Porter's Golden Section, Experimentally

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Abstract

The paper relates the story of "The Porter Code". Long standing research by the two co-authors, about Fairfield Porter's paintings on one hand, and mathematical considerations about the golden section on the other hand, came together through discussions at several Bridges conferences. The experiment initially seemed to re-enforce Porter's use of golden ratio harmony in his composition, but the co-authors re-thought their original conclusion that the audience recognized golden proportions. Perhaps, if the paintings are indeed based on phi relationships then the test subjects' choices of dominant divisions of space and subject matter may just naturally appear to corroborate a golden ratio composition. The authors also recognize the experimental methodology employed in this initial collaboration requires careful revision to dispel skepticism over the significance of the results.

1. Introduction

Fairfield Porter (1907–1975) was a political activist, a bourgeois bohemian, and a bisexual–a renegade individualist whose life and art shared parallel paradoxes. He gained notoriety as a heretical realist in a period when abstraction was in vogue, combining painterly technique with intriguing figurative subject matter. Articles about Porter usually emphasize his search for "naturalness", where the paint relays the perceptual experience and the artist becomes an objective observer. As a diligent intellectual, this also meant he would have to suspend cerebral analysis and conscious deliberation for direct sensory experience, assuming an almost Zen-like attitude to the experiential nature of painting, a kind of artistic anarchism. Thus he is a controversial example to demonstrate the use of sophisticated geometry in art, because he and his painter friends seem to deny any system of organization in his pictures. Indeed, co-author Chris Bartlett showed that, true to the contradictions that seemed to define his life, Porter employed a precise compositional syntax. Certainly, an important principle of design is repetition, so creating any structure such that the elements of a painting are aligned within self-similar areas would satisfy the goal of unity and provide a harmonizing geometry. For Porter, a carefully articulated arrangement of forms in accordance with the tenets of time-honored systems of design and harmony the golden ratio preceded what seems spontaneous and improvisatory. Hidden below the surface this geometry, the formal construction of his paintings, was accomplished with inspired esthetic intelligence and an invisible mastery of composition. [1,2,3]

While presenting papers on the geometric analysis of Porter's paintings, demonstrating the presence of the golden section as their organizing principle, it was asked if this could have been simply coincidence or just an a posteriori interpretation (beauty in the eye of the beholder?). Regardless of whether the golden section makes Porter's paintings more visually attractive, Bartlett believes he has convincingly established that the construction of Porter's paintings uses phi, but he wondered if the viewers of the pictures could actually detect/ perceive/ experience these golden section compositional proportions. He began looking at the literature on the experiments on preferences of golden section proportions. The growing refutation of the principle of beauty or visual harmony in the application of the golden ratio proportioning in art, suggested results may have been skewed by stimulus range and experimental bias. [14] So rather than enter into the controversy as to whether golden section proportions are the test subjects' preferred ratio, it seemed a more useful approach would be to test the geometric analysis of Porter's composition with an experiment that would provide more convincing empirical data.

Speculating what kind of procedure would avoid the criticisms leveled at previous studies of golden section perception, co-author Huylebrouck proposed an experiment where the paintings are viewed and participants are asked to mark important points on the pictures which could then be plotted and the ratios determined using statistical analysis and celestial mechanics just as astronomers do with their data.

2. A statistical approach about art observations

In celestial mechanics, Carl Friedrich Gauss removed the most skeptical doubts about how to find the best curve fitting to given data, when he found the "lost planet" of Ceres, which astronomer G. Piazzi had briefly observed in January of 1801, and which no other astronomers could relocate during the rest of 1801. Campbell and Meyer gave an excellent account on this famous curve-fitting story, which we adapt here for the data given as in the illustration on the left, while we are looking for the closest ellipse as in the illustrations on the right. [5]

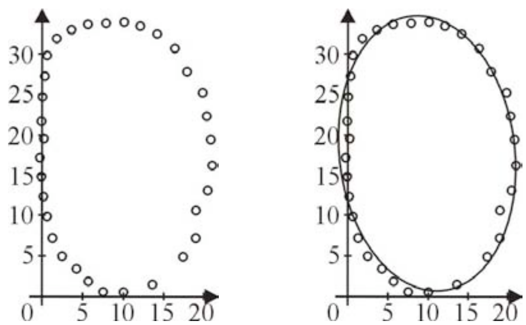


Fig 1: *Curving fitting*

Campbell and Meyer tell us to collect the data

x1 = 17.496; y1 = 4.871; x2 = 19.129; y2 = 7.143; x3 = 19.200; y3 = 10.551; x4 = 20.549; y4 = 13.107; x5 = 21.259; y5 = 16.160; x6 = 21.046; y6 = 19.426; x7 = 20.549; y7 = 22.337; x8 = 20.052; y8 = 25.248; x9 = 18.064; y9 = 27.804; x10 = 16.573; y10 = 30.644; x11 = 14.372; y11 = 32.490; x12 = 12.313; y12 = 33.413; x13 = 10.396; y13 = 33.910; x14 = 8.195; y14 = 33.839; x15 = 5.852; y15 = 33.697; x16 = 3.722; y16 = 33.058; x17 = 1.947; y17 = 31.993; x18 = .740; y18 = 29.792; x19 = 0.456; y19 = 27.378; x20 = .243; y20 = 24.822; x21 = 0.243; y21 = 12.397;
x22 = 0.811; y22 = 9.912; x23 = 1.450; y23 = 7.427; x24 = 2.657; y24 = 5.013; x25 = 4.361; y25 = 3.380; x26 = 5.710; y26 = 1.889; x27 = 7.688; y27 = 0.571; x28 = 10.224; y28 = 0.571; x29 = 13.805; y29 = 1.541.

in matrices

$$= \begin{bmatrix} x_1{}^2 & y_1{}^2 \\ x_2{}^2 & y_2{}^2 \\ \vdots & \vdots \\ x_4{}^2 & y_4{}^2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} \frac{1}{a^2} \\ \frac{1}{b^2} \end{bmatrix}, \mathbf{j} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}.$$

In their matrix notation, the curving fitting problem implies **X.b-j** should be minimal, and the numerical values can be found by computing the product of the so-called "generalized inverse" of **X** and **j**. This is easily done using today's software, such as $\text{Mathematica}_{\text{TM}}$. It results in an ellipse

$$- 0.0078x^2 - 0.0033y^2 + 0.1686x + 0.1241y - 0.0009xy = 1$$

However, in the present case, the data were not based on astronomical observations, but on measurements made on a reproduction of the "Mona Lisa". With the same determination as in Gauss' case, it could now be concluded that a rectangle with a proportion of 1.54... provides the closest fit (and this is closer to, say, 1.5, than to the golden number of 1.618...). The word "closest" can be substantiated, since it can be computed to fit "at 99.55%".

In a publication about this approach, similar examples were given based on reproductions of architect Gaudi's Palau Güell, and it was concluded a hyperbolic cosine function approximated it better than a parabola. [8] However, for Gaudi's Collegio Teresiano, a parabola could be chosen with the same degree of accuracy, and this was a rather surprising result, in view of the widespread association of Gaudi

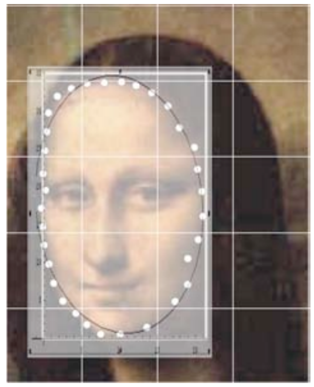


Fig 2*: The closest Mona Lisa ellipse.*

and catenaries. Later on, during the "Mathematics and Design 2007" conference in Blumenau, Brazil, co-author Huylebrouck was surprised to learn Barcelona Prof. Amadeo Monreal shared this view, based on his profound knowledge of Gaudi's work. [13] Understandably, co-author Huylebrouck supposed he could turn to co-author Chris Bartlett, whose prime research is the geometry of Porter's composition, to find what could be "The Porter Code" …

3. Geometric Analysis of Porter's Composition

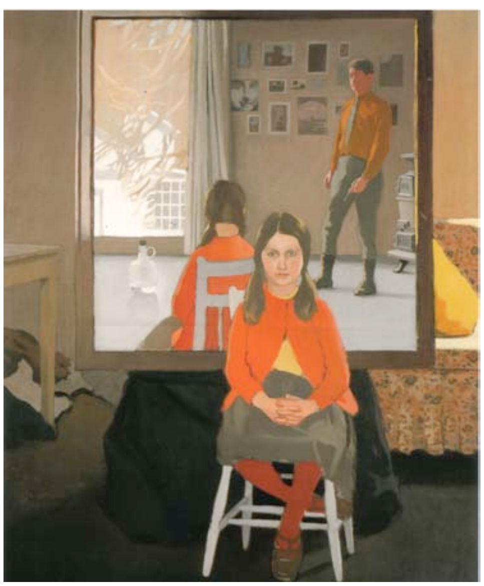


Fig. 3a*: The Mirror,* 1966, 72" x 60" (The Nelson-Atkins Museum of Art, Kansas City, Gift of the Enid and Crosby Kemper Foundation). **Fig. 3b**: *View of the Barred Islands,* 1970, 40" x 50", (on loan to Rose Art Museum, Brandeis University from the Herbert W. Plimpton Foundation).

The two Fairfield Porter paintings chosen for this project, one an interior with some obvious linear divisions and the other more organic, one a horizontal format and the other vertical had not been previously analyzed by co-author Bartlett. *View of the Barred Islands* is typical of the many paintings done from his family summer home on Great Spruce Head Island in Maine. It's a familiar view from the porch of the house overlooking the Island's small harbor and looking out towards this group of islands. *The Mirror* depicts the interior of Porter's Southampton studio in which his daughter, Elizabeth, is sitting on a chair in front of a large, square mirror. Behind is the reflection of Porter, brush in hand, who stands between a tall window and a wood stove in front of a wall filled with reproductions of paintings (the "Mona Lisa" among them).

A number of golden rectangles emerge from the visual evidence of the painting. Starting with the outer edges of the mirror frame, (the largest and clearest square) from half the height of that square up the left edge and swinging the diagonal formed by that point with the lower right corner of the mirror, in the classic golden-rectangle manner, we find it hits the bottom edge of the canvas forming a large, centrally-placed golden rectangle. Fig. 4a. In the space beneath the mirror (the smaller golden rectangle), the height from the mirror frame to bottom edge of canvas makes a square with the axis in the center of the girl, leaving another smaller golden rectangle to the right of that axis. That further subdivides as square and golden ratio rectangle at the level of the ruffle and the horizontal fold in the skirt. The horizontal of the girl's eyes forms a line with the base of the window, which in turn creates a Φ rectangle with the vertical of the right side of the curtain. That line continues down to define the juxtaposition of real and reflected images of the girl creating another Φ rectangle with the outer frame of the mirror. This Φ rectangle is "reflected" (repeated) on the right side of the girl's vertical axis.


Figs. 4a, 4b: *Co-author Bartlett's study of the elements of composition in "The Mirror" and "View of the Barred Islands"*

The *View of Barred Islands* is a 1:1.27 root golden ratio rectangle, which means the sides and diagonal are in a geometric progression of 1, √Φ, Φ. Dividing the long side successively by √Φ yields the short side and all the same Φ proportions as each other. Here though, with organic subject matter the analysis is more problematic. The clear verticals in the subject matter are the tallest tree and the right edge of the trees. Fig. 4b. The width and height are the same forming an implied square. Using the same method described in *The Mirror,* the diagonal swung to the right from the base of half the square hits the bottom edge of the canvas to define the vertical on which Porter placed the boat. The length of that diagonal is also the distance from the top canvas edge to the boat. The boat is on a line with the nearest islands to form a clear horizontal. This horizontal divides the short side of the canvas by Φ proportions. The boat although tiny, because of its isolation, becomes a strong focal point. Porter positioned it horizontally the same distance from the tallest tree, as the tree is high forming another overlapping square and creating two smaller Φ rectangles within the ends of the larger.

4. The Porter experiment

Although a discussion of the literature on perception and Gestalt psychology would be useful here it is beyond the scope of this short paper, but hopefully can be discussed in a future paper.

University art faculty and art majors of co-author Bartlett together with majors in other disciplines were selected as test subjects. The two paintings were presented to these test subjects as large, very high quality reproductions of the originals. A question was formulated that would tend to deter a focus solely on subject matter, or picking patches of bright color, arresting shapes, contrasting values, etc., but was also phrased so as not to bias test subjects into picking out any pre-conceived relationships and to have minimum constraints. A question giving totally free choice, such as “pick important points in the painting” was dismissed, since respondents are not sure what is being asked of them. The question purposely did not have any reference to rectangles, composition, design or aesthetic preference. Participants were asked to examine the pictures as closely as they wanted, individually, without conversation or question and directed by the question:

“Using just single dots/points, not lines, mark *important* actual or suggested horizontals and verticals, and intersections within or at the edges of the painting, and any other points you think could be significant in the painting. Use as few or as many dots as you want.”

Fig. 5: *Test subjects examining the pictures and marking the points they chose.*

Each person was given an approximately 8”x10” color reproduction of each of the two paintings and asked to mark their chosen points accurately and carefully on these reproductions. They were given no time limit, but they took about 10-15 minutes to mark points for both paintings. They were asked to put their names, major and concentration within the major on the back. The small reproductions were collected and organized into majors and concentrations. For *The Mirror* 188 respondents were collected and 202 for *View of the Barred Islands*. Using a numbered grid co-author Bartlett tabulated the horizontal and vertical co-ordinates for each individual’s set of points on each painting. The co-ordinates were then entered into an Excel spreadsheet for each painting. The sheets of Excel data were then forwarded to co-author Huylebrouck in Belgium.

5. First outcomes of the “experiment”

To get an initial grasp on the abundance of data, some graphical representations were tried out before starting the computational aspects. Initially some difficulties with the data were encountered until it was realized co-author Bartlett was using a grid with the zero axis at the top rather than the bottom left. However, collecting just three groups, a large set of data was at our disposal. The data were plotted on the painting, and straight lines became clearly visible, no statistics whatever being needed. The obtained golden rectangles seemed to have a straightforward explanation too: a large one (shown by a - . - .- white line) almost surrounds the painting, as to give the general disposition of the canvas. Four vertical ones (- - - black line) and two horizontal ones (… line, one black, one white) give the vertical subdivision and the composition of the main subjects of the painting.

Clearly, even the skeptical mathematician can imagine a painter making such a golden section composition. He must have placed the girl on her chair in the desired position with respect to the mirror behind her. The fact that the window has a golden proportion is not surprising either.

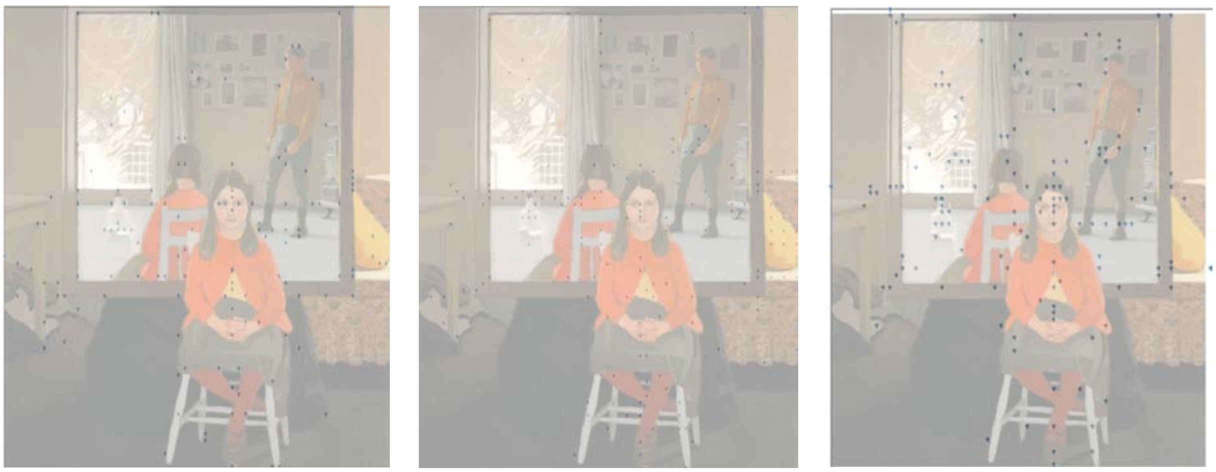


Fig. 6c. Business **Fig. 6d**. Fine arts **Fig. 6e**. General

The data of three groups, fitting well to primary points on the painting.

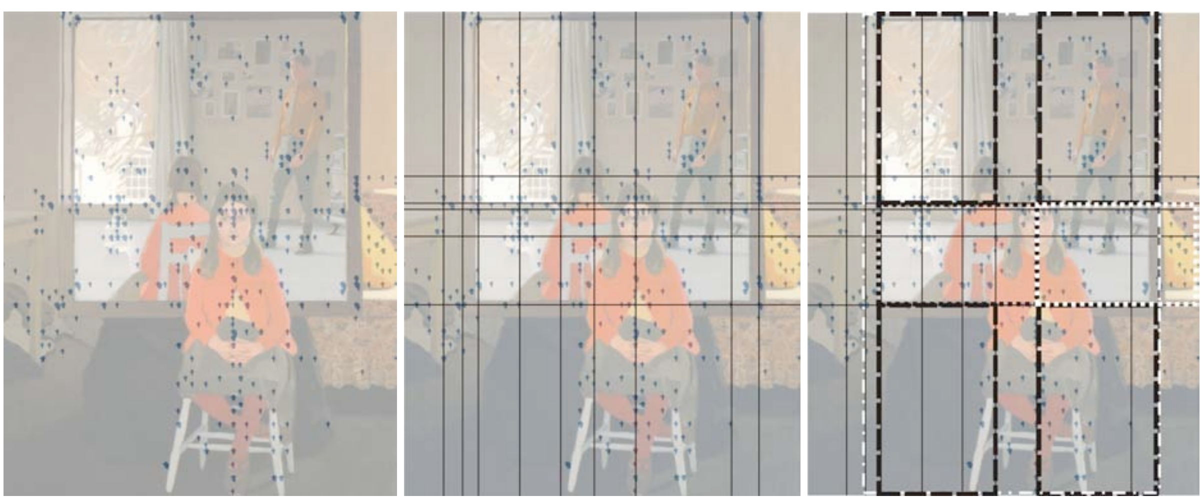
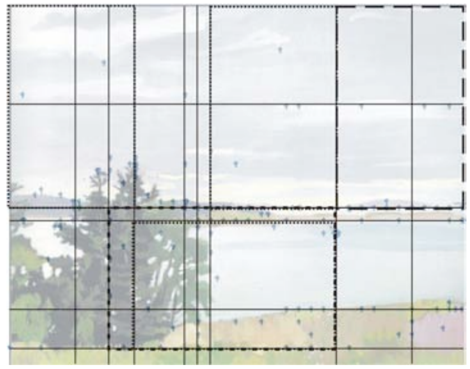


Fig. 7a: *The data on the painting.* **Fig. 7b**: *The obvious straight lines.*

Fig. 7c: *7 undeniable golden rectangles (5 vertical, 2 horizontal).*

However, the second painting proposed by co-author Bartlett was a landscape, and at first sight it would be very surprising to discover a golden section in an empirically derived view of an island and trees by an artist who appears to be making a direct, observational rendering of nature. To save time for this first draft of an experimental approach, the group of business students was chosen. Nevertheless, even this smaller selection of points tended to align easily into vertical and horizontal lines, and their intersections again turn into rectangles, many of which have the 1:1.618 proportion. Strangely, none of the students identified points at the lower edge of the painting, but the boat in the bay attracted their attention, as most indicated. Surprisingly, the tree seems to form a golden rectangle (even two) with the boat. Presumably, the painter was walking along the bay, admiring the landscape, and he may have been struck by a particular point and time of observation where that tree and boat would form a golden ratio rectangle in a composition. Figs. 8a, 8b.



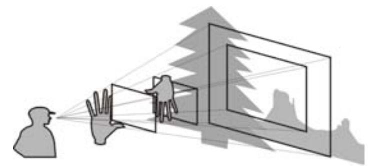


Fig. 8a: *Left: the data on the View of the Barred Islands.*
Fig. 8b: *middle: some golden rectangles (3 vertical, 2 horizontal).*

Comments

It is not hard to see why this paper would create some controversy. The co-authors are very familiar with the literature refuting the so-called "myth of the golden ratio", and agree that many applications of the golden ratio may be flawed. [6,7,11,12] On the other hand, there is also a body of literature on painting detailing the use of golden ratio proportions in composition [4] and in many other fields as well. [9,10,15] In Porter's work a golden ratio composition at first seemed supported by the results of the experiment, particularly in *The Mirror*. However, after a skeptical review the authors were obliged to re-evaluate their

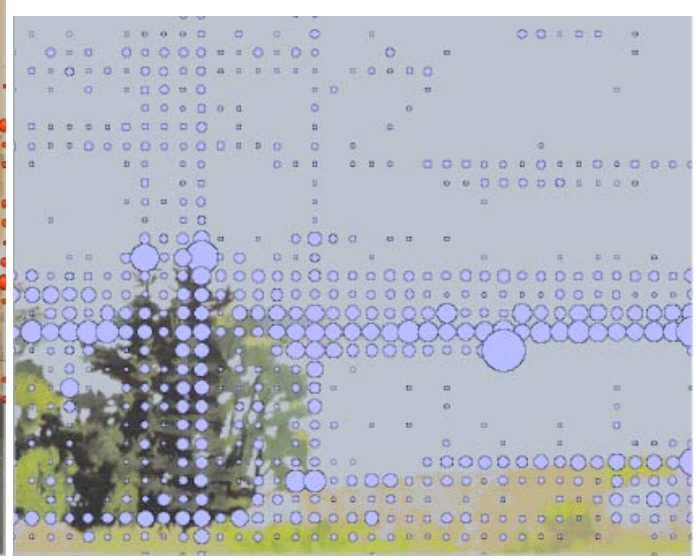


Fig. 9a, 9b. *Two new diagrams since the review have been included to present a compilation of all the data gathered to demonstrate how the dominant points (most "hits") and lines emerged.*

procedure, data and conclusions. First, they had omitted to mention in their original submission that to help avoid "a self-fulfilling prophesy" with an arbitrary selection of rectangles, only points with the most "hits" and those with contiguous positions on the grid forming lines were used. Also, the two authors did their analysis independently, not seeing each other's results or commentary until the first draft of the paper. Thus the data seemed to show sufficient linear correspondences to arguably reinforce co-author Bartlett's analysis. So, initially the authors were under the impression that their experiment proved that the phi rectangles were significant. However, after the review the authors rethought their original conclusion that the test subjects recognized golden rectangles. Since, if the paintings are indeed based on phi relationships then the test subjects' choices of dominant divisions of space and subject matter may just naturally appear to corroborate co-author Bartlett's analysis.

Co-author Huylebrouck wanted to use his technique given for the "Mona Lisa" example in the Porter case, but it seemed that with the given precision, the statistical technique would have involved quite a mathematical overkill, as the golden section rectangles seemed to emerge in this case without it. The authors see this experiment as a first tentative step and concede that the experiment and the data interpretation can be significantly improved. It is hoped that with more background on perception in art, by asking more appropriate questions, by changing the experimental set-up slightly, and by limiting researcher interpretation, such a statistical study for the detailed analysis could indeed make sense. For instance, the "experimental question" could be to provide the coordinates of the most clearly distinguishable rectangle delimited as such, in the opinion of the observer. Then, the celestial mechanics procedure could be applied on these data, as was done for the "Mona Lisa" case.

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