The SpHidron Conjecture

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Abstract

The SpHidron is a curved development of the original Spidrons [1-6], which were composed of plain triangles. The flat-foldability of the SpHidron surface is debated. Some of the opponents[1] who contend that it is impossible say that this makes the original Spidron deformation - which was proven mathematically in 2004 by Lajos Szilassi - even more interesting. In my paper I present arguments for the flat-foldability of the curved and twisted SpHidron vortices. I hope that these ideas can lead to the mathematical solution of the problem.

The problem of the flat-foldability of curved edges, bent and twisted surfaces

The restrictions on the flat-foldability of torsionless planar surfaces, as described by authors Robert Lang, Eric D. and Martin L. Demaine and Toshikazu Kawasaki and others, are quite well-known, although the conditions of flat-foldability in the case of curved edges and bent, twisted surfaces are quite mysterious. [7]

According to Jonathan Sneider's definition of the torsionless deformation of origamis [8]: *"The paper is never cut nor chemically manipulated; its size, shape, and flatness are never altered; nothing is ever added or taken away. Only its position in space is affected."*

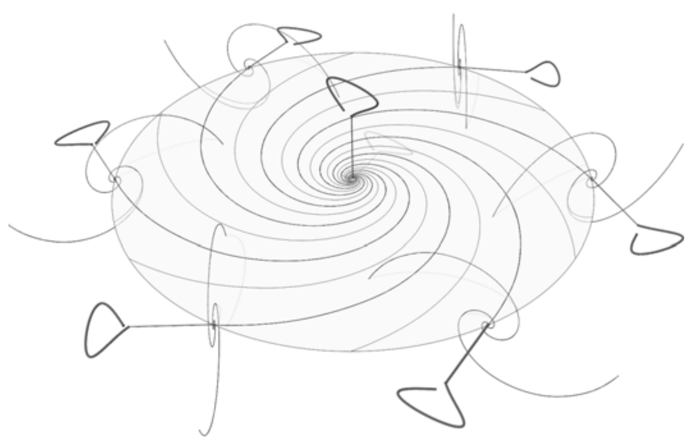

Figure 1: *The can-openers show the rotation of the spiral arms. The further they are from the centre, the more they turn the arms, i.e. themselves, around.*

[1] Namely Mr. Emil Molnár and Ms. Márta Szilvási, professors of the Geometry Department at the Budapest University of Technology and Economics, Hungary and the Spidron Team members Paul Galiunas and Walt van Ballegooijen.

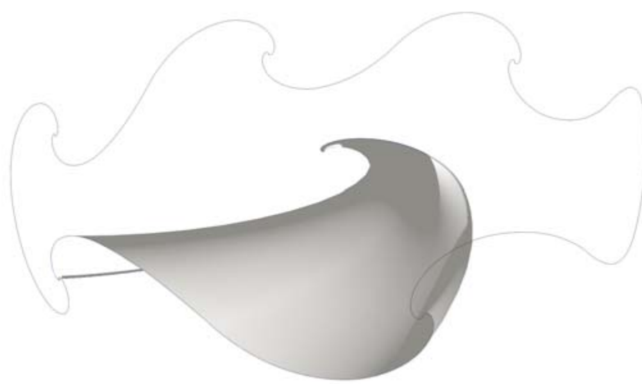

Figure 2: *Between the so called sharp arms the surfaces are smoothly bending and curling*

Figure 3: *Deformed SpHidron disc*

Axial torsion in the SpHidron arms

We have to observe the axes of this deformation in greater detail. If we pay attention, we can distinguish three different types of rotational center. Two of them are similar. The main difference between them is the arms running into them. On the periphery it is always two arms that meet in the axis, while in the center of the model we can have as many pairs of arms as we wish. In our case there are 12 arms running in spirals from the periphery to the center. Every even arm is a smooth arm and every odd arm is a ridged arm. The ridges are similar to the horizontal one in the center of the disk. As logarithmic arms never reach the center, we need to imagine a slightly different shape to be able to imagine what is going on inside the curling surfaces. For better understanding, we tried to illustrate the shape of the edge at the periphery (**Figure 4.a**) and the vortex's centre (**Figure 4.b** and **4.c**) separately.

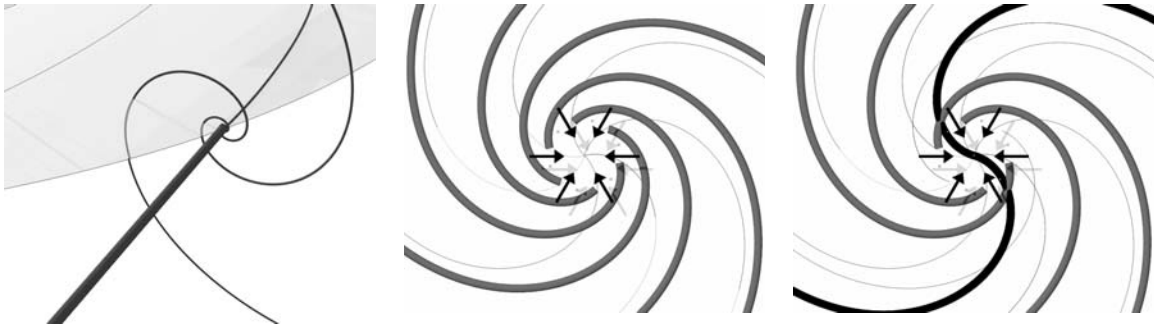

Figure 4: **a–c** *The arms can meet without breaking the logarithmic spiral curves*

It is surprising that during the deformation process, the meeting point of the curves are rotating and rotating, while they are approaching the ideal spiral shape, and the separate spirals which are approaching points are simultaneously approaching each other. The end position of the asymptotic approach of the curve is identical with the temporary meeting point of any of the connected curves. In my opinion, that is why this deformation can take place in physical reality.

Figure 5: **a–b** *At the centre, the arms don't ever meet, but we can force them for demonstration purposes*

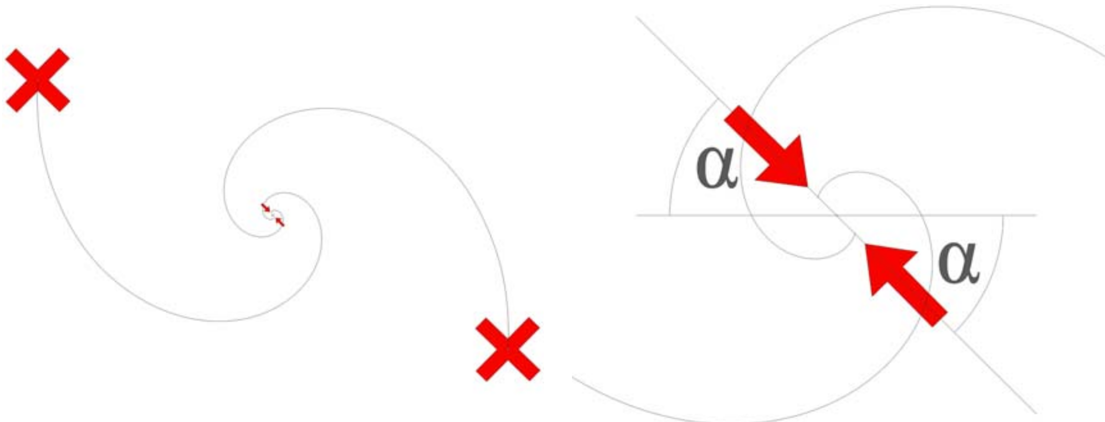

Figure 6: **a–b** *It is not trivial to see, but by pushing these ends towards each other, the distance between their other ends increases*

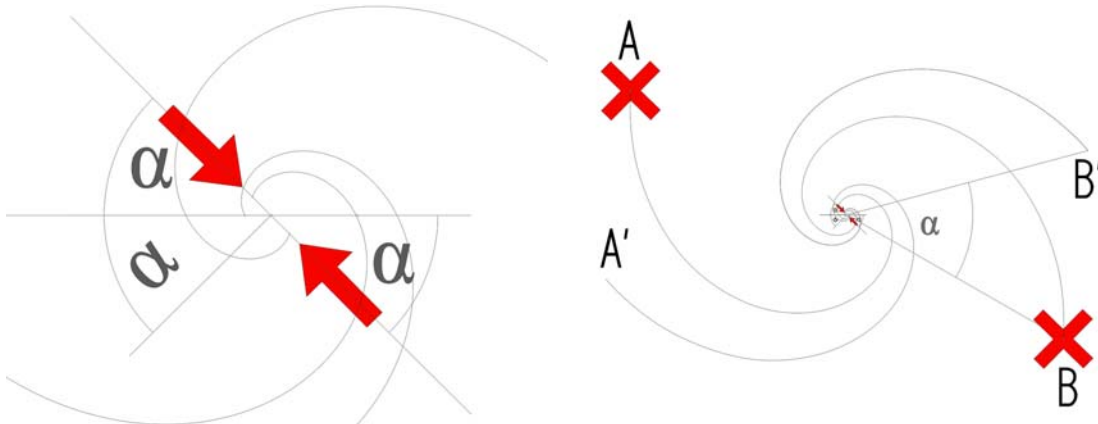

Figure 7: **a–b** *Images showing the effect of a rotation by alpha at the center and at the periphery*

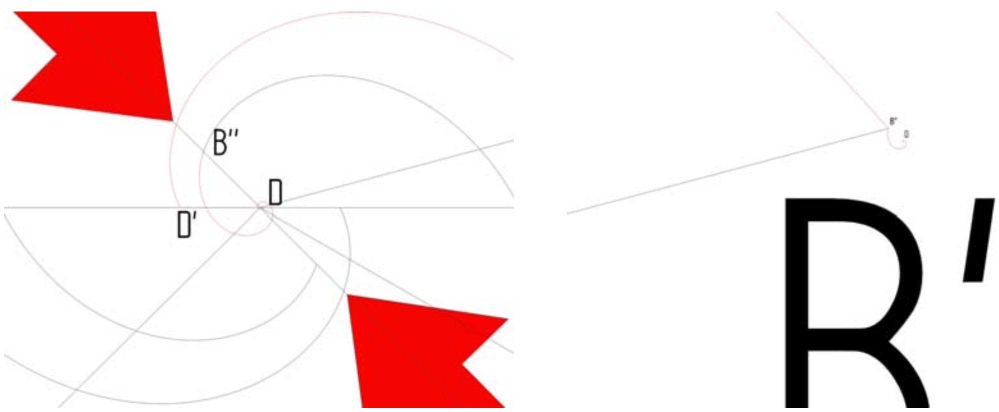

Figure 8: **a–b** *Here we extended the B' arm with a tiny B" D spiral. It's length – say 2 mm – must be subtracted from the arm at the periphery to ensure constant arm length.*

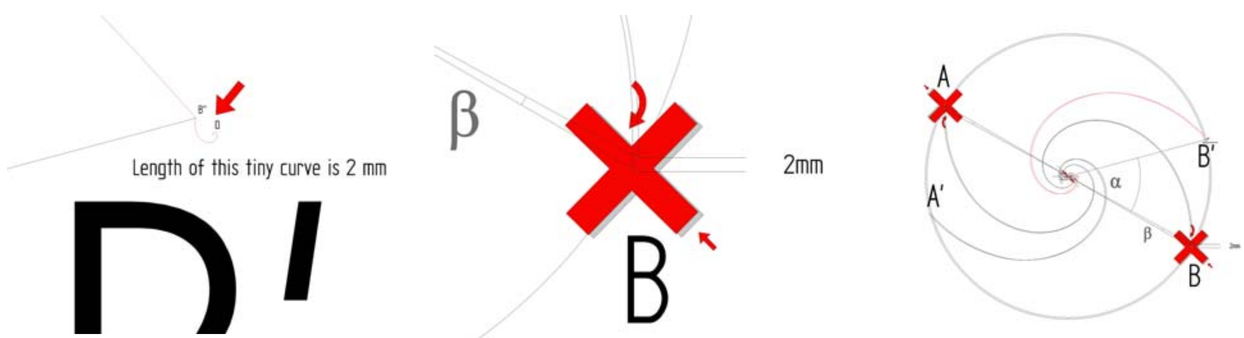

Figure 9: **a–c** *Rotation by an angle beta, shrinking the projected shadow of the disk, and increasing the level of curling at the centre and – perpendicularly to the central one – at the periphery. These are simultaneous processes that keep going until the extreme state, which is the "perfectly curled up" SpHidron surface. This process is a description of a stable solid shape. It can be seen that it is not a deformation at all, but it seems to be an appropriate way to describe the infinite SpHidron surface.*

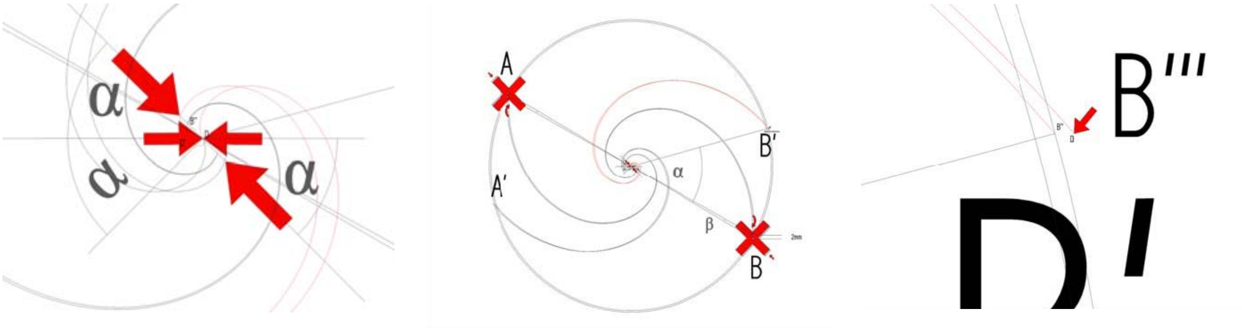

Figure 10: **a–c** *Here you can see the extension B" D of the arm resulting from joining the ends in the center*

It is my conjecture that the shrinkage resulting from curling up and the enlargement resulting from translation of the arms balance each other out.

As the composing logarithmic SpHidron arms (at least every second one, the "smooth" ones) always remain in the baseplane, it is possible for the corresponding points of the circular disk to remain on and run along the arms.

SpHidron Creatures

The deformed circular disks can be cut into hexagonal items, which fill the plane as a very nicely and playfully moving relief.

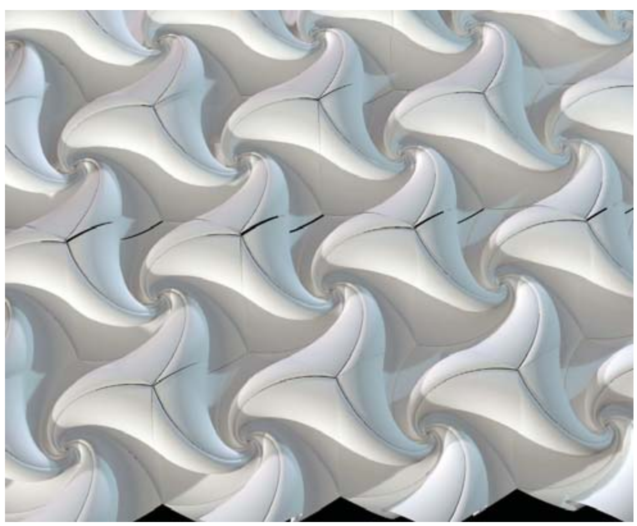

Figure 11: *We can tessellate the 2D plain with hexagonal SpHidron tiles*

You can see in the next picture that in the case of the Spidron, the can openers' axes point towards the center, while in case of SpHidrons they are tangential to the logarithmic spirals i.e. the axes of deformation. This deformation is centrally and spherically symmetric. The most interesting property of this change is that if we release the tightening rotation, the shape is becoming similar to smaller and smaller parts of the original surface. We can calculate precisely the radius of the projected surface the following way: if the original radius is **r**, and we rotate the can openers by an angle of **φ**, we can find a similar figure on the original surface (as it was before we rotated the can-openers) with a radius **r(1/φ)** *if and only if* all our spirals are identical golden logarithmic spirals.

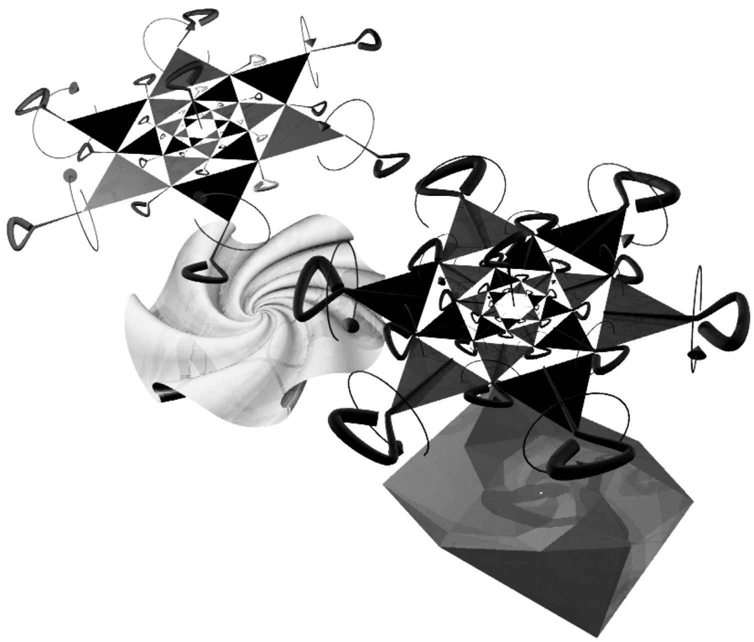

Figure 12: *Two kinds of deformation*

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