Mirrors and Spheres:
The Geometry within the "Tall Tree and the Eye"

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Abstract

The "Tall Tree and the Eye" is an experimental project of AGU in collaboration with Anish Kapoor, exhibited at the London's Royal Academy of Fine Arts in 2009. This paper describes the geometrical studies underlying the design and manufacturing of this sculpture. Using digital form-finding techniques that simulate gravity force and explicit history tools together with the study of sphere packing and curved mirror reflections, the AGU developed a geometrical model that could adapt and change accordingly to the design and structural progress from the initial stage to the construction phase.

Introduction

The Team. The Advanced Geometry Unit (AGU) at Arup is a specialist cross-discipline team of structural engineers and architects. Founded in 2000, the Unit has grown out of the ARUP culture combining the firm's engineering tradition with innovative outlook on structure, form and aesthetics. With its multi-disciplinary skills set on a project-by-project basis, working independently or collaborating with other architects, artists and engineers, the AGU has been increasingly invited to create solo- exhibitions and installations.

Figure 1: *The "Tall Tree and the Eye" sculpture in the Royal Academy of Arts courtyard, London, 2009.*

The Description. Seventy-three mirror polished stainless steel spheres stacked to a height of 14m and a width of 5m create the appearance of weightless floating bubbles rising into the sky (Figure 1). Each sphere has approximately 1 meter diameter and a wall thickness of maximum 2mm – a fragility that presented one of the major challenges of the design process.

For this reason the sculpture has required an inner structure of three carbonated steel masts linked together by curved bracing elements (Figure 2) and connected to a steel base frame at ground. The hollow spheres have been fabricated with a high pressure water technique and then mirror polished [1].

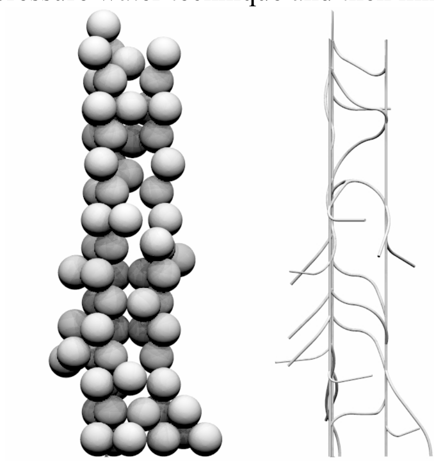

Figure 2: *The spheres and the inner structure.*

Sphere Packing

The Process. When approached by artist Anish Kapoor with a concept on stacking several mirror spheres the AGU examined and developed multiple methods and design options. A first approach was based on the analysis of spherical packing theory in order to achieve an ideal design that avoids visibility of connections between the spheres or of any structural element, that presents the minimum necessary quantity of structure and that is based on a simple construction sequence.
As soon as the design process began, the focus was set on the most efficient way of packing spherical object and whether there is a rule that facilitates constructing and controlling tangencies between numerous spheres.

Regular Packing. To provide an insight into regular packing systems, we'll imagine to fill a box with marbles. Once a first line of marbles is placed on the bottom of the container's edge we can then lay the second row following a lattice arrangement and resulting in a symmetrical pattern called Regular Packing. Once the container's base is full with this first regular layer we can set each marble of the second layer either on top of three or on top of four marbles. As a result we will obtain the two most efficient Regular Packing systems (where a density arrangement of 0,74 can be achieved [2])**.** These are:

a) the hexagonal close packing (HCP) = where the sphere's centers are located in the tetrahedron vertices' (Figure 3)
b) the face centered cubic packing (FCC) = where the sphere's centers are located in half-octahedron vertices' (Figure 4)

Packing with Octahedron and Tetrahedron. As a consequence, an effective way of controlling tangencies between all spheres would be placing their centers on either an octahedral or tetrahedral packing configuration. Bearing in mind that together, the octahedron and tetrahedron pack space (their dihedral angles are supplementary), but neither of them can do so alone.

Irregular Packing. Let's now imagine to randomly fill a container with marbles and shake it throughout. A configuration called Random Close Packing [2] will be obtained; the more you shake the container and the more space is gained. This explains why the irregular packing of spheres is a stable configuration for compression. This is a peculiarity because through compression you usually gain a regular packing (for

example compression of circles in 2d space will give a regular packing of circles). The maximum density arrangement that can be achieved with this Irregular Packing is generally 0,64, lower than the one obtained through a Regular Packing.
To understand the effects of an Irregular Packing arrangement, we therefore used a gravity-force computer simulation applied to spherical objects of different sizes inside a variable cone shape container (Figure 5) and analysed the results from an aesthetic and structural point of view.

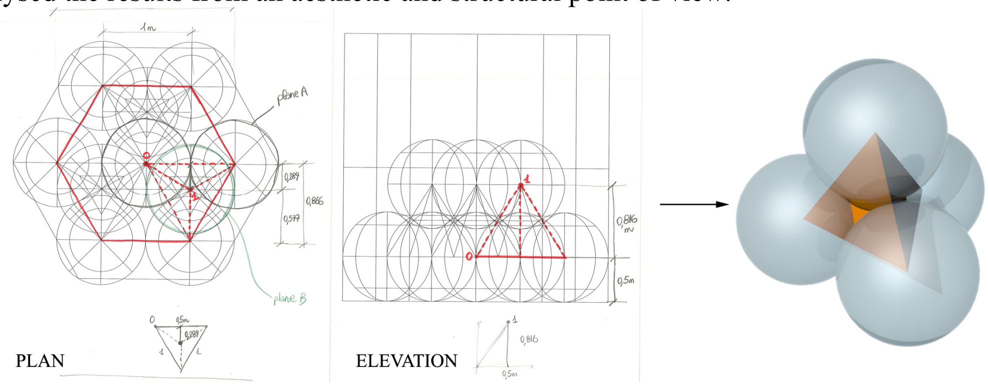

Figure 3: *A Regular Packing of spheres: the Hexagonal module.*

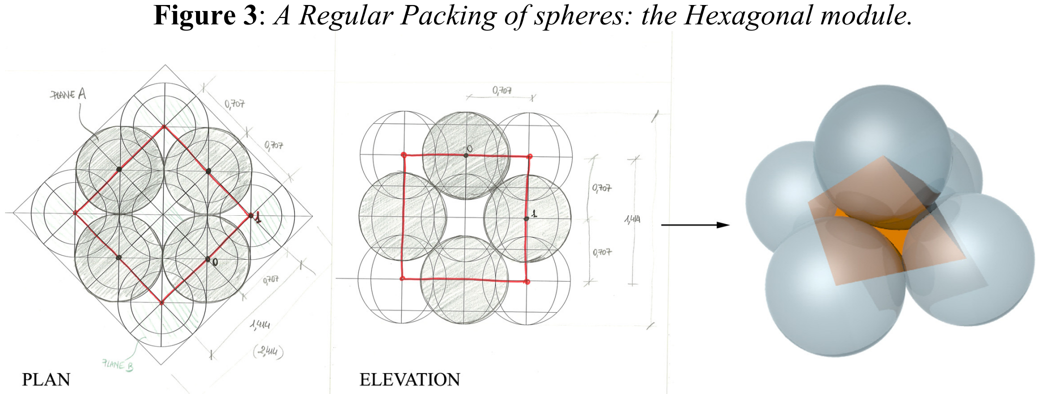

Figure 4: *A Regular Packing of spheres: the Face centred cubic module***.**

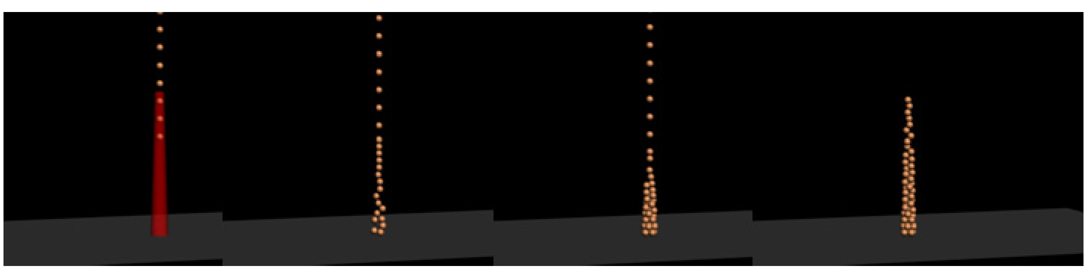

Figure 5: *Video sequence for an Irregular Packing.*

Conclusion. Through the results of these investigations on both Regular and Irregular Packing the sculpture’s design has been led towards a geometrical outline that combines the two systems together, for both structural and aesthetical reasons. The Regular Packing would provide a modular system that facilitates constructability but at the same time would give a far too obvious regular arrangement. The Irregular Packing, on the other hand, would provide the visual effect of a casual layout but, at the same time, would require an extra structure to provide the equal forces of the virtual container that held the spheres.

Reflection Analysis

Studies on the reflection properties of mirror spheres have been carried out to enhance the control of visual impact that the sculpture would have on future observers. Through the analysis of basic convex mirror properties, through three-dimensional computer models and computer-generated images, it has been possible to visualize the effect of reflection on multiple tangent spheres. Interactive photo-realistic renderings gave us images that have been then verified on site as very close to reality.

Curved mirror surfaces. It has been necessary to examine curved mirror properties [3] in order to understand how the final sculpture would have looked like once built. Through general questions such as why does the reflection depend on the observer, how do angles and distances between spheres affect the reflection or how will the context be distorted in the reflection (Figure 6) have been the starting point for further considerations.

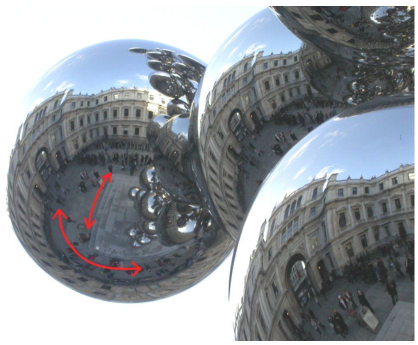

Figure 6: *Reflection on a top sphere: the courtyard edge becomes a curve.*

Reflection on 2 or 3 spheres. As can be seen in the following image (Figure 7) different angles, relative position and diameters in more than one sphere can affect the result of the reflection. One of the multiple findings was that we can achieve far more interesting reflections when the sphere's centers lie on a non-orthogonal plane to the viewer (Figure 8).

Figure 7: *Study of reflections on mirror spheres with different diameters and relative angles.*

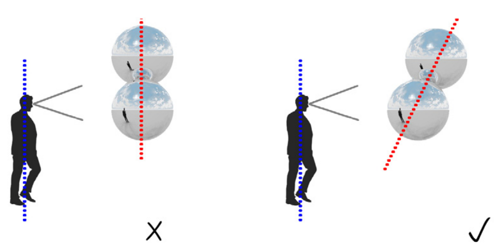

Figure 8: *The angle towards the viewer.*

Hyperbolic Geometry. The research for the underlying principles behind mirrored reflection on multiple spheres conducted to a study of the Hyperbolic Space [4]. The transformations that apply to the Hyperbolic Plane explain at best the effect of the spherical mirror surface [5].
The reflections on multiple mirror spheres can be related to the principles of *Inversion*. As in two-dimensional inversion with respect to a circle, it becomes more evident in the three-dimensional space with a set of tangent spheres under inversion to each sphere [6]. As with inversions the multiple tangent spheres of equal diameter will produce a fractal reflection as shown in the computer rendered image as well in the photograph of the sculpture in Figure 9.

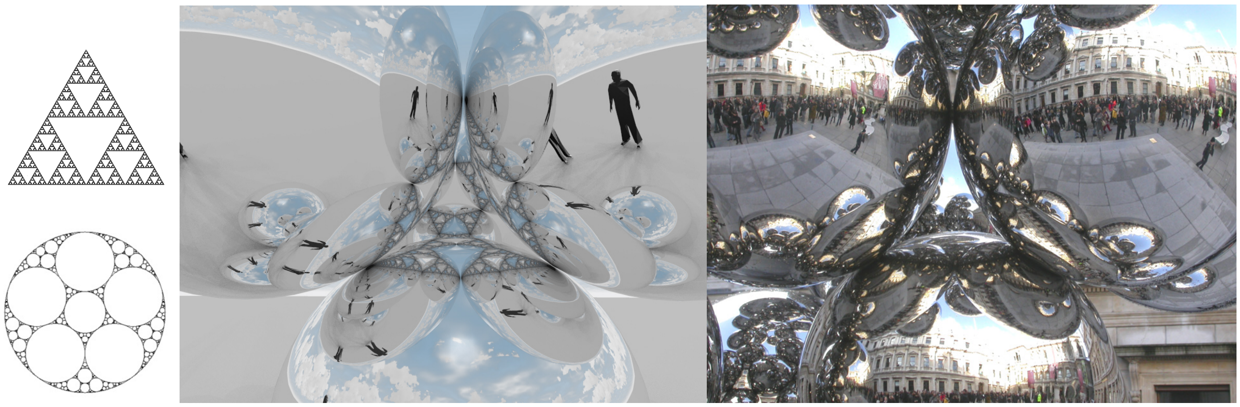

Figure 9: *The Sierpinski and Apollonian Gasket, a tetrahedron of rendered spheres and a photograph of the sculpture.*

As a consequence to these casual explorations, the sculpture has been based on the observation that the reflection of perfectly reflective tangent spheres into each other generates an infinite fractal pattern. The reflection would then become increasingly complex and rich when people and surrounding buildings are introduced in the space and when the packing is carried out in an irregular form. An opening at the lower level of the sculpture has also been designed in order to allow the visitor to enter the sculpture and gaze up at the endless reflections as if into infinity (Figure 10).

Figure 10: *The viewer can step inside the sculpture and gaze up to Infinite reflections.*

The Dialogue between Structure and Geometry

The form-finding process of the conceptual design had to integrate aesthetic requirements but most of all geometric and structural ones. Once a three dimensional layout succeeded aesthetically, both from a reflection and a volumetric point of view, it would be exported into a structural analysis software to verify local and global stability of the sculpture. These results would then be used as feedback for further changes to the three dimensional model that, in a back and forth process, would generate further considerations resulting in a new model to be once more analysed structurally.

Sphere Types. To provide an insight into the geometrical attributes of the "Tall Tree and The Eye" sculpture a classification of the spheres is necessary. They are all characterized by a different number of tangencies with other spheres and by the type of structure contained within. As can be seen in Figure 11:

a) *the mast spheres* are the spheres that conceal the three carbonated steel masts and have at least two points of tangency with other spheres
b) *the inner bracing spheres* are the spheres that enclose the curved bracing structure (linked to the three masts) and have at least three points of tangency
c) *the cantilever spheres* are the spheres that will be supported by a curved cantilever structure connected to just one mast
d) *the top mast spheres* are three extra mast spheres that have been inserted on site on the summit of the masts
e) *the ground spheres* are particular spheres that have been added at ground level for improved stability and enhanced reflections

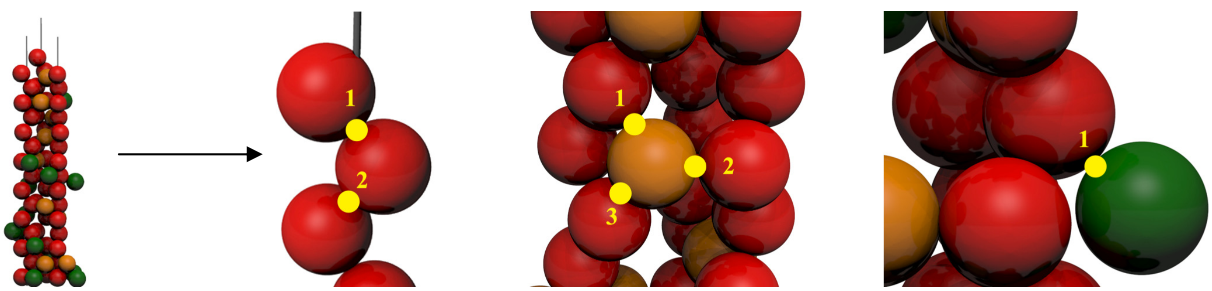

Figure 11: *Some of the sphere types: mast spheres, inner bracing spheres and cantilever spheres*

Geometrical Requirements. Several geometrical constraints had to be established in order to fulfill visual and structural requirements.
The first priority was to avoid visibility of the inner structure throughout the sculpture. For this reason all spheres had to be tangent to one up to four others and the diameter of the three masts had to be as small as possible.
Furthermore, the three masts had to be located at an equal distance from the global point of origin (see O(0,0,0) in Figure 13) not exceeding 1m, so to enable tangencies between the mast and the inner bracing spheres.
It has then been crucial selecting the mast spheres that would be connected to the inner bracing spheres because their location would determine the position and shape of the curved structural bracing and therefore the structural performance of the sculpture.
Multiple alternatives for inserting and connecting the spheres on the structure were provided bearing in mind few rules. First of all a double steel bracing in the same sphere was avoided as it would be challenging to bolt the sphere to the mast. Then each inner bracing steel tube (Figure 12) had to consist of three different tubes with constant curvature for fabrication purposes. In the intersection points (see A and B in Figure 12) the tube centerline had to be perpendicular to the tangent of the spheres. While the spheres

had to necessarily have elliptical holes with a minimal dimension that would allow them to run through the bracing.

Finally, once the geometrical model was built and finalized, the polar coordinates of each single spheres were exported into a spreadsheet in order to enable the fabricator to control and rebuilt the three-dimensional model with any software.

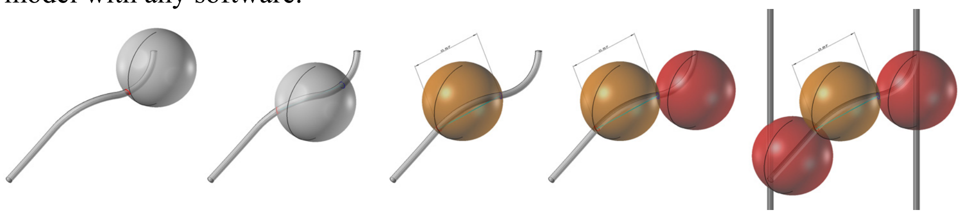

Figure 12: *The curved bracing steel tube inside 2 mast spheres (red) and 1 inner bracing sphere (orange).*

The Parameters. Once established the basic geometrical requirements for building a model in a 3d modeling software (Rhino) it has been necessary to assign parameters that could define completely the geometry. These parameters would enable to control any required change in an efficient way.

a) After designating the distance of the 3 masts from the global origin (as previously explained) it was essential to determine the rotation of the 3 masts in relation to the global x axis (angle α in Figure 13).
b) The mast spheres inserted along the three structural poles could all rotate in relation to the mast axis (angle β in Figure 13) or in relation to the sphere below (angle δ) so to interrupt the visual effect of linearity between the spheres that belonged to the same mast.
c) One of the first parameters to be specified was the angle between mast spheres along the global z-axis (angle γ in Figure 13); it would inevitably vary whenever the mast spheres had different angle δ.
d) The height of the mast spheres would also be related to the mast thickness; the wider the diameter of the mast was and the lower the mast spheres would need to be, so to cover always the structural mast.
e) Finally, two alternatives appeared whenever a fourth tangent sphere was required for the inner bracing and therefore it was possible to manipulate just a confined portion of the sculpture.

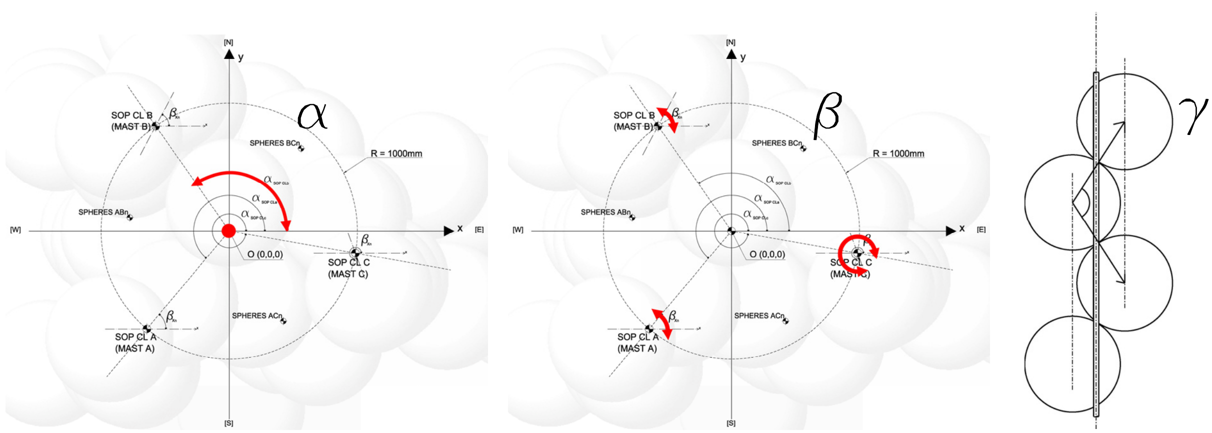

Figure 13: Plan view for angle α and β; elevation view for angle γ

The Tool. Having defined the parameters, the need of a three dimensional model that could adapt and change quickly accordingly to the design and structural progress could be achieved either through implicit or explicit history tools tightly integrated with Rhino's 3-D modeling tool. The best solution for this project has been provided by the explicit history tool named *Grasshopper* = a graphical algorithm editor (see [8]) where parameters could be assigned and linked through several components (Figure 14) that controlled the three dimensional model.

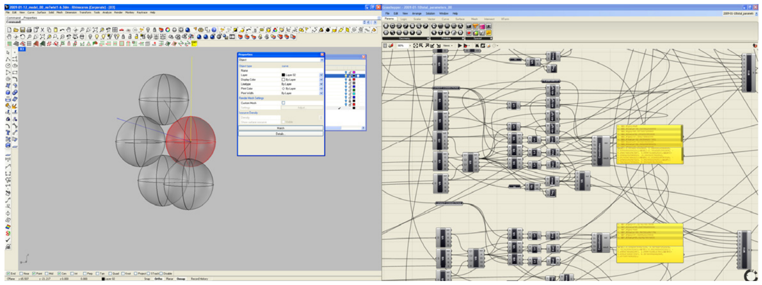

Figure 14: *A screen capture of the Parametric model.*

The best characteristic of this tool has been the fact that, differently form the implicit history tools, it could provide an immediate visual feedback and full control on each single component and stage of the process created by the user. Whenever one of the parameters would change all the model would adjust consequently to suit the initial requirements. In this way any geometrical variation required from aesthetic or structural reasons could be rapidly exported into analysis models for structural tests.

Conclusion

As a conclusion, although every single sphere weighs approximately 45kg, the result of all the design process from concept to construction phase has been a stable and yet light structure appearing to be almost weightless and rising in the sky. As one of the most recent works of AGU, this installation has been a great example of close collaboration between artist, architect, engineer and fabricator and through new computational technologies this design could be an innovative example of connection between art, geometry, architecture and engineering.

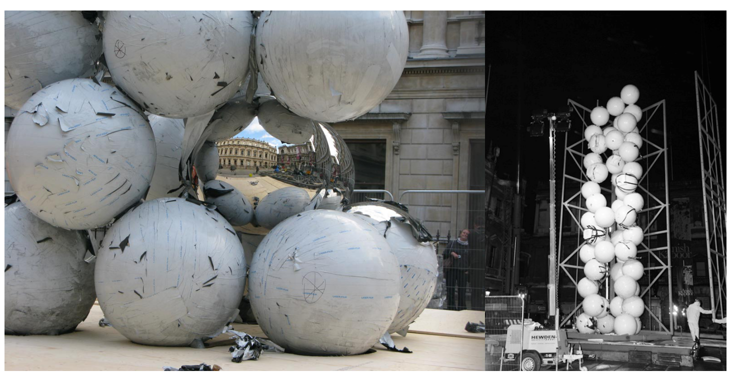

Figure 15: *the "Tall Tree and the Eye" arrives on site.*

References

[1] EngineerOnLine Ltd – http://engineeronline.ws/
[2] Mathworld, http://mathworld.wolfram.com/SpherePacking.html
[3] Wikipedia, http://en.wikipedia.org/wiki/Curved_mirror
[4] S.Santos, - *private communication*. 2009
[5] D.N.Arnoldand, J.Rogness, *"Moebius Transformations Revealed"*, http://www.ima.umn.edu/~arnold/
6] S. Kalajdzievski, *Math and Art, An Introduction to Visual Mathematics,* CRC Press, pp.143-167, pp.216-218. 2008.
[7] Dick Termes, Painting the Total Picture. In *Bridges Leeuwarden:Mathematical Connections in Art, Music, and Science,* Sarhangi R., Séquin C., Tarquin Publications, pp. 363-368. 2008.
[8] Grasshopper. *Plug-in for Rhino Software,* developed by David Rutten http://www.grasshopper3d.com/