Create a Mathematical Banner Using the Lute, the Sacred Cut, and the Spidron

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Abstract

This activity-based workshop will enhance the mathematics teachers' collection of fascinating constructions. Each construction is useful for a variety of student levels. The Lute of Pythagoras, named for Pythagoras of Samos (ca. 580-500 B.C.), demonstrates the interesting relationship between the pentagon and the Golden Triangle. The Sacred Cut, a construction dating to the sixth century B.C., was found in excavated brick remains in the ancient Roman port of Ostia. The Spidron, a spiraling polygon, is a figure popularized by Daniel Erdely in the early 1970's. Each construction will be shown and then will be used as a design element to create a mathematical banner. All materials will be provided by the teacher.

1. Introduction

Constructions using a compass and straightedge add interest to the mathematics classroom. Students enjoy constructing figures. The tools are portable. Constructions invite the student to critically analyze if the marks can be justified appropriately. While reinforcing the theorems of geometric development, constructions add a hands-on motivating facet to the classroom. Further, a brief history of constructions will be included.

2. The Lute of Pythagoras

The Lute construction will begin with the construction of the Golden Triangle BCL. From the Golden Triangle, a "ladder" will be built on the legs of the isosceles triangle BCL , using the base of the triangle as the starting point. In figure 1, BC = CD = BE. Arcs are used repeatedly to continue the construction upward using the base of the previous pentagon to determine the size of the new arcs. Once DE is determined, then DE = DG = EF, and so on. In the Lute, we find a series of Phi relationships throughout the figure. One example is BD: DF is Phi.

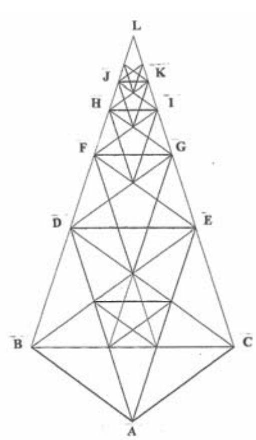

Figure 1: *The Lute of Pythagoras*

3. The Sacred Cut

The Danish scholar Tons Brunes coined the term *sacred cut*. According to an article "A Roman Complex" see Watts [1]:To ancient geometers, the circle symbolized the unknowable part of the world (since the circumference was proportional to the irrational number Pi) while the square represented the comprehensible world. Squaring a circle was a means of expressing the unknowable through the knowable, the sacred through the familiar. Hence the term *sacred cut.*

In Ostia, Italy, the sacred cut appears to have been used to design the overall dimensions of the courtyards of the garden homes. Scales at various levels were used to also design rooms, tapestries, and even the positions of objects within the courtyard. Our square
ADPM is the reference square. To initiate the construction, a quarter circle is swung from points A, D, M, and P. The quarter circles must meet at the center of the reference square. From here, points B and N are connected, C and O are connected, E and H are connected, and finally I is not connected to L, but the segment JK that would be part of IL is formed.

We now have 7 polygons. Students will find it interesting to find the proportions of the poloygons: If AM =1, then ME = square root of 2 divided by 2. The side of the Sacred Cut Square can be found to be (square root of 2) – 1. A nice problem for many classrooms to investigate.

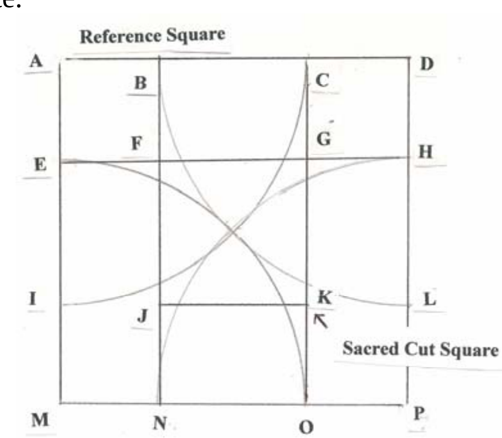

Figure 2: *The Sacred Cut*

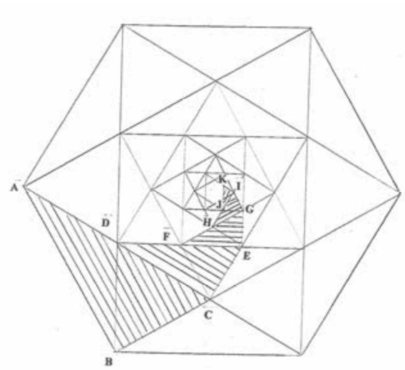

Figure 3: *The Spidron*

4. The Spidron

The Spidron is a totally fascinating set of equilateral triangles and right triangles nesting inside a hexagon. The angles in the triangles in the Spidron are 30, 60, and 90 degrees. These triangles are perfect for study and investigation at a variety of levels.

References

[1] Watts, D. J. and Watts, C., "A Roman Apartment Complex." *Scientific American*, vol. 255, no. 6, 132-140 (December 1986).

[2] Kappraff, J., *Connections—The Geometric Bridge Between Are and Science,* 1991, McGraw-Hill, 28-32.

[3] Kappraff, J., *Beyond Measure—A Guided Tour Through Nature, Myth, and Number,* 2002, World Scientific Publishing Co., Pte. Ltd, 125-129, 217-223.

[4] Boles, M. and Newman, R., *Universal Patterns—The Golden Relationship: Art, Math and Nature,* Pythagorean Press, 1990, 86-87.

[5] Peterson, I., "Swirling Seas, Crystal Balls—Spirals of Triangles Crinkle into Intricate Structures", *Science News*,vol. 179, October 21, 2006.