

## Workshop on Mathematics and Dance

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### Abstract

The presenters will offer a workshop for Bridges participants on how to integrate mathematics and dance in the classroom as well as on stage. They will incorporate several mathematical topics, including symmetry, counting principles, and the mathematics of rhythm. Participants will create, practice, and perform short dance phrases, and simultaneously explore mathematical principles and critique the work from the point of view of both the mathematics and the artistry involved.

### “Math Dance”

Dance and mathematics share many essential concerns: (1) Both deal with pattern recognition and manipulation. (2) Both involve defining a problem and seeking a solution. (3) Both begin with concrete problems and progress to abstract ideas – or vice versa. (4) Both involve aesthetics and are integrally connected to cultural values and biases. (5) Both can make you sweat!

**The Purpose of the Workshop.** The purpose of this workshop is to give participants a palpable experience to help them understand connections between mathematical and choreographic concepts. Prior experience in either mathematics or dance is not required. The workshop alternates between creative problem solving and reflection/discussion. Workshop participants will (1) solve problems physically in groups of two to four, (2) discuss problems in smaller groups and as a class, (3) will first explore creatively the subject matter and then examine formalization of the concepts, (4) Discuss the use of these activities in K-12 and college classes.

**Background.** Judith Lynne Hanna [1] provides arguments for the role dance education plays in stimulating thinking, self-expression, and problem solving. The study of dance, according to Hanna, helped with perceptual awareness and encourages exploration of space, time, dynamics, gesture, phrasing, and motif. The article cites a ten-year study of low-income youths. Regular study of the arts improved the youths’ academic performance and increased their abilities in self-assessment. I. S. Yakimanskaya [2] has analyzed constituent aspects of spatial thinking, and explored connections that spatial thinking has to education. In the 1960s and 70s Zoltan P. Dienes pioneered the use dance, games, and the arts in mathematics education [3].

Mathematicians view their work as having a creative component. Dancers/choreographers note the judicious thinking and attention to form which their art requires. Contrary to the perception of the two disciplines as opposites, they work well together. The activities in this workshop were published by authors of this article, along with co-author Scott Kim, in 2001 [4].

**Is This For Everyone?** These activities are for everyone and do not require special training in dance or mathematics (although training or interest in these subjects can be helpful). The activities begin by addressing the most basic universal elements of dance and mathematics: recognizing and remembering patterns, walking, counting, moving, making shapes. The activities are designed to be flexible and can be extended to suit the level of the participants.

**Everyday Movement.** Everyday movements, like shaking hands, are a great way to involve non-dancers in doing and creating movement sequences. Although many types of dance, such as classical ballet, take years of practice to master, others are built on everyday movements that anyone can do. For instance the musical performance Stomp creates dance out of everyday actions like sweeping the floor. Many hip-hop moves began as everyday gestures. All over the world folk dances are made out of the everyday movements of work and play.

In the world of modern dance, many choreographers during the 1960's used untrained dancers performing ordinary movements in their works. For example, "Esplanade," one of choreographer Paul Taylor's most popular works, is built out of walking, running, and jumping movements. Yet it takes concentration and focus to perform even the simplest of these dances well.

**Clap Your Name.** *In this activity participants create and perform rhythmic clapping patterns using their names, and then convert the patterns to movement sequences.* The activity helps develop number sense and addresses the concept of least common multiple. Participants look at basic choreographic principles of sequencing and transitions by creating and manipulating dance phrases. Participants also explore the relationship between pedestrian movement and dance. The connection between polyrhythm and least common multiple is also revealed.

Clap Your Name is an excellent "break the ice" activity and gently progresses from a simple clapping activity to whole body movement. Participants pair up for the first exercise and can work in chairs (or for students, at their desks.) However, for the movement portion we will need a clear, open space, and in the classroom students will need to push desks to the side of the room.

**Symmetry.** *In this activity, participants explore that several types of planar symmetries (see Figure 1), then create sequences of group shapes that exhibit the symmetries.* Symmetry has long been applied to a variety of subjects such as mathematics, architecture, and visual arts. In dance, symmetry is a powerful guide to creating rich and challenging spatial relationships between dancers and also as a means of analyzing dance forms from around the world.

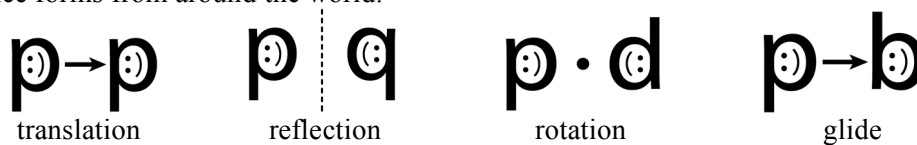
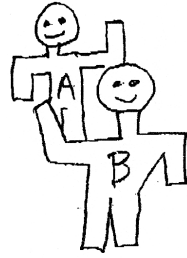
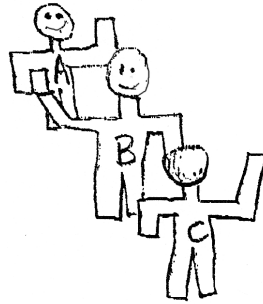


Figure 1. *Four types of planar symmetries*

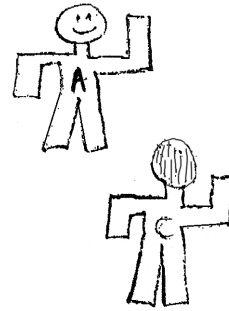
*Next participants investigate how symmetries combine to produce new symmetries.* In the process they glimpse the rich spatial structures that underlie all human movement, and observe how these structures may be examined more closely by translating them into symbolic form and back again into movement. (See Figure 2.) Participants will observe how the Klein Four Group, generated by translations, mirror reflections, glides, and 180 degree rotations, is exhibited in the movement studies they perform.



*A and B in glide symmetry*



*B and C in rotational symmetry*



*Result: A and C in mirror symmetry*

**Figure 2.**

**How Many Ways to Shake Hands?** *In this activity participants create sequences of handshakes. Then they are asked to find the number of possible handshakes.* Resulting discussions lead to an understanding of counting, combinations, and problem definition. When working with a phrase of movement, manipulating that phrase by altering sequence and certain aspects of the movement is an essential part of tradition form known as theme and variation. On a broader level, defining the problem (in terms of subject matter, movement theme, visual theme, etc.) is a major aspect of the choreographic process. The question, "What belongs and what does not belong?" has far-reaching mathematical and artistic implications.

This activity invariably leads to groups generating different answers, which then leads to discussions (sometimes heated!) of which count is correct. Aside from the mathematical and choreographic ideas which are inherent in this activity, it also challenges participants' ability to interpret rules and explain why an answer is consistent with their interpretation.

**The larger picture.** Two quotes by William Thurston seem applicable to the larger purpose of these activities [5]:

"An interesting phenomenon in spatial thinking is that scale makes a big difference. We can think about little objects in our hands, or we can think of bigger human-sized structures that we scan, or we can think of spatial structures that encompass us and that we move around in. We tend to think more effectively with spatial imagery on a larger scale: it's as if our brains take larger things more seriously and can devote more resources to them."

"One-on-one, people use wide channels of communication that go far beyond formal mathematical language. They use gestures, they draw pictures and diagrams, they make sound effects and use body language. Communication is more likely to be two-way, so that people can concentrate on what needs the most attention. In talks, people are more inhibited and more formal...in papers people are still more formal. Writers translate their ideas into symbols and logic. And readers try to translate back... Mathematics in some sense has a common language: a language of symbols, technical definitions, computations, and logic. This language efficiently conveys some, but not all, modes of mathematical thinking."

**Background.** Karl Schaffer and Erik Stern had been choreographing works together for three years when they began to discuss the similarities between the processes which underlie mathematics and dance. The performance which resulted, "Dr. Schaffer and Mr. Stern, Two Guys Dancing About Math," premiered in 1990, has been performed over 500 times throughout North America, and led to the creation of numerous other performances exploring the connections between mathematics and dance.

In 1993 Schaffer and Stern collaborated with Scott Kim, noted software designer and mathematician, on the performance "Dances for the Mind's Eye." As a natural outgrowth of their work on stage, Schaffer, Stern and Kim created workshops which allowed students to experience in the classroom the connections between dance and mathematics. The workshops have been given for fifteen years to tens of thousands of educators, artists, and students of all levels, from kindergarten through college. In addition, both the performances and workshops have been requested and delivered at scores of conferences, such as mathematics education conferences (including the National Council of Teachers of Mathematics national meeting) and arts education conferences (including the Dance and the Child International Festival).

In 2001 the three collaborators collected their extensive classroom experience and the handouts they had been using and wrote the book *MathDance with Dr. Schaffer and Mr. Stern* [4]. Schaffer and Stern are on the Touring Artists Roster of the Kennedy Center for the Performing Arts Partners in the Arts program, and travel extensively through that program doing teacher development work. Their work has also garnered five National Endowment for the Arts awards for their math/dance choreographic and educational work. Scott Kim, a collaborator in creating these activities, is an internationally renowned puzzle-master and a mathematics educator and enthusiast who has a Ph.D. in Computer Science from Stanford University.

### References

- [1] Judith Lynne Hanna, *Learning Through Dance: Why Your Schools Should Teach Dance*, American School Board Journal, Vol. 187 n6, pp. 47-48, June, 2000.
- [2] I.S. Yakimanskaya, *The Development of Spatial Thinking in Schoolchildren*, translated from Russian and published by the National Council for Teachers of Mathematics, 1991.
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- [4] Karl Schaffer, Erik Stern, and Scott Kim, *Math Dance with Dr. Schaffer and Mr. Stern: Whole-Body Math Movement Activities for the K-12 Classroom*, published by MoveSpeakSpin, 2001.
- [5] William P Thurston, "On Proof and Progress in Mathematics," in *18 Unconventional Essays on the Nature of Mathematics*, ed. By Reuben Hersh, Springer, New York. 2006.