

Cyclic Symmetric Multi-Scale Turing Patterns

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Abstract

In his 1952 paper “The Chemical Basis of Morphogenesis” Alan Turing suggested that a system of chemical substances, reacting together and diffusing through a tissue, would account for the generation of patterns in multi-cellular organisms. In this paper a simplified computer model is derived from his idea, successively elaborated and used for the production of artworks reminiscent of electron microscope images of diatoms.

Simple Turing Patterns

Spots and stripes are a recurring motif in the animal world, for example the spots on whale sharks and the stripes on zebras. Is there a common fundamental process by which they form? In his 1952 paper “The Chemical Basis of Morphogenesis” Alan Turing mathematically demonstrated that a system of diffusing substances which produce and consume each other would naturally give rise to such patterns, see Turing [1]. The simplest case involves “activator” and “inhibitor” substances. The activator causes the production of more of itself as well as of the inhibitor, and also causes pigmentation. The inhibitor causes the destruction of the activator and of itself. The substances diffuse through the tissue, with the activator diffusing more slowly than the inhibitor and thus having a smaller range of effect before destruction. Spots of a size dependent on the diffusion rates arise spontaneously in this model from an almost homogeneous initial state, and space themselves into regular arrays.

The first unambiguous experimental evidence of a Turing pattern in a physical system was reported in 1990 by Castets et al. [2] in a system of five chemicals in a gel reactor. The system was modeled by Lengyel and Epstein [3] who derived an equivalent two variable model and analysed its spatial behaviour.

Models derived from Turing's idea are widely used in computer graphics and known as “reaction-diffusion systems”. The most seminal paper is Turk [4], who showed how to produce a non-square mesh conformal to the geometry of a model, and produce Turing patterns on it for texture and bump maps.

In this paper a simple model is derived from Turing's idea, then elaborated and applied in a series of mathematical artworks. In this model there is only one “substance”, which takes the role of pigmentation, activator and inhibitor. It is represented by a single floating point number for every element of a rectangular array. The elements of the array may be considered to be equivalent to the cells in the discrete model put forward by Turing in his paper. The floating point number representing the concentration of the substance becomes the pixel value in the resulting gray scale image. Diffusion is simulated by taking an average of the values of a certain area around each element, a smaller area for the activator and a larger area for the inhibitor. A simple rule is iteratively applied to all of the array elements for (usually) hundreds of time steps: If the average concentration in the smaller area is greater than the average concentration in

the larger area, increase the value by a small amount, otherwise decrease the value by a small amount. At each time step the resulting values are re-normalised to the somewhat arbitrary range -1.0 to 1.0, to avoid “run away”. Periodic boundary conditions are imposed to avoid edge effects.

Starting from an initial condition of pseudo-random values in the range -1.0 to 1.0, we see characteristic Turing patterns emerging at time step 20, becoming more defined at time step 100 and well defined at time step 10000, see figures 1, 2 and 3 below. The system slowly relaxes over time to a more smooth and regular state.

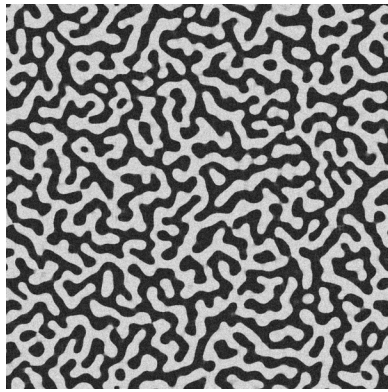


Figure 1: *Time step 20.*

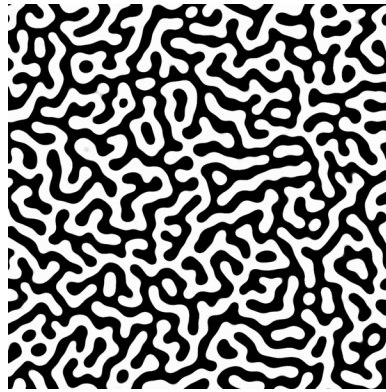


Figure 2: *Time step 100.*

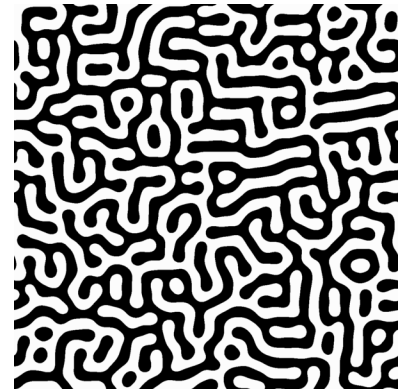


Figure 3: *Time step 10000.*

Compound Turing Patterns

The model described so far spontaneously produces stripe patterns somewhat reminiscent of those seen in the natural world. Being symmetric in its behaviour towards low and high concentrations it doesn't produce fields of dots, which are asymmetric in terms of light and dark. It produces structure at only one scale, which is rare in natural processes. A straightforward elaboration is to add together multiple copies of the simple model (as a weighted sum, with the possibility of negative weights), operating simultaneously at different scales on the same data. The scale is given by the areas over which averages are taken to determine the concentrations of activator and inhibitor. Resulting patterns are shown in figures 4, 5 and 6 below.

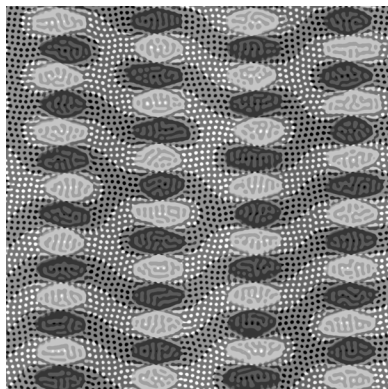


Figure 4

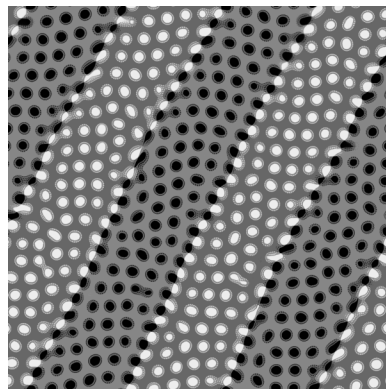


Figure 5

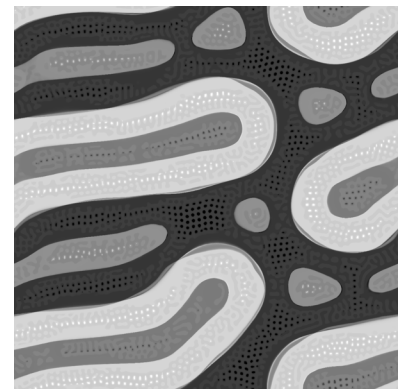


Figure 6

Here we see a locally broken symmetry that allows regions of light dots balanced by regions of dark dots, within a larger stripe structure. Histograms of the images show multiple sharp peaks, possibly indicating stable sub-populations of cells where the various rules are “firing” in recurring short sequences.

Lingfa Yang et al. [5] provide an analysis of “interacting Turing modes” (referred to here as “compound Turing patterns”). They note that fish skins provide examples of Turing patterns of more than one wavelength or characteristic scale, and provide a set of partial differential equations describing two Turing patterns of different scale coupled together. Some of their simulations result in dots arranged in stripes, similar to figure 5 above. Turk [4] produced compound patterns in a different manner, by running a large scale simulation, “freezing” parts of the pattern and then decorating it with a smaller scale simulation.

Multi-Scale Turing Patterns

A different way of combining Turing patterns of multiple scales (rather than simply adding them together) is to switch between Turing instabilities of different scales at different time steps. In figure 7 below, for each pixel and at each time step a decision about what scale Turing instability to deploy is made based on the amount of variation in the image around the pixel at that scale. The Turing instability of the scale with the least variation “wins” and “fires”, increasing slightly the variation at that scale. The resulting image has variation at multiple scales in a fractal-like manner, and strangely resembles an electron microscope image. Lighter areas appear raised, and darker areas recessed. The 3D shading effect is an accident of the purely 2D algorithm.

In figures 8 to 12 below a cyclic symmetry is imposed by averaging the concentrations of "activator" and “inhibitor” at each point with its counterpart points at $1/n, 2/n, \dots, (n-1)/n$ times 2π radians around the circle. There is an interesting juxtaposition of a strict symmetry and a complex organic looking surface. The images are somewhat reminiscent of electron microscope pictures of diatoms.

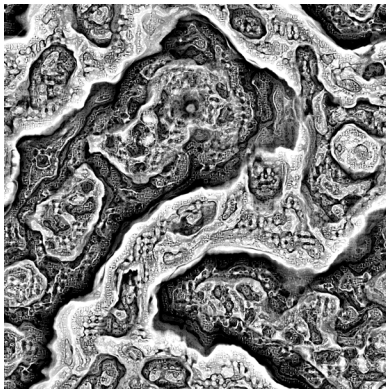


Figure 7

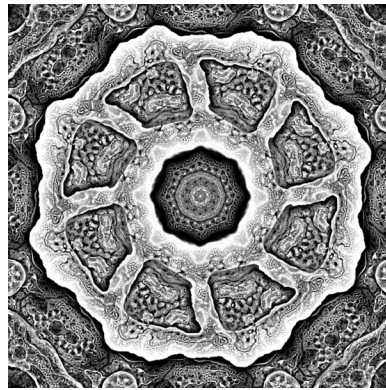


Figure 8

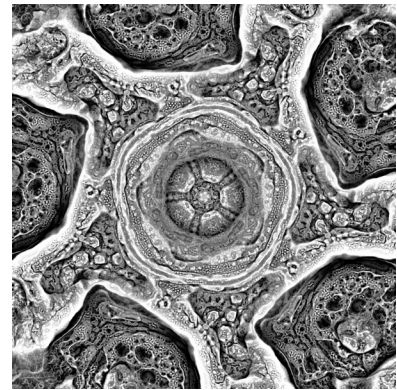


Figure 9

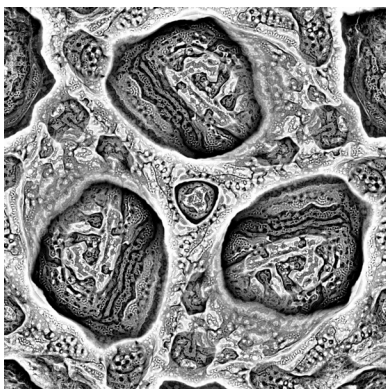


Figure 10

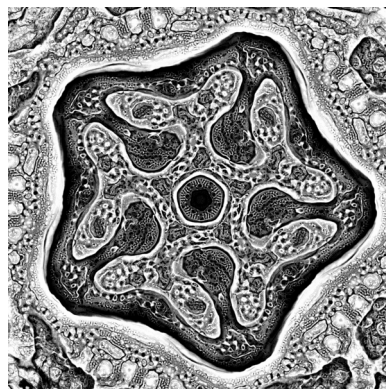


Figure 11

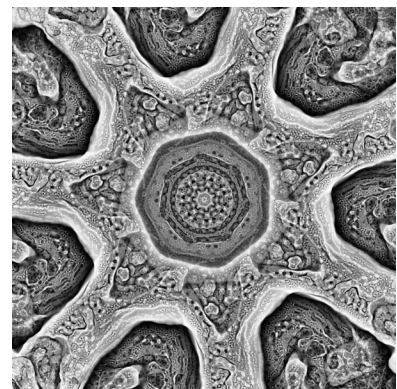


Figure 12

In figure 13 below compound Turing patterns are combined in the manner of multi-scale Turing patterns, giving a synthesis of the methods put forward in this paper. Figure 14 shows a mixture of symmetries, with 3-fold symmetry imposed on the small scale Turing instabilities and 9-fold symmetry on the large scale. We can see differing short scale details within larger repeating patterns.

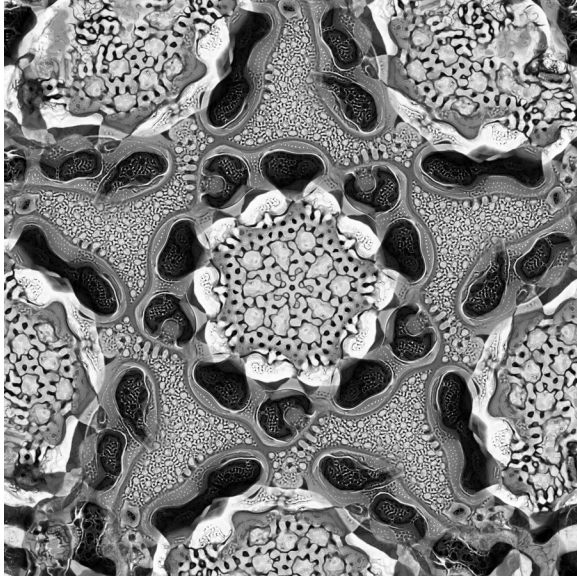


Figure 13

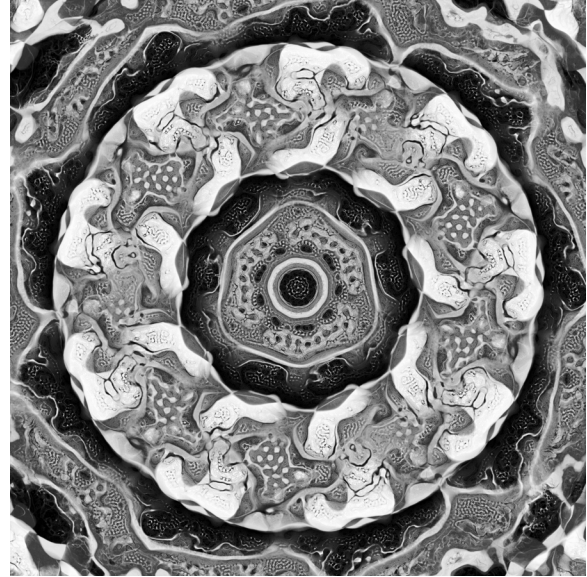


Figure 14

Future work will include an exploration of what sorts of symmetries can be imposed on the Turing instabilities and the application of an aesthetic selection genetic algorithm to the free parameters of this model, to explore the implicit space of potential forms.

References

- [1] A. M. Turing, *The Chemical Basis of Morphogenesis*, Philosophical Transactions of the Royal Society of London. Series B, Biological Sciences, Vol. 237, pp. 37-72. 1952.
- [2] V. Castets, E. Dulos, J. Boissonade and P. De Kepper, *Experimental Evidence of a Sustained Standing Turing-type Nonequilibrium Chemical Pattern*, Physical Review Letters Vol. 64, pp. 2953–2956. 1990.
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