Cylinder Anamorphosis of Impossible Structures

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Abstract

Cylinder Anamorphosis is known from the middle of the seventeenth century. Traditionally portraits of well-known persons are used. This article describes a method, using current computer hardware and software to make Cylinder Anamorphosis of all kind of structures, even impossible structures.

Introduction

Cylinder Anamorphosis is popularized by Jean-Francois Niceron (1613-1646). The reflecting cylinders would usually show the portrait of a prince or another well-known person. These portraits were always identifiable. Easily identifiable are also the three structures in the figures 1, 2 and 3. These models were initially used in Finite Element Method computer programs to analyze displacements and stresses. However they can be reused in computer programs to make Cylinder Anamorphosis Figure 1 showed the tensegrity, exposed at the Bridges 2008 conference in Leeuwarden [1]. Figure 2 is one of Lajos Szilassi's [2] toroids (polyhedra with 8 faces) and figure 3 is a simple crate which will be used later in this paper.

Figure 1: *Tensegrity*   **Figure 2**: *Torus*   **Figure 3**: *Crate*

Derivation Formulas

Structures are always tree dimensional. In Finite Element Method programs one uses spatial node coordinates and lines to describe the structure to the program. The traditional method of approach (figure 4) is not suitable to handle this 3D model. For this a more exact method (figure 5) is desired.
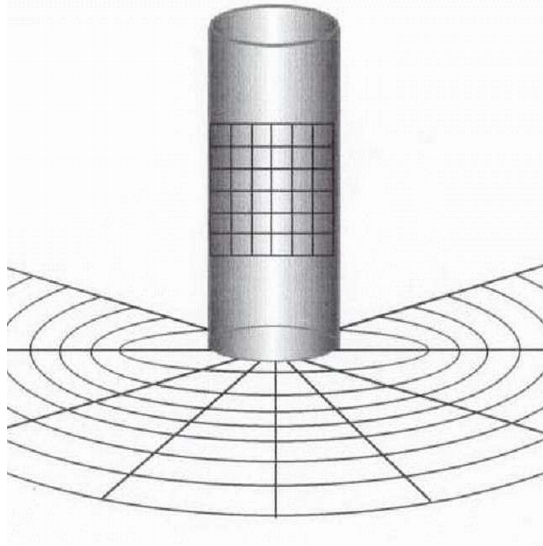

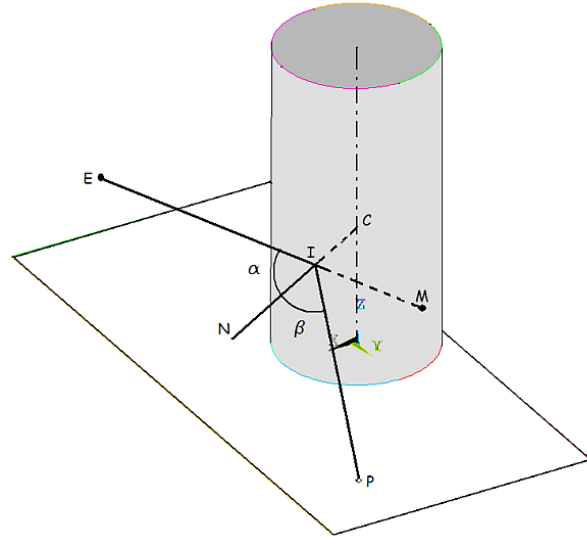**Figure 4**: *Traditional method*   **Figure 5**: *Exact method*

The general procedure is:
● The model to be warped is scaled in a way to fit in the cylinder.
● For a given point on the line between nodes, the deformations are calculated.
● These points are plotted and optionally post processed (for instance by using AutoCad)

Crucial to the process is determining the projection on paper from each point of the model. An orthogonal coordinate system XYZ is used (figure 5). The imaginary line between eye E and model M intersects the cylinder at point I. At this point there is a line I-N perpendicular to the reflective cylinder. The angle β (between the lines N-I and P-I must be equal to the angle α (between the lines N-I and E-I) Moreover P-I-N and E-I-N are in the same plane. From this data the coordinates for the projection point P can be determined. This procedure is shown in the BetterBasic program below. This procedure is developed in the early eighties of the previous century. It was (and remains) the core of a BetterBasic program to drive a HP 7550A Graphics Plotter. Instead of an HP plotter one can use several other devises to view the HP-Graphics-Language file, produced by the BetterBasic program.

```
Input  Radius Cylinder: RC  ,  Coordinates Eye: XE,(YE=0),ZE  ,  Coordinates Model: XM,YM,ZM
  t1=YM*YM/(XM-XE)/(XM-XE)
  XI=(XE*t1+SQR(t1*RC*RC-XE*XE*t1+RC*RC))/(t1+1) : YI=SQR(RC*RC-XI*XI)
  t2=ATN(YI/XI)+ATN(YI/(XE-XI)) : t3=SQR((XM-XI)*(XM-XI)+(YM-YI)*(YM-YI))*COS(t2)
  t4=SQR((XE-XI)*(XE-XI)+YI*YI)*COS(t2) : ZI=(t3*ZE+t4*ZM)/(t3+t4)
  t5=ZI*t3/(ZI-ZM) : t6=TAN(t2)*t5 : t7=ATN(t6/(RC+t5)) : t8=SQR(t6*t6+(RC+t5)*(RC+t5))
  XP=t8*COS(t7+ATN(YI/XI)) : YP=t8*SIN(t7+ATN(YI/XI))
Output Coordinates Projection point : XP,YP,(ZP=0)
```

The coordinates of the eye, mainly the height (ZE), determine the size of the Anamorphosis. Depending on the available paper size ZE can be calculated so that the entire Anamorphosis can be drawn.

Impossible Structures

How to make Cylinder Anamorphosis of impossible structures? It is M.C.Escher himself who gives the solution. At the Belvedère engrave [3] a man is seen, sitting on a bench with an impossible structure in his hands (figure 6). In front of this man there is a drawing at the ground with the solution! At two places the "front" and "back" are swapped incorrectly. See the circles at that drawing. Returning to figure 3, the crate, we can make the same "mistakes" resulting in the Cuboid as shown in figure 7.

Using the same model of the crate, rotating around a vertical Z-axis until the diagonal is exactly in the direction of view and enlarging the beams results in an impossible structure of the famous Hungarian artist Istvan Orosz [4] (figure 8).
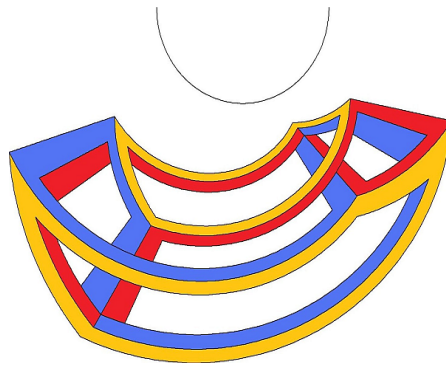

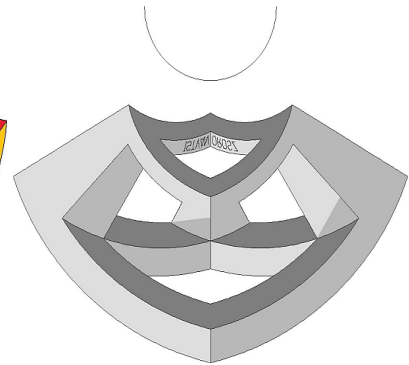**Figure 6**: *Detail*   **Figure 7**: *Cuboid*   **Figure 8**: *The Wall III*

"Really" Impossible Structures

The above structures are not really impossible; they arise because of a few mistakes in the drawings. It becomes really impossible when one line is used in two structures. An example: Using only one horizontal and two vertical beams of the structure *The Wall III* then figure 9 arises. The dimensions are chosen so that the vertical lines are equally spaced. Because of this, instead of two squares at the upper side, one can also draw three circles at the upper side! Now we have the famous Trident (figure 10)

An other example of using circles is given in figure 11. Again, there are some "mistakes" made in de post processing.
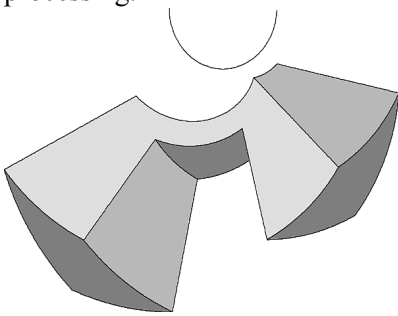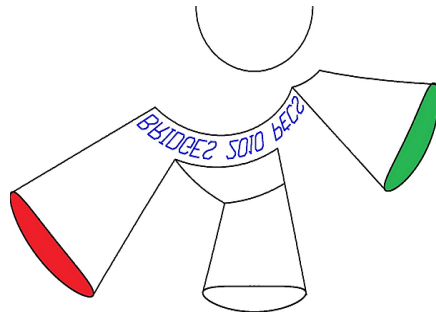

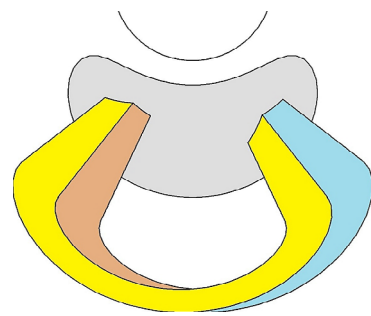**Figure 9**: *Twobeam*   **Figure 10**: *Trident*   **Figure 11**: *Arch*

Figure and Ground

What part of a drawing is figure and what is ground? Douglas R. Hofstadter gives in his book Gödel Escher Bach [5] a nice example. Between numbers of ink splashes, one can read de word MAIL BOX. To treat the ink splashes as a "model" in the BetterBasic program, it is possible to make Cylinder Anamorphosis of these splashes. "Easy" readable are the words when they arise horizontal in the reflecting cylinder. This is done for two cases as shown in figures 12 and 13. Can you read the words?
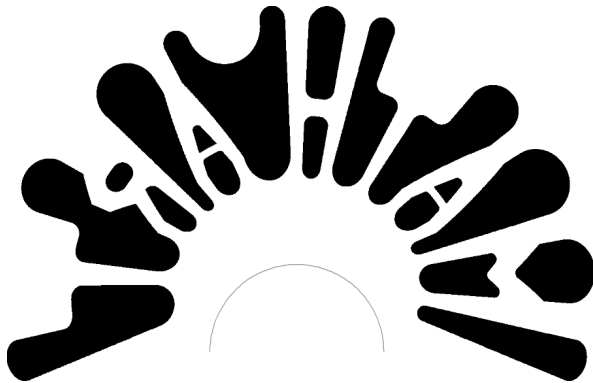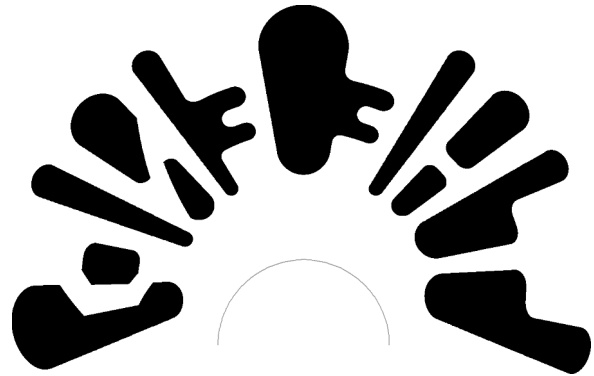

Figure 12   **Figure 13**

Conclusion

The proposed method offers the possibility to deform existing digital models to Cylinder Anamorphosis. Once in the BetterBasic program imported models are easy to rotate for viewing them from different sides.

Future work

Using this theory and software, it must be possible to make 3D Cylinder Anamorphosis (using red/green glasses). Instead of one viewpoint at coordinates XE, 0, ZE the process of calculating the projection points must be done twice. Depending on the distance DE between the left and right eye, the viewpoints are XE,-DE/2, ZE and XE, +DE/2, ZE. The impression will be: One can look "through" the mirror! Hopefully the results of this approach can be presented at the next Bridges conference.

References

 [1] bridgesmathart.org/carlo-mirror/BRIDGES2008/Tensegrity/index.html

[2] homepage.mac.com/dmccooey/polyhedra/RegularHexagonalToroid8_1.html

[3] J.L.Locher. *De werelden van M.C.Escher*, Meulenhoff Amsterdam, 1971, page 230.

[4] im-possible.info/english/art/orosz/orosz6.html

[5] Hofstadter, D. R. *Gödel, Escher, Bach: An Eternal Golden Braid.* New York,1979, page 64.