Connecting Mathematics and the Arts through the Magic of Escher for Elementary School Students

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Abstract

According to Discipline Based Art Education, the teaching of Art as an academic subject should focus on the content of Art Production, Art History, Art Criticism and Art Aesthetics. Likewise, according to the National Council of Teachers of Mathematics, the teaching of Mathematics should integrate both content and process standards. Escher’s art can be a motivating catalyst for blending the study of visual arts with mathematics in the elementary classroom, especially in relation to tessellating patterns through the use of transformational geometry.

Introduction

In the early 1980’s, the J. Paul Getty Trust, a private foundation located in Los Angeles, California dedicated to the visual arts, developed Discipline-Based Art Education (DBAE) as a conceptual framework for the study of the visual arts in K-12 education [1]. The DBAE framework communicates an encompassing view of art through the study of an artist’s work(s) or types of work using four distinct disciplines adapted to specific grade levels:

• ART PRODUCTION: Students learn skills and techniques in order to produce personal, original artwork.
• ART HISTORY: Students study the artistic accomplishments of the past and present as motivation, examples of style or technique, and as discussion topics, especially in relation to cultural, political, social, religious, and economic events and movements.
• ART CRITICISM: Students describe, interpret, evaluate, theorize, and judge the properties and qualities of the visual form, for the purpose of understanding and appreciating works of art and understanding the roles of art in society.
• ART AESTHETICS: Students consider the nature, meaning, impact, and value of art, to formulate reflective, “educated” opinions and judgments about specific works of art, and examine criteria for evaluating works of art [1, pp 3-4].

Therefore, the goal of discipline-based art education is to develop and enhance students’ abilities to understand and appreciate art. Instruction in the four disciplines should deal with a) production-processes and techniques for creating art, b) history-contexts in which art is created, c) criticism—a basis for valuing and judging art and d) aesthetics-conceptions of the nature of art [2, p 135]. For the elementary teacher, this means facing the challenge of implementing art education classroom objectives for elementary school children so as to incorporate:

1) The making of art and the components of making art through the use of tools, media, form and expression;
2) Past and present history through works of art and artists’ lives;
3) Style, principles of design, and elements of art through critical analysis and discussion; and
4) The philosophy of art through aesthetic questioning--what is art? What is good art? What is beauty?-given the understanding there are no right or wrong answers.

Given that DBAE is an instructional approach that integrates the visual arts with higher order, critical thinking skills through art history and an appreciation for art, then a corollary aspect is to integrate other curricular areas such as mathematics, science, social studies and/or language arts. The visual arts can act as a medium for understanding mathematical skills and concepts imbedded in the National Council of Teachers of Mathematics Principles and Standards for School Mathematics [3]. The five Content Standards (Number and Operations, Algebra, Geometry, Measurement, and Data Analysis and Probability) describe the understanding, knowledge and skills students should acquire from pre-kindergarten through grade 12 and the five Process Standards (Problem Solving, Reasoning and Proof, Communication, Connections, and Representation) emphasize ways students should acquire and use content knowledge.

According to the Connections Process Standard, elementary (grade 3-5) students should participate in instructional programs that enable all students to:

1. Recognize and use connections among mathematical ideas,
2. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.
3. Recognize and apply mathematics in contexts outside of mathematics [3, p. 200].

Statement number 3 clearly mandates the need to address how, when, where and why mathematics is used in real world settings. And, according to the Geometry Content Standard, elementary (grade 3-5) students should be able to “apply transformations and use symmetry to analyze mathematical situations” as well as “recognize geometric ideas and relationships and apply them to other disciplines.” In addition, elementary students in grades 3-5 are expected to apply the principles of transformational geometry given the expectation that students should be able to “predict and describe the results of sliding (gliding), flipping (reflecting) and turning (rotating) two-dimensional shapes” [3, p.164].

The fascinating works of M.C. Escher provide an intriguing catalyst for integrating art and mathematics as described by the DBAE approach to art instruction and NCTM’s process standard for connections and content standard for geometry. According to Severin [in 4], Escher is a “remarkable and original artist who was able to depict the poetry of the mathematical side of things in a most striking way” [4, p. 25]. Many of Escher’s works tessellate patterns through the use of non-rigid tessellations, e.g., the application of dilation whereby he shrinks or expands images. Even so, his art can serve as a motivational tool to investigate and explore the mathematical connections between the creation of tessellating patterns and the basic principles of transformational geometry. In an effort to expand elementary teachers’ perspective of how mathematics may be connected to the visual arts, the works of Escher as well as his profound use of tessellation to cover a surface including associated transformational aspects will be explored through classroom-based activities designed for elementary (grade 3-5) students.

**Activities**

The following DBAE activities begin with an historical review of Escher, suggest samples of his works so as to address the aspects of criticism and aesthetics, and conclude with mathematical productions of tessellations. Using a selection of Escher’s works, teachers
introduce elementary students to an historical perspective of the man and his works. Select other samples of Escher’s works and incorporate the elements of criticism and aesthetics.

**Art History.** It’s not unusual for elementary students to want to know what an artist’s life was like. Introduce Escher to elementary students through a synopsis of his early life. This can be done through storytelling or in electronic format such as a PowerPoint. Be sure to include personal photos of the artist such as Escher’s *Self-Portrait*, a lithograph made in 1929.

Maurits Cornelis Escher was born on June 17, 1889 in Leeuwarden, Holland. He was the youngest son of George A. Escher, a hydraulic engineer. Unlike his father, Escher was not a very good student. His grades were poor, but he excelled at drawing. His father wanted him to become an architect, but Escher preferred the decorative arts. He studied with de Mesquita and mastered the printmaking technique of making woodcuts. After two years of study, he left school in 1922 and traveled to Italy where he lived until 1935[4].

Tell students that Escher, a famous graphic artist, was known for his woodcuts and lithographs, two ways of making prints. In a woodcut, Escher carved his design on a block of wood, applied ink to it, and then stamped the design onto paper such as his work entitled *Self-Portrait in a Chair* (1920). To make a lithograph, he used chemicals and water to create an image on a metal plate, e.g., *Relativity* (1953). The plate was then used to stamp his design onto a piece of paper.

**Art Criticism.** Have elementary students explore and experience the analytical aspect of criticism by first sharing examples of Escher’s work along with anecdotal narratives. Conclude each presentation with critical questions related to the specific works of art. As students respond, listen for and encourage the use of 1) elements of design, the visual tools artists use to create art such as line, shape, form, texture, color, and space, and 2) principles of art such as balance, harmony, variety, emphasis, rhythm and movement[5]. This type of activity might also work well with middle and high school students.

Interestingly, even though Escher left Italy due to political unrest, his works were not directly influenced by the times in which he lived but rather what he found interesting. His early works (1922 to 1937) were mostly landscapes of Italy and Mediterranean coastlines. During his journeys he created hundreds of drawings and sketches such as the *View of Atrini*, a colorful seaside drawing completed in 1931.

Criticism questions for discussion should include describing, analyzing, interpreting, or judging a piece of art. Using samples of Escher’s drawings and sketches, discuss the following criticism questions for each work of art.

- **Describe:** What do you see when you look at this work?
- **Analyze:** How is this work of art organized?
- **Interpret:** What is the artist saying to me?
- **Judge:** Is the work successful?

Escher’s later works were seldom created directly from his observations and travels, but rather from images created in his mind. One of his fascinations with graphic images was the effect of having 3-D grow out of 2-D which created prints of “impossible reality.” For example, his work entitled *Reptiles*, a lithograph completed in 1943, looks very realistic, but could not actually happen in the real world.

Using samples of Escher’s graphic, “impossible reality” images, discuss criticism questions such as the following for each work of art.

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Describe: What does this work of art remind you of?
Analyze: What do you think is happening in this picture?
Interpret: What title would you give this picture and why?
Judge: Why do you think other people should see this work of art?

Art Aesthetics. Have elementary students explore and experience the philosophical aspect of aesthetics by first sharing examples of Escher’s work along with anecdotal narratives. Conclude each presentation with philosophical questions related to the specific works of art.

Escher became a master at using mathematics to create tessellations. Tessellations are patterns of pictures that fit perfectly together without any spaces or overlaps. In his youth, Escher experimented with tessellating patterns. He became discouraged when his products did not match his mental visions. For a while, he abandoned his works with tessellations and concentrated on drawings from nature [6].

In 1922 and again in 1936, Escher visited the Alhambra, an old Muslim castle, in the southern mountains of Granada, Spain. The castle walls, ceilings and floors were covered with colorful, tessellating tiles, the same style of art he experimented with as a youth. He sketched these examples of Islamic art, studied the patterns and taught himself how to design his own style of tessellating art by using geometric grids as the basis for his preliminary, beginning sketches [6].

Applying the mathematical properties of tessellating with geometric shapes, Escher created tessellating patterns with designs from nature, mostly with animals and sometimes people. His work entitled Sun and Moon, a woodcut completed in 1948, exemplifies his unique style of tessellating art. Using samples of Escher’s tessellations, discuss aesthetic questions such as the following:

Why do you think Escher created this work?
Must you understand Escher’s intentions to appreciate his work?
If Escher were alive today, how do you think his tessellation works might have evolved?

One of Escher’s unique expressions of artistic design is the morphing of tessellating images. In his Sky and Water, a woodcut completed in 1938, the tessellating birds in the sky morph into the “impossible reality” of fish in the water. Using example’s of Escher’s morphing tessellations, aesthetic questions for discussion may include those related to the viewer’s personal thoughts, ideas, or beliefs.

Can art be an imitation or an interpretation of nature?
How does this piece of art make you feel?
Would you want to hang this piece of art in your home?

Art Production: Have elementary students produce one or more of these tessellation projects to celebrate Escher’s work while concretely exploring geometric principles and properties associated with tessellations and transformations. Begin with simple geometric tessellations that use the transformational property of sliding (gliding). As students’ abilities develop and or interests’ evolve, introduce the visual effect of flipping (reflecting) or turning (rotating) geometric shapes or other images/shapes.

1) Tessellate geometric shapes such as rectangles, squares, rhombuses or parallelograms. Use the following steps to create a rectangular tessellation.
   a) Create a master pattern of a 2x3 inch rectangle on card stock paper and cut it out very carefully.
b) Mark one side of the rectangle with an “X” to consistently trace the same side.
c) Trace the 2x3 inch rectangle across the top of a sheet of 9x12 white construction paper viewed with the 12-inch length across the top and bottom.
d) Slide the rectangle under the first row of traced rectangles and trace a second row of rectangles.
e) Slide the rectangle under the second row of traced rectangles and trace a third row of rectangles.
f) Cut off the last inch from the 9x12 white construction paper for a finished 8x12 tessellation of rectangles.
g) In the first row, color in every other rectangle.
h) In the second row, do not copy the first row, but color in alternate rectangles.
i) In the third row, copy the first row by coloring in every other rectangle.
j) Sample finished product (see Figure 1).

![Figure 1](image1)

k) Experiment with other rectangular variations such as sliding the first rectangle in the second row to the midpoint of the first rectangle in the first row (see Figure 2).

![Figure 2](image2)

l) Try using squares, parallelograms or rhombuses to replicate steps a-k for other geometric tessellations. Or, challenge students to tessellate with other polygons such as equilateral triangles whereby flips (reflections) or turns (rotations) must be utilized (see Figure 3).

![Figure 3](image3)

2) Tessellate shapes made from a square or rectangle.
   a) Begin with a 3x3 or 2x3 inch card stock paper square or rectangle.
   b) Across the top or bottom, from corner to corner, cut a free-form shape.
   c) Slide the cutting to the opposite side of the polygon and tape the cutting into place.
   d) Down the right or left side, from corner to corner, cut a free-form shape.
   e) Slide the cutting to the opposite side of the polygon and tape the cutting into place.
   f) Look for an image within the space such as an animal, person or thing.
g) On a piece of paper 5x7 or larger, fill in the page by tessellating the pattern across in rows or down in columns.

h) Color in the results to illustrate the image determined in step f (see Figure 4)

![Tessellated shape](Image)

**Figure 4:** Tessellated shape [7]

Once students have made a work from tessellating a geometric shape or an image created from a rectangle, have students share their artistic product. Encourage students to answer the following evaluative questions about their tessellation [8]. For visual arts ask: Does the work look the way they imagined it? What do they like about the work? What improvements or changes could be made? For mathematics ask: Does the shape cover the paper without any gaps or overlaps? What transformations were used to tessellate the shape?

**References**


