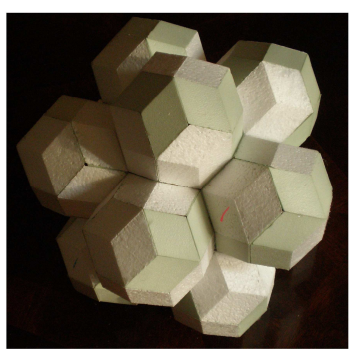


***Fig. 2**: StyroBlock model of the structure*

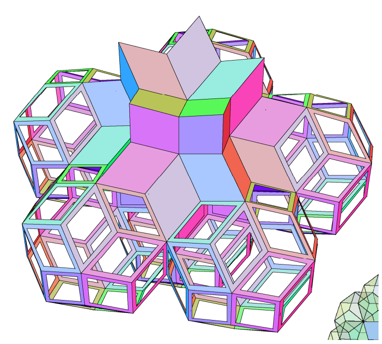


***Fig. 3**: Interpretation of the structure with Wolfram Mathematic graphics*

The history of quasicrystals began in 1984, when D. Shechtman *et al.* demonstrated a clear diffraction pattern with a fivefold symmetry [4]. Not very long after that, the Hungarian Ágnes Csanády managed to photograph crystals (Fig. 4), the shape of which is very similar to that of RH [5, 6]. One of such image also appeared on the cover of a physics journal in Hungary in 1986 (Fig. 5).

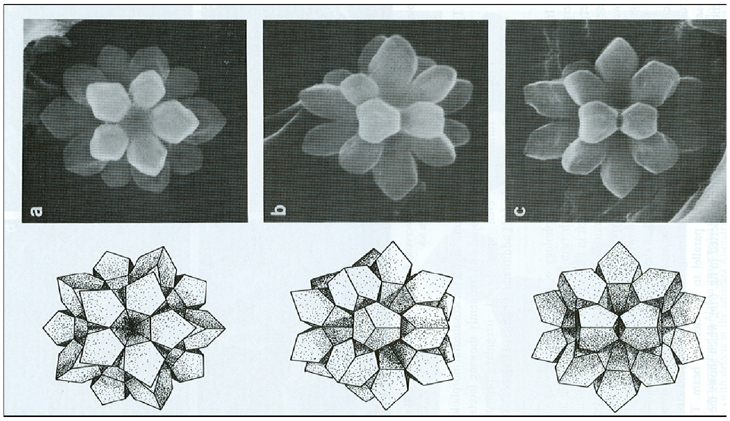


Fig. 4: Images by A. Csanády et. al.

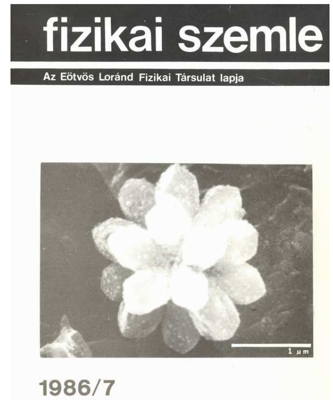


Fig. 5: Cover page of Physics Bulletin with the image produced by A. Csanády

RH on Cover Images

The cover of book Regular Polytopes by H.S.M. Coxeter shows a great rhombic triacontahedron, the shape of which partly coincides with RH (Fig. 8). The author's images related to RH and created with Wolfram Mathematica appeared several times on publications (Fig. 9 ... Fig. 12).

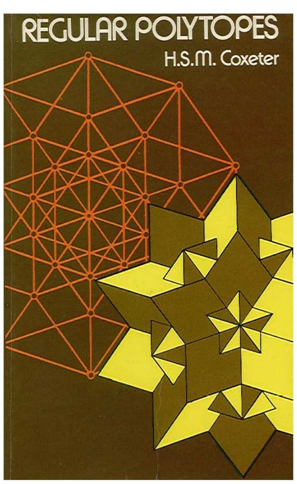


Fig. 8: The Great Rhombic Triacontahedron

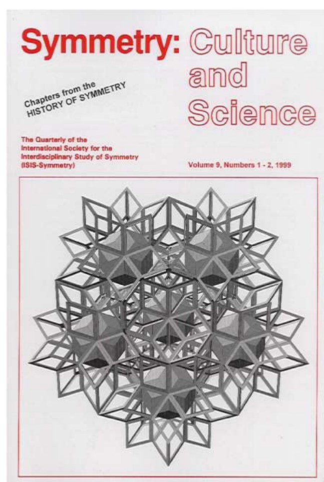


Fig. 9: Cluster of RHs

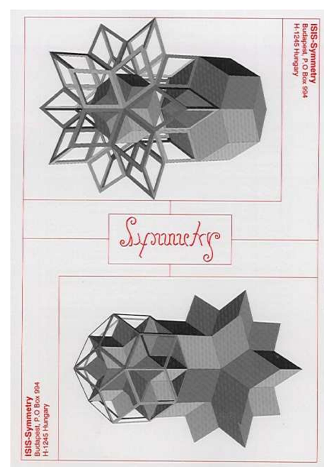


Fig. 10: Pairs of RH and RT

The 204-page book Rhombic Structures includes lots of images of RH (Fig. 13) [7]. Several ways of constructing RH is also explained in the author's other book [8]. The Wolfram Demonstration Project includes 17 demonstration at this moment, in which RH is mentioned [8]. These were created mostly by the author and by Izidor Hafner, who is a prominent researcher of rhombic geometry [10]. Branko Grünbaum also studied RH extensively [11, 15], and it is also dealt with by H.M.S Coxeter, as well as by Michael S. Longuet-Higgins [12], Koyi Miyazaki and George Hart [13]. Wolfram Research selected RH for the logo of the novel Wolfram Alpha "Computational Knowledge Engine" (Fig. 14), and there is a short description of RH in its blog (What is in the Logo?) [14]. Wolfram also produced a paper model of RH (Fig. 15).

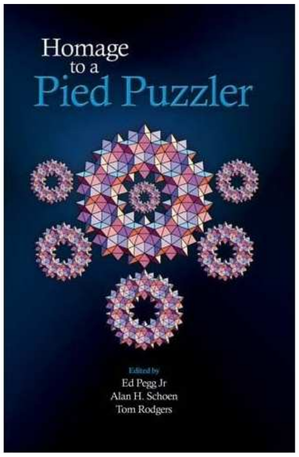


Fig. 11: *Rings of RHs*

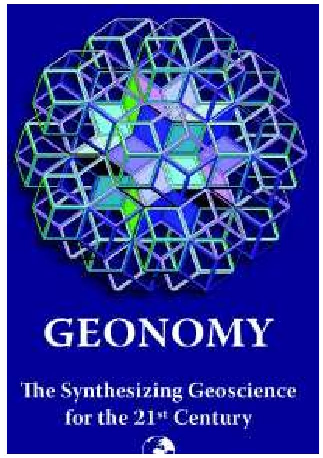


Fig. 12: *12 RT around an RH*

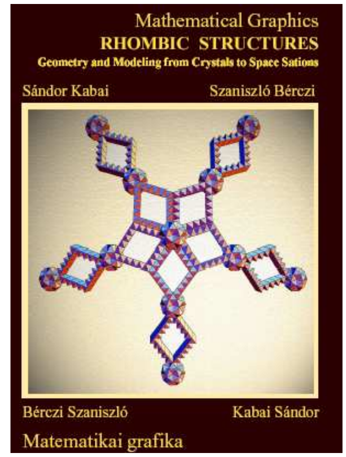


Fig. 13: *Rhombic structures*

Models of RH can be made with the plastic-magnetic Rhombo patented by Longuet-Higgins. The hollow paper templates produced by Daniel Suttin are also good for modelling RH structures.

Fig. 14: *Wolfram Alpha logo*

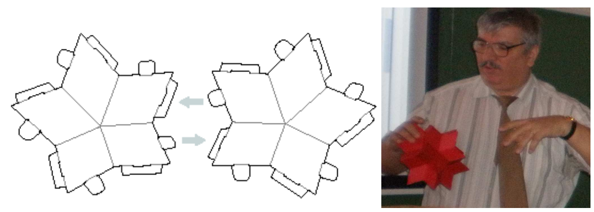


Fig. 15: *Wolfram paper model of RH*

Construction of RH

A Wolfram Demonstration can show how the faces of rhombic triacontahedron (RT) are extended until an RH is produced. (Fig. 16). This is an illustration of the concept of polyhedron stellation. The stellations of RT are detailed in an article by Messer [16].

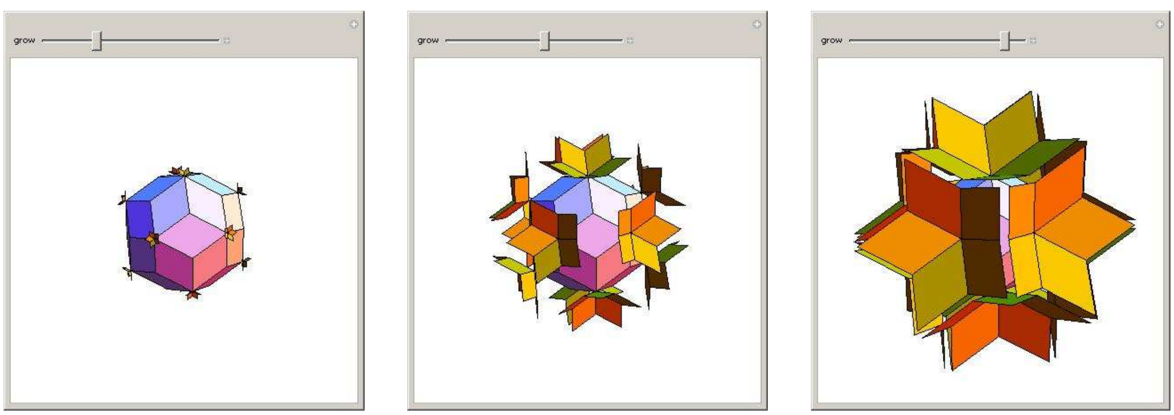


Fig. 16: *RH as a stellation of RT*

20 prolate golden rhombohedra can make a solid RH. (Fig. 17). 20 golden octet truss (GOT) placed on the faces of an icosahedron also produces an RH (Fig. 18). If half of the PGRs are used, then they can be placed on the hexagonal faces of a truncated icosahedron to form an the outline of RH (Fig. 19). Alternatively, the half rhombohedra can be adhered together to from an RH with a void shaped like a buckyball.

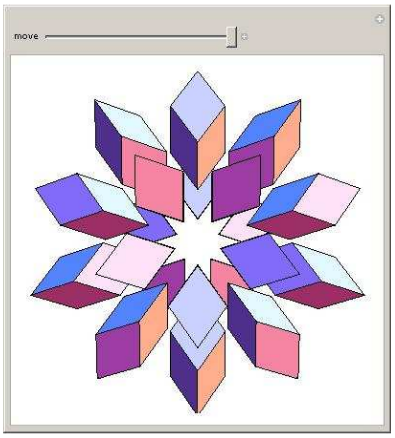


Fig. 17*: Twenty PGRs make an RH*

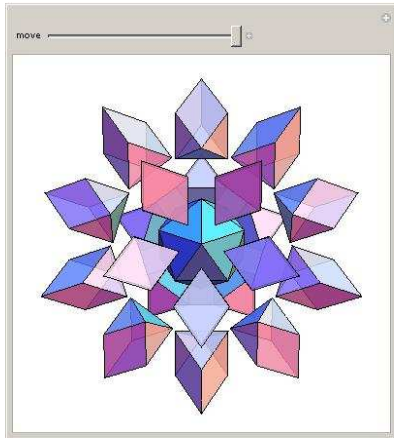


Fig. 18*: Twenty golden octet trusses.*

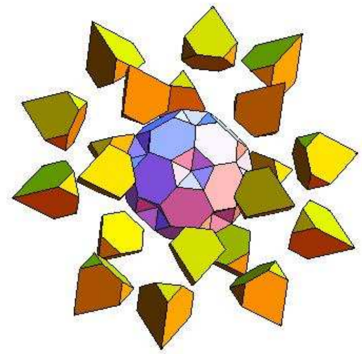


Fig. 19*: Twenty half PGRs and a bukyball*

Placing a set of 3 rhombi, with the central rhombus coinciding with RT faces, results in an RH (Fig. 20). Hollow templates (12 of them) could be assembled with paper clips into an RH (Fig. 21). Twenty templates of three rhombi can also be used to produce a paper model (Fig. 22).

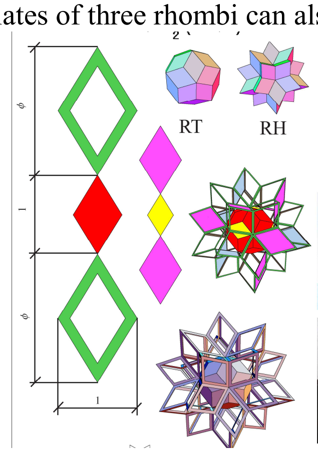


Fig. 20*: Rhombi on RT*

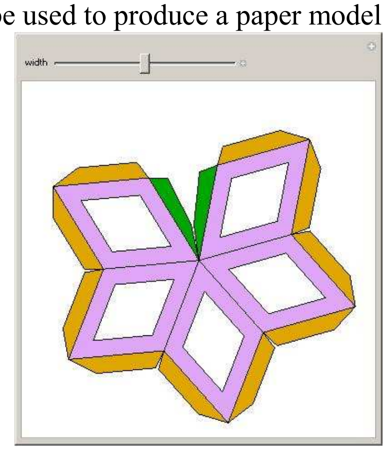


Fig. 21*: Hollow template of five rhombi*

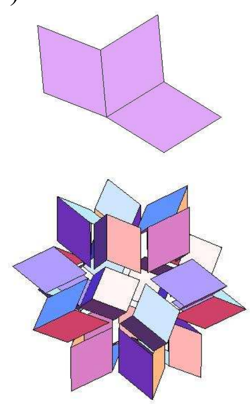


Fig. 22*: Twenty templates of three rhombi.*

Internal parts of RH can be selected for practical purposes, e.g. a bucky dome supported by legs (Fig. 23). If the edges are regarded as struts, then a lattice structure is produced, which could be used to support parabola antenna (Fig. 24). A model made by the author reveals the relationship of RH to various objects, such as cube, truncated icosahedron (fullerenes), Penrose tiling as a projection of RH (Fig. 25).

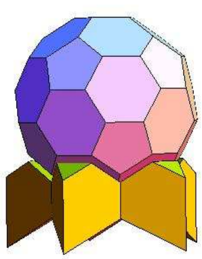


Fig. 23*: Selected parts of RH*

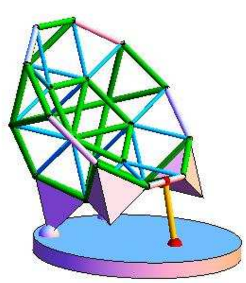


Fig. 24*: Parabola antenna support.*

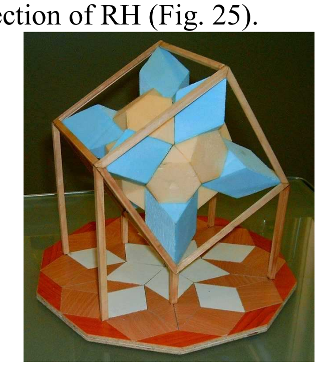


Fig. 25*: A model made by the author*

Internal structure of RH

Let us place an object (e.g. sphere, dodecahedron or truncated icosahedron) at each node and edge of a frame consisting of 20 golden rhombohedra forming an RH. There is one object at the centre, 12 at the vertices of an icosahedron, 12 again at the vertices of another icosahedron, 60 at the vertices of a truncated icosahedron, 30 at the vertices of an icosidodecahedron, 60 at the vertices of another truncated icosahedron, and 20 at the vertices of a dodecahedron, 195 altogether. This can be done with a Wolfram Mathematica demonstration (Fig. 26), with the Zome system (Fig. 27), and with Wolfram Mathematica graphics imitating the Zome system (Fig. 28).

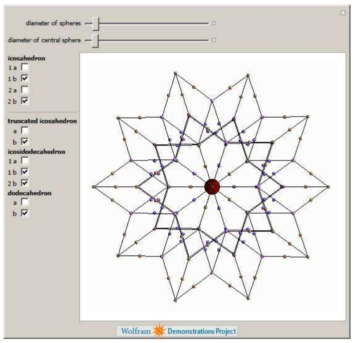


Fig. 26*: RH frame with Wolfram demonstration*

Fig. 27*: RH frame with Zometool*

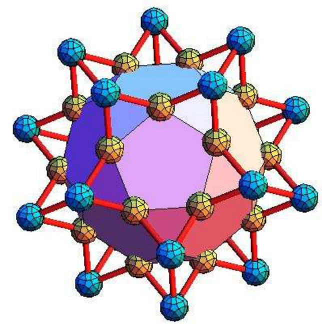


Fig. 28*: RH frame with Zome balls.*

When we start with solid rhombohedra, then we can construct structures with icosahedral symmetry using parts of the rhombohedra. For instance GOT or Golden Octet Truss consists of a tetrahedral part and an octahedral part established by removing a tetrahedral part from a golden rhombohedra (Fig. 29). This polyhedron is good for creating many different shapes, including linear and spatial structures. Twenty GOTs forms a truncated RH) (Fig. 30), which is actually part of an RH (Fig. 31).

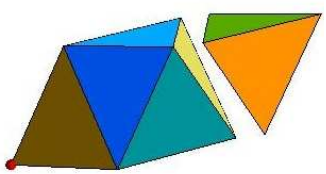


Fig. 29*: GOT*

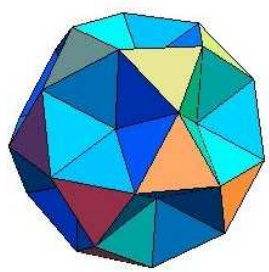


Fig. 30*: Truncated RH*

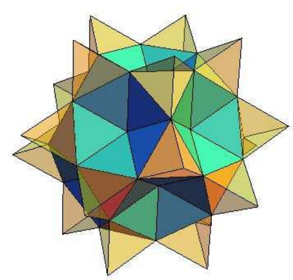


Fig. 31*: RH*

RH with matching RT are aligned along the edges of a golden rhombus (Fig. 32). Such golden rhombi can be placed on rhombic polyhedra such as RH, RT, etc (Fig. 33). Instead of RT, the RHs could be connected with sticks. This can be a model of a novel construction set. (Fig. 34).

Construction with RH Units

RH is placed at each vertex of small rhombicosidodecahedron (SRID), then these are connected with RTs (Fig. 35). One RH can be placed at each of the 32 vertices of RT. The gaps between the RHs (along the edges of RT) are filled with RT or RT-RH pairs to create a frame RT (Fig. 36).

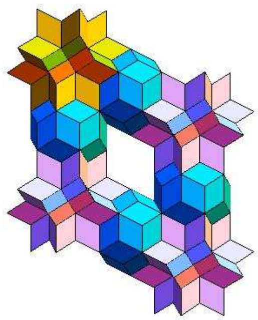


Fig. 32: *RH and RT along edges of a rhombus*

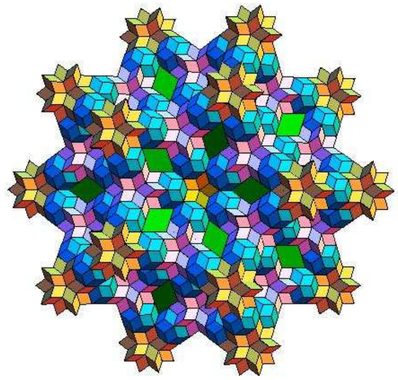


Fig. 33: *RT and RH along the edges of a larger RH*

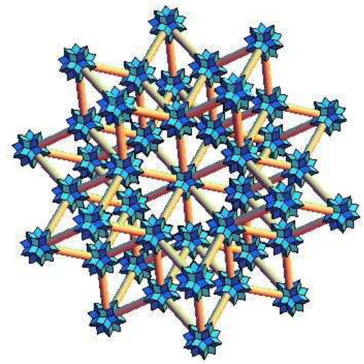


Fig. 34: *RH and the nodes of a larger RH.*

The same can be done with the truncated icosahedron (TICO), then matching RHs and RTs can be added (Fig. 37) to start a construction that could be continued ad infinitum.

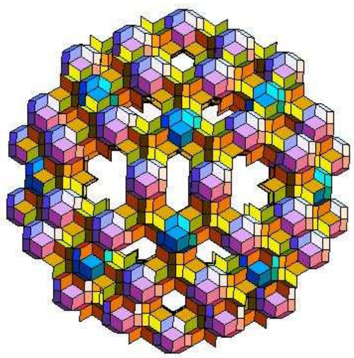


Fig. 35: *RHs at the vertices of SRID*

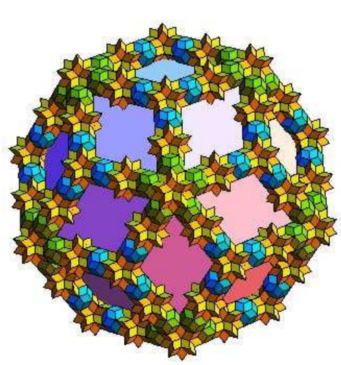


Fig. 36: *Frame RT.*

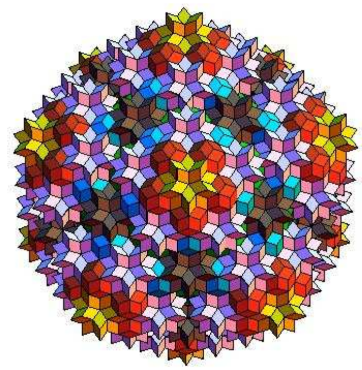


Fig. 37: *RHs around a TICO*

RH fits into a golden rhombus (Fig. 38), as well as into a dodecahedron (Fig. 39). In turn, dodecahedra can be placed with face to face connection along the edges of an RH (Fig. 40).

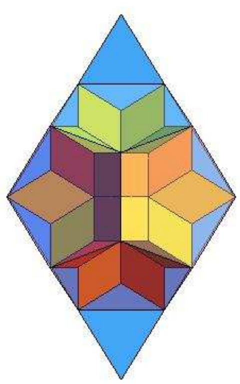


Fig. 38: *RH in a rhombus*

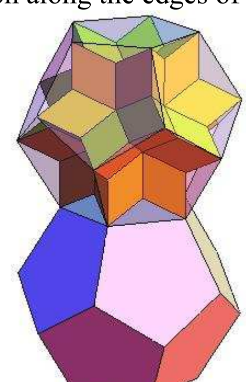


Fig. 39: *RH in a dodecahedron*

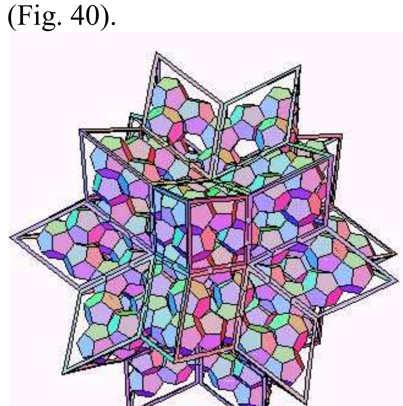


Fig. 40: *Dodecahedra along the edges of RH*

Twelve polar zonohedra (PZ) can be constructed around an RH (Fig. 41). This can be done also with the Zometool (Fig. 42, 43). This arrangement is related to "zonohedral completion" dealt with by Russel Towle.

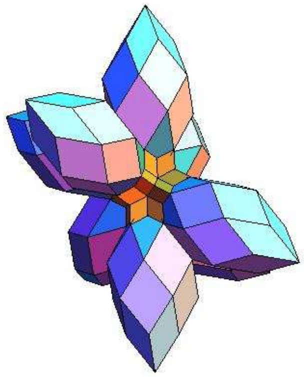


Fig .41*: RH with PZ around it*

Fig. 42*: RH with PZ*

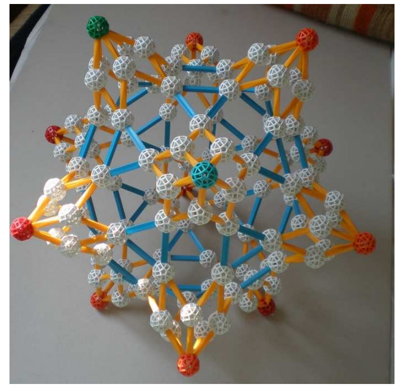


Fig. 43*: Tips of 12 PZs*

The 12 PZs could support clusters of 30 rhombic dodecahedra (RD) (Fig. 44), or 30 cubes (Fig. 45, 46).

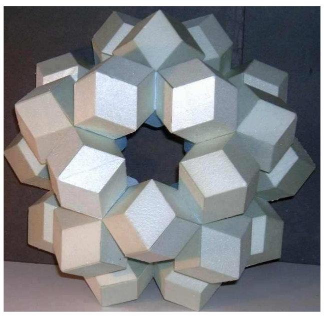


Fig .44*: 30 RDs*

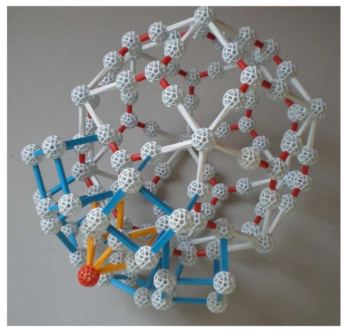


Fig. 45*: Initial stage of 30 cubes construction*

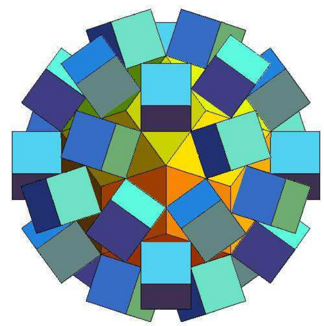


Fig. 46*:30 Cubes*

Modelling with RH

RH can also be interpreted as a member of an infinite family of deltoidal hexecontahedron (DH) (Fig. 47).

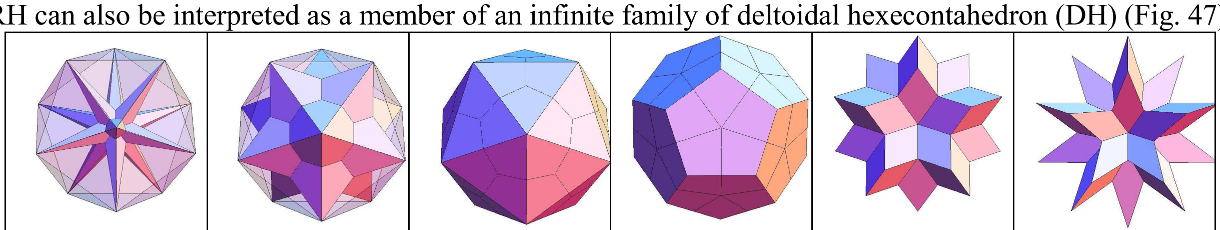
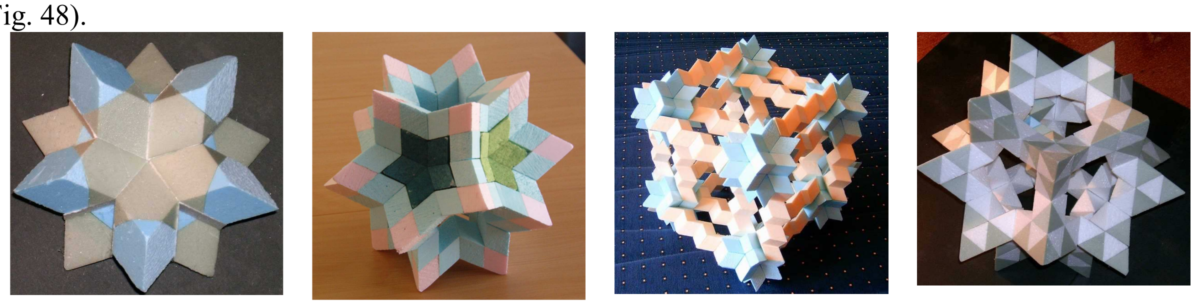


Fig. 47: Various DHs produced with a continuous transition by moving the vertices. Wolfram Mathematica is able to produce these images in an interactive manner

The Author's StyroBlock system is very suitable for producing of structures from the derivatives of RH (Fig. 48).



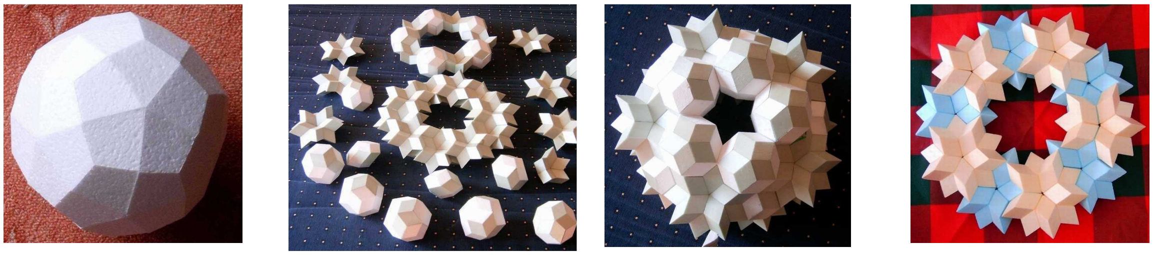


Fig. 48: *StyroBlock models based on RH*

With the many capabilities (interactivity, rotation of objects on the screen, zooming) of Wolfram Mathematica it is easy to visualize various structures, such as the creation of RH in a frame of icosahedron (Fig. 49). It is also possible to place patterns on the faces (Fig. 50), or printing the patterns and making paper models (Fig. 51).

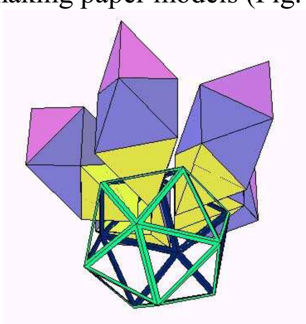


Fig. 49: *RH with rhombohedra in icosahedron frame*

Fig. 50: *Mathematica image of RH with pattern on the faces*

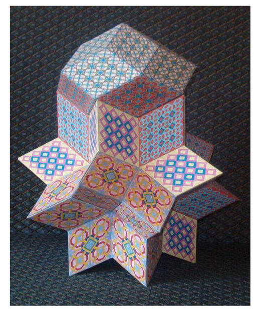


Fig. 51: *Paper models with patterns on the faces*

Summary

This paper contains only a small fraction of what one can do with rhombic hexecontahedra. Wolfram Mathematica software, the Zometool construction kit, and the author's StyroBlock system are useful tools for understanding and creating new structures inspired by the rhombic hexecontahedron.

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