Geometric Transformations in Design Generation

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Abstract

We present a method for transforming a simple plane figure or motif into another, which is related to it in a definite way, by applying a group of systematic geometric transformations. The method creates certain classes of star motifs from a basic plane figure. We demonstrate that surface design is an 'atomic' and a 'self-assembly' process, and expansion, figure and motion are the only properties in design which can be directly represented geometrically. By applying a group of fundamental point transformations of the plane, and rearranging the axis of the reflection, we transform a simple figure into a series of complex star motifs. Our ultimate aim is to develop the invariants of design and creative process, and to bring the formal side of art within the purview of mathematics.

Self-Organized Motifs

In order to gain a quantitative and mechanistic understanding of creative design, it is important to understand the recurrence relations and sequential moves in the transformation process. Although the use of symmetry and geometric transformations to make patterns is well-understood and applied to practical applications at a certain extend [2,4], however, there is no established mechanism for creating a range of aesthetic designs and highly extended structures from first 'building blocks'.

Any figure, drawn, painted, printed, embossed, woven, or otherwise executed on a surface, is termed a 'motif', provided there is at least one possible motion which moves the figure, as a whole but not point for point or part for part, back to its initial position [2]. If A and B are two (not necessarily distinct) sets, then a mapping of set A onto set B in which distinct elements of A have distinct images in B will be a transformation (or one-to-one mapping) of A onto B [3]. In three consequent stages, Figures 1, 2 and 3 shows how this method can be applied to construct chains of two dimensional surface designs in which successive designs are derived from the initial figure and earlier ones in the chain. These designs are all composed entirely of one single very basic rectilinear geometric figure and nothing else. A given figure or the first 'atom' is transformed into a new figure by applying certain transformation or symmetry rules. In Figure 1, if O be a fixed point of the plane figure and θ the angle of rotation, we first apply rotation $R(O, \theta)$, the figure is rotated in the plane through a certain angle, 180º in this case, about a point, and it turns as a whole to its initial position if we apply an inverse transformation. In this case, the individual points are rotated through this angle about the point. In the second step, we apply mirror superimposed transformation, thus a type of reflection symmetry $R(O)$ about the same point. In order to achieve expansion, in final two steps, we rearrange the point of transformation into another point (O') and apply rotational mirror symmetry $R(O', \theta)$, to achieve the final design.

Figure 2 shows the next step and application of first, superimposed mirror transformation $R(O')$ of the final motif of the Stage I, and then rotational transformations $R(P, \theta)$, to the resulting motif, where P is another fixed point outside of the final motif. An arrowhead star and cross design is reproduced and rearranged from the building blocks in its whole expanse. Figure 3 shows the application of additional superimposed mirror symmetry $R(O')$ to the final motif of Stage II and then the application of rotational mirror symmetry $R(P', \theta)$ to the resulting surface design, where P' is the new fixed point on the plane.

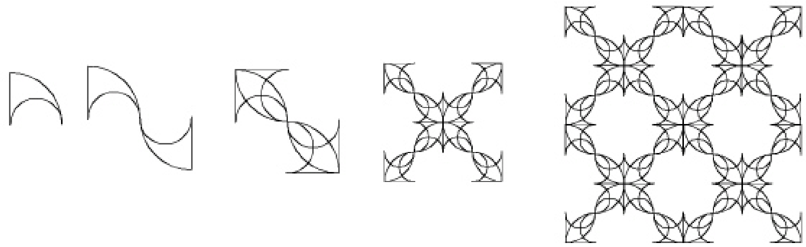

Figure 1: *Geometric transformations in self-organized four-armed arrowhead motif (Stage I)*

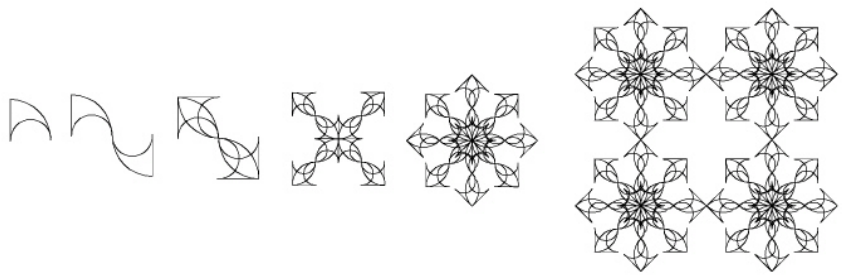

Figure 2: *Geometric transformations in self- organized arrowhead star and cross motif (Stage II)*

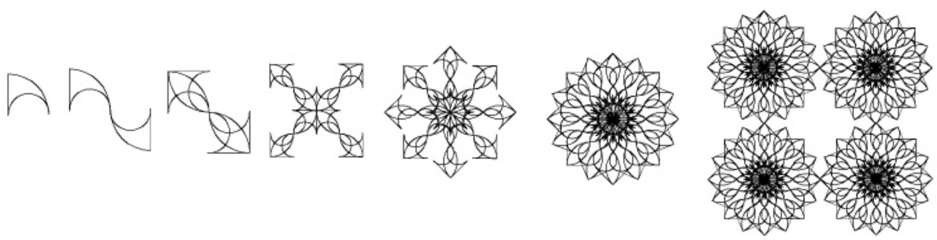

Figure 3: *Geometric transformations in self- organized star motif (Stage III)*

Geometric transformations, as indeed groups and symmetry in mathematics [4], are useful not only for solving problems and discovering new facts, but also in creating new designs [6,8]. This paper shows that the concept of symmetry as understood in light of transformation theory has several advantages in its applicability to many design problems. The process which resembles very closely the self-reproduction of biological molecules also sheds light on uncovering the mathematical mystery in different cultural practices and artifacts[5], such as the geometric decorative motifs in Islamic arts and early Turkish tile and carpet design motifs [1,7].

References

[1] C. Alexander, A Foreshadowing of 21st Century Art: The color and geometry of very early Turkish carpets, Oxford University Press, New York-Oxford, 1993.
[2] G. D. Birkhoff, Aesthetic Measure, Harvard University Press, Cambridge, MA, 1933.
[3] H. W. Eves, A Survey of Geometry, Revised Edition, Allyn and Bacon Inc., Boston, 1972.
[4] D.W. Farmer, Groups and Symmetry: A Guide to Discovering Mathematics, Universities Press, 1998.
[5] P. Gerdes, Geometry from Africa: Mathematical and educational Explorations, The Mathematical Association of America, 1999.
[6] M. Hallett and U. Majer(Eds), D. Hilbert's Lectures on the Foundations of Geometry 1891-1902, Springer, 2003.
[7] C.S. Kaplan and D.H. Salesin, *Islamic Star Patterns in Absolute Geometry,* ACM Transactions on Graphics, Vol. 23 , No.2 , pp: 97 – 119, 2004.
[8] R. Penrose, *The role of aesthetics in pure and applied mathematical research,* 10, pp.266-272, 1974.