Through and Around instead of Over and Under. Another Way of Weaving.

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Abstract

In Figure 1 you can see an iron window lattice that is constructed with bars: horizontal bars are threaded through holes in vertical bars and vice versa. You can call this a weaving, but here the 'threads' are going through and around each other instead of over and under as in normal weaving. In this paper I will investigate the possibilities of this kind of weaving. There is a close relationship between the through and around weaving and the Borromean rings. In the three dimensional setting each ring goes through one of the other rings and is around the third ring.

1. Introduction

1.1. Through and Around. In Italy you can find many examples of barred windows as shown in Figure 1 and Figure 2a. These iron window lattices can be taken apart by just sliding out the horizontal straight bars. So the lattice is built with two types of bars, straight bars and bars with holes, as can be seen in Figure 2b.

Figure 1: *Window Lattice.*

Figure 2a: *Lattice 4 x 4.*

Figure 2b: *The eight bars of the 4 x 4 lattice.*

This may be the most obvious way to make a barred window, but when you think about it you may ask whether it would be possible to make barred windows with only one type of bar. This could have an economic as well as a practical advantage. And indeed when you look around in Italy you will find many variations of the barred windows among which the example of Figure 3a is built with only one type of bar (Figure 3b). The choice of using only one type of bar has an unexpected consequence: the way you have to assemble the barred window is more complicated.
You can not add the bars one by one as in the window lattices of Figures 1 and 2a, but you first need to assemble two groups, which are then slid together to make the final construction (Figure 3c).

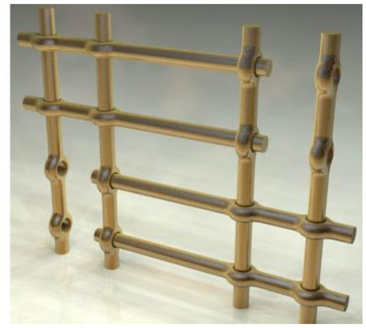

Figure 3a: *One type of bar.* **Figure 3b**: *The eight equal bars.* **Figure 3c**: *Two groups of 4.*

Characteristic for this category of barred windows is the mid part: the holes seem to be divided in four groups situated around the centre (Figure 4). Some variations can be made in design of the bars. You can vary the number of holes in the bars as well as their position. When we limit ourselves to the 4 x 4 bar grids there are exactly seven types of bars that can be used for bar grids which can be slid together. In the example of Figure 5 six different types are used and this appears to be the maximum number [1]. It needs some thinking in which order you have to slide the bars together but it can be done.

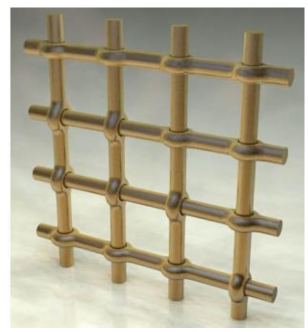

Figure 4: *One type of bar.* **Figure 5a**: *Six different bars.* **Figure 5b**: *Assembled.*

2. Escher's Barred Windows

2.1. Belvedere. Escher lived in Italy from 1924 till 1935 and most probably has seen these barred windows.

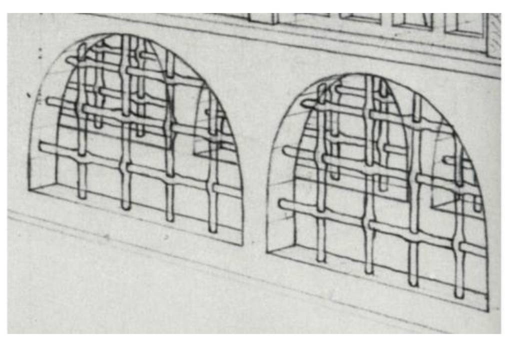

Figure 6a: *Sketch for Belvedere* **Figure 6b**: *Detail: the barred windows*

In the sketch for his print ‘Belvedere’ we can see that he paid special attention to the construction of the barred windows. And, surprisingly enough, he found his own variation: his barred window can not be disassembled! You can not slide the bars apart. There is just no way to do it.

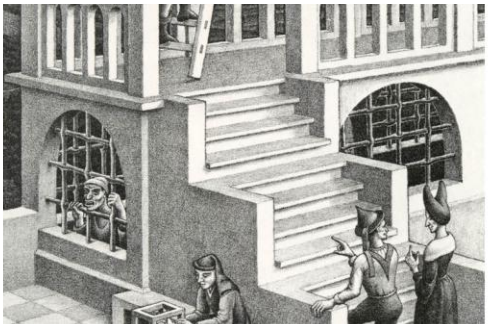

Figure 7a: *Barred windows in ‘Belvedere’ (1958).*

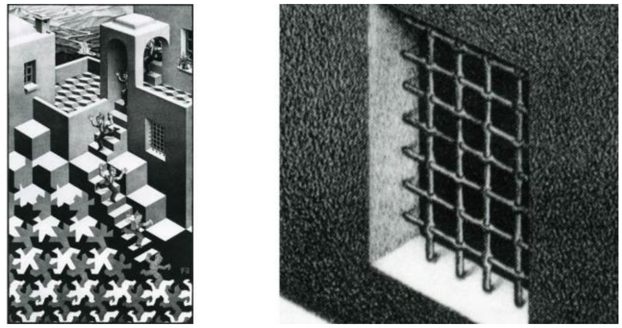

Figure 7b: *Barred window in ‘Kringloop’ (1938).*

There is one other print in which Escher used his version of the barred window. This print is ‘Kringloop’ and was made in 1938, the period in which Escher started to use mathematical patterns in his prints.

3. Circular Bars

3.1. Circular Bars - vertical. In Figure 8 a new type of bar is introduced: a circular bar with holes. There are two main ways to make holes in circular bar: it can be done in the plane of the circle of the bar or perpendicular to the plane of the circle of the bar. The last case we will call ‘bars with vertical holes’. And now using Escher’s idea of distributing the holes on the bars we can make some nice ‘weaving’ patterns with circular bars. The examples show eight, six and five holes respectively (Figures 9 - 13).

Figure 8: *Circular bar.*

Figure 9a: *Ring, 8 holes.*

Figure 9b: *Ring, 6 holes.*

Figure 9c: *Ring, 5 holes.*

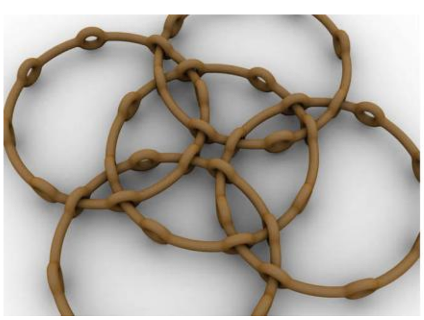

Figure 10a: *Connecting rings ‘8’.*

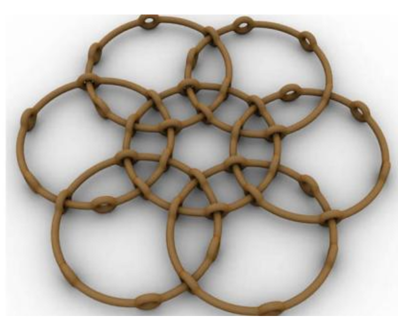

Figure 10b: *Connecting rings ‘6’.*

Figure 10c: *Connecting rings ‘5’*

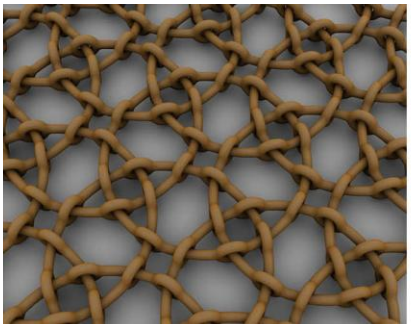

Figure 11: *Weaving pattern '8'.*

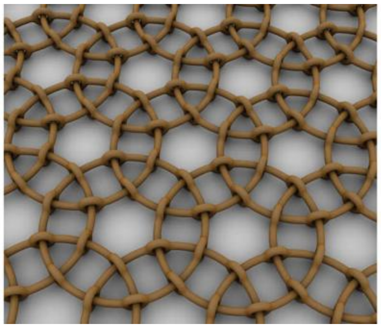

Figure 12: *Weaving pattern '6'.*

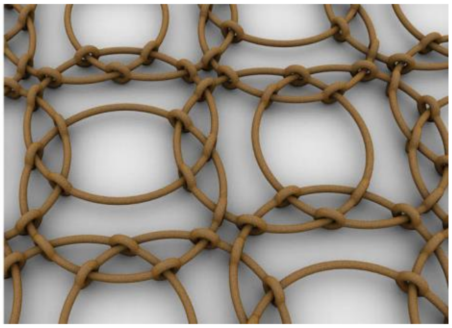

Figure 13: *Weaving pattern '5'.*

3.2. Circular Bars - horizontal. Connecting rings with horizontal holes will result in 3-dimensional objects. To make a design for a spherical ring structure you can start with a regular or semi regular polyhedron. There are, except for the anti prisms, five polyhedra that can be used to make a ring patterns on the sphere (Figure 14). The smallest of them, based on the octahedron, is a ring pattern with three rings, with four crossings each. This leads to the bar grid construction of Figure 15. Each ring has two horizontal holes (Figure 16).

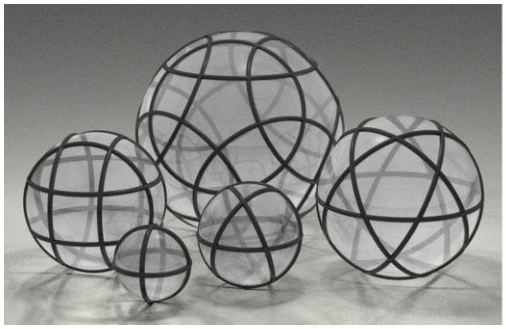

Figure 14: *Ring patterns on the spheres.*

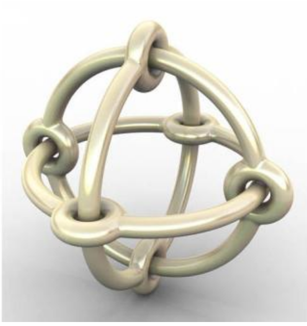

Figure 15: *Three rings.*

Figure 16: *Two holes.*

This is just a basic set up of a 'through and around weaving' with circular bars. Varying the design of the elements can give a very different look (Figure 17 and 18). But mathematically speaking it is the same object.

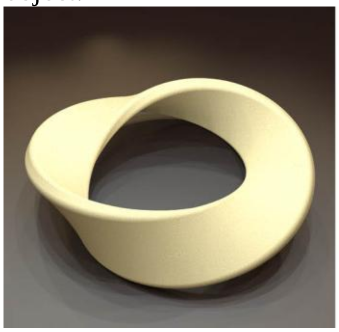

Figure 17a: *Twisted band.*

Figure 17b: *Band with holes.*

Figure 18: *Final object.*

The twist in the band is needed to get a nice 'through and around weaving'. When you have an odd number of twists, which is the case when you start with the right most ring pattern of Figure 14, the icosidodecahedron, the rings becomes Moebius bands. We need six of these rings with five holes in each ring to construct the bar grid structure of the icosidodecahedron (Figure 20 and 21).

Figure 19: 6 *Twisted bands.*

Figure 20: *Ring with holes.*

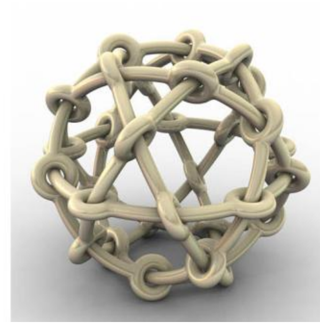

Figure 21: *Icosidodecahedron.*

There are two ways to connect rings with five holes in such a way that we get the structure of an Archimedean polyhedron as the final object (Figure 22). When we start connecting the rings as shown in Figure 22b we will need twelve rings and end up with a circular bar construction based on the rhombicosidodecahedron (Figure 23). In the example shown in Figure 24 you may not directly recognise the basic polyhedron. But it consists of four pairs of parallel rings, and the structure is based on the cuboctahedron (Figure 25a). When you use only single rings you will get the object of Figure 26.

Figure 22a: *Connection 1.*

Figure 22b: *Connection 2.*

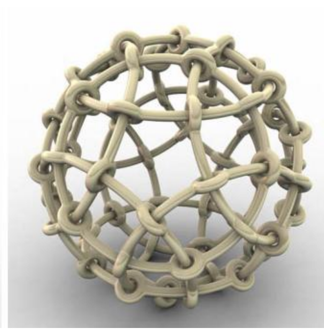

Figure 23: *Final object.*

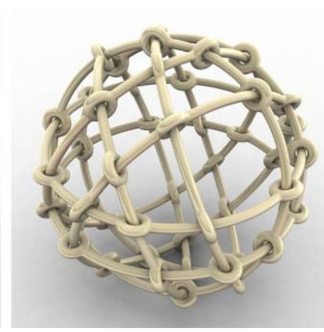

Figure 24: *8 Rings.*

Figure 25a: *Cuboctahedron and the twisted band.*

Figure 25b: *Band with holes.*

Figure 26: *Final object.*

4. Infinite Ring Structures

4.1. Rings with three Holes. Four rings with three holes each can be used to construct the cuboctahedron structure of Figure 27. But we can also place the four rings on four of the hexagonal faces of the truncated octahedron as in Figure 28.

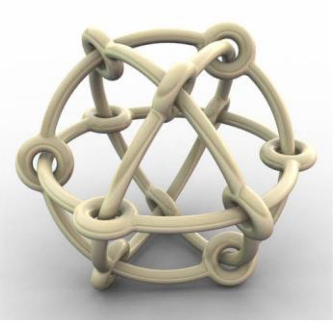

Figure 27: *Cuboctahedron.*

Figure 28a: *Separate rings.*

Figure 28b: *Truncated Octahedron.*

And now, because of the space filling property of the truncated octahedron, we can build a infinite space structure with the rings by connecting groups of rings as is shown in Figure 29. A few pictures of the final structure are presented in Figures 30a and 30b.

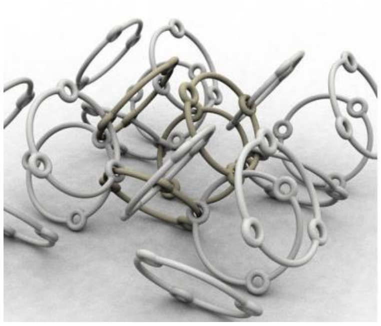

Figure 29: *Connecting groups.*

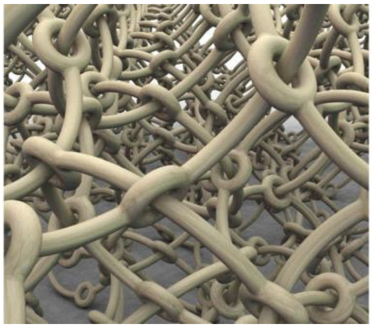

Figure 30a: *Infinite structure.*

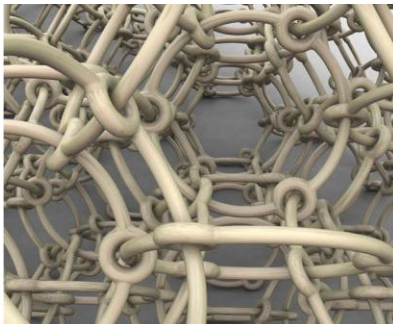

Figure 30b: *Infinite structure.*

4.2. Rings with two Holes. When we look at the steps we have taken to construct this structure, it is easy to understand that a similar structure can be made by starting to place rings with two holes on the square faces of the truncated octahedron. And this leads to the infinite structure of Figure 31.

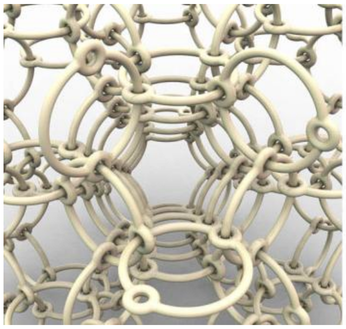

Figure 31: *Rings with two holes.*

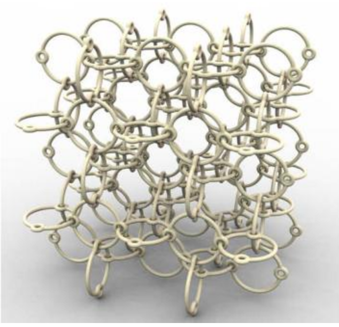

Figure 32: *Infinite structure* .

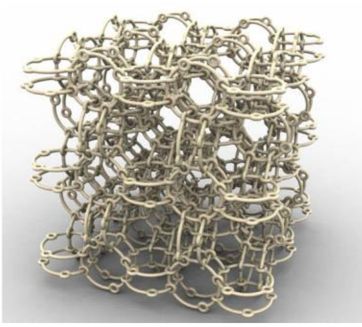

Figure 33: *Rings with four holes.*

Rings with two holes can be used in more than one way to construct an infinite structure. In Figure 32 a second possibility is shown. And from here on we can create the structure of Figure 33 by replacing each ring by a pair of parallel rings with four holes each. It is in the same step that you need if you want to develop the ring structure based on the rhombicuboctahedron (Figure 34). You start with the three ring structure of Figure 15, and replace each ring (with two holes) by a parallel set of rings with four holes each.

5. Borromean Rings

5.1. Other Connections. This brings us back to finite constructions, based on polyhedra. And when we change the construction of the object in Figure 34 by replacing each ring by a pair of elements as shown in Figure 35, we are also back to our grids built with bars. The only difference with the bar grids in Section 1 is that the bars are bent. We can choose where we want to make the bending in the bar and this will give the object a different look. The object in Figure 35c comes close to the shape of a cube.

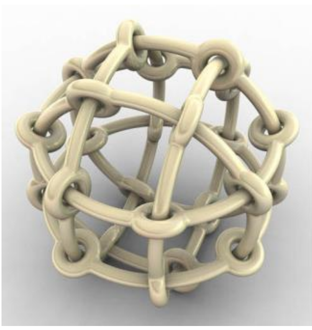

Figure 34: *6 Rings.*

Figure 35a: *12 Bars.*

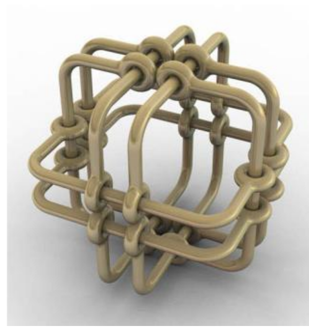

Figure 35b: *12 Bars.*

Figure 35c: *12 Bars.*

5.2. Borromean Rings. A next step is to change the connection of the three bars that meet in a corner point of the cube. The midpoint of each hole in the bars is now situated exactly on the corner point of the cube. To make it fit we had to change the shape of the hole from circular to elliptic. It is still a 'through and around weaving' as defined in Section 1. The same connection is used in the dodecahedron structure of Figure 37. This special connection is in fact nothing more than the well known Borromean Rings (Figure 38).

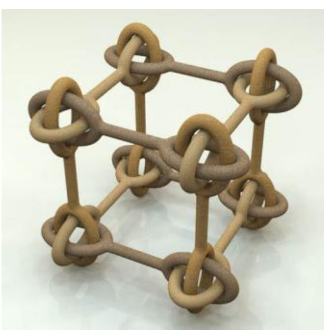

Figure 36: *Cubic structure*

Figure 37: *Dodecahedron.*

Figure 38: *Borromean Rings.*

5.3. Borromean Patterns. Starting with the Borromean Rings of Figure 39 we can develop the Borromean Pattern of Figure 40. Each ring in the pattern is used twice in a Borromean Ring connection.By changing the shape of the element as can be seen in the construction of Figure 41 these two connections are separated. In fact the element now connects two holes that can be used for a Borromean Ring connection. In the Borromean Joint of Figure 41 you can see very clearly that each of the elements is threaded through the hole of another element. With this element we can develop the 3-dimensional Borromean Pattern of Figure 42. The element can also be used to make the structure of Figure 43. An important difference between the two structures is that the structure of Figure 43 can be disassembled whereas the Borromean joint (Figure 44) can not be taken apart.

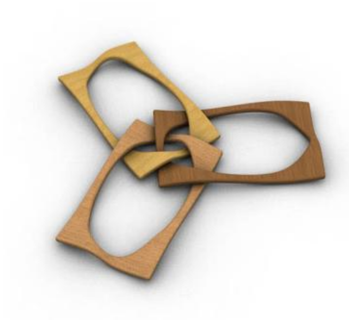

Figure 39: *Borromean Rings.*

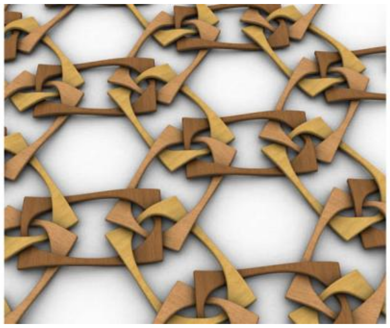

Figure 40: *Borromean Pattern.*

Figure 41: *Borromean Joint.*

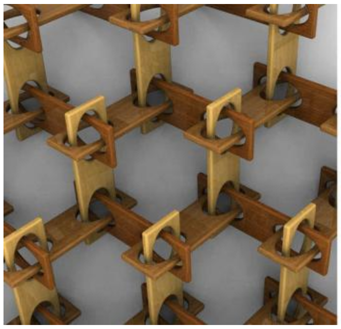

Figure 42: *Borromean Pattern.*

Figure 43: *Mortise and Tenon Joint.*

Figure 44: *Borromean Joint.*

6. Borromean Polyhedra

6.1. Mirrored Corners. The Borromean Joint can be used to create polyhedral constructions. The first example is the cube. The cube in Figure 45a is made of twelve straight elements with holes at both ends. When you look close you will see that two neighbour connections are each others' mirror image. When we want to have each of the connections exactly the same we have to use either curved elements (Figure 45b) or bent elements (Figure 45c).

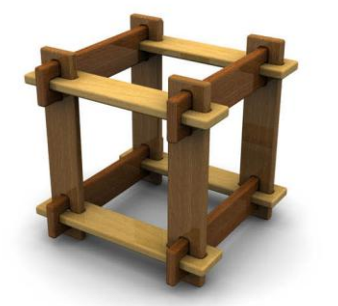

Figure 45a: *Borromean Cube.*

Figure 45b: *Cube – curved elements.*

Figure 45c: *Cube – twisted elements.*

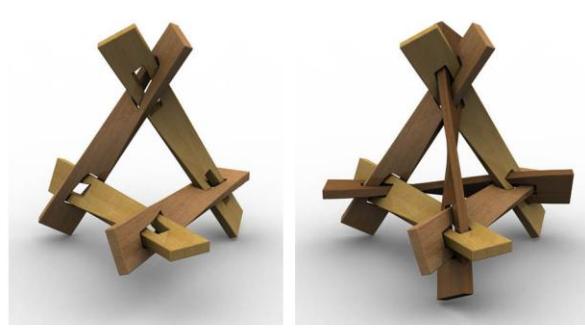

Figure 46: *Tetrahedron – curve elements.*

Figure 47: *Tetrahedron – twisted elements*

6.2. Equal Corners. The next example is the tetrahedron. Because of the odd number of edges of each of the faces we can not have two different corner connections as in the cube of Figure 45a. So we have to either use the curved elements (Figure 46) or the twisted elements (Figure 47). This is also the case when we want to construct the dodecahedron (Figure 48b). Each face of the dodecahedron has five edges which is an odd number. The biggest number of elements that can be slid together without problems automatically creates an Hamilton path on the dodecahedron (Figure 48a). In Figure 49 you can recognise the truncated icosahedron. Also here it is necessary to use bent elements.

Figure 48a: *Hamilton path.*

Figure 48b: *Dodecahedron*

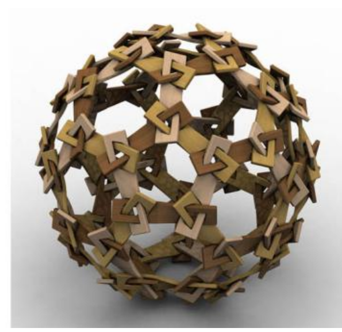

Figure 49: *Tr. Icosahedron.*

6.3. Straight Elements. Besides the cube there are three more polyhedra which only have faces with an even number of edges: the truncated octahedron (Figure 50), the rhombitruncated cuboctahedron (Figure 51) and the rhombitruncated icosidodecahedron (Figure 52)

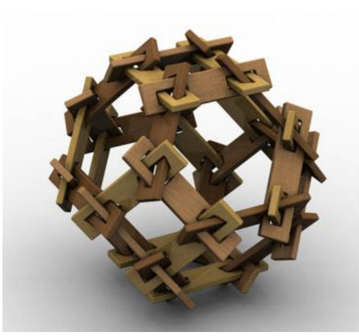

Figure 50: *Polyhedron 1.*

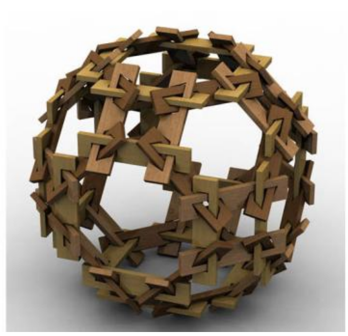

Figure 51: *Polyhedron 2.*

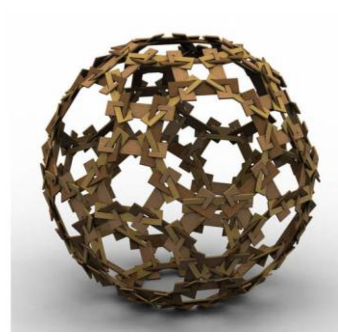

Figure 52: *Polyhedron 3.*

6.4. Borromean Rings. In Section 5.3 we have seen that we can make more than one Borromean Ring connection with each ring in a standard Borromean Ring pattern. In the examples below you can see how this idea can be used to build polyhedral with just rings. The tetrahedron (Figure 53) and the dodecahedron (Figure 55) are now build using rapid prototyping techniques and in the models the rings are not connected but it is not possible to change the structure.

Figure 53a: *Single Ring.*

Figure 53b: *Three Rings.*

Figure 53c: *Tetrahedron.*

Figure 54: *Borromean Cube.*

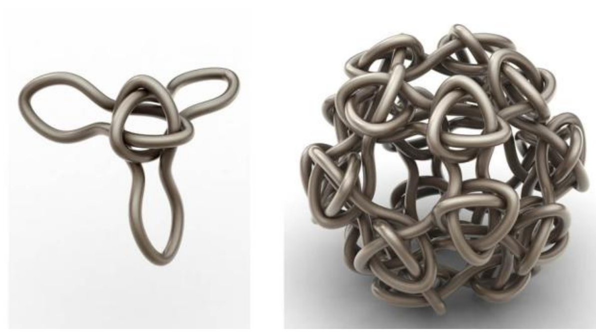

Figure 55: *Borromean Dodecahedron.*

7. Conclusion.

7.1. Conclusion. I think we may say that 'Through and around weaving' is as inspiring as 'Over and under weaving'. Escher did some experiments in this field most probably inspired by the Italian iron window lattices. Nice constructions can be made with this technique especially in combination with the Borromean ring structure.

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