The Maths of Churches, Mosques, Synagogues and Temples

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Abstract

Places of worship are a rich source for creative mathematics. The wealth of symbolism such places contain forms a way of bridging the gap many school students experience between mathematics and other ways in which we reflect on, and make sense of, our experience. The starting point of this paper is the way in which people of all cultures invest symbols with meaning. This meaning might be cultural or religious; it might also be mathematical. The meaning given to a symbol may well depend on simple mathematical properties of the shape chosen as a symbol. Examples of mathematical shapes which are frequently found in places of worship, and have acquired meaning because of their specific mathematical properties, are symmetric patterns and circles, and these are explored further. I have used these ideas in several workshops with school students, and the workshop which this paper accompanies will be an opportunity for participants to explore some of the ideas given here for classroom work. Such activities can be used to support both the regular curriculum and as cross-curricular enrichment activities.

Introduction

All around us, whichever country we live in or culture we are part of, are buildings used for worship. These buildings contain many different mathematical features as part of their architecture or decoration. Places of worship are also full of symbols, many of which use mathematical objects to create meaning.

Looking at places of worship with a mathematical eye gives school students an opportunity to see how mathematics can be used to express religious ideas, and also enables them to use their imaginations in creating their own mathematics. Ideally a topic like this would involve at least one visit to a local place of worship. Such a visit would have a cross-curricular focus, with students finding out more about the building and the faith to which it belongs, and also investigating the symbolism of what they see. Students should also be encouraged to find their own mathematics in what they see, stimulating them to think creatively about the connections between mathematics and the world around them.

Symbols

Figure 1 shows a collection of shapes. When asked what these shapes mean to them, school students frequently interpret many of them as religious symbols or national flags, the precise identifications depending on what colour the various shapes are and where the students live. If pressed for further meanings, they may also observe that there are addition and multiplication signs.

Figure 1: *Symbols*

They very rarely say that what they see is just a collection of lines or shapes, and yet that really is all that these are! We imbue simple shapes and configurations of lines with meaning which varies with context, but which those who share a culture will readily recognise and accept.

Symbols like these provide us with a form of shorthand. For instance, if a group of children from the UK see the elongated cross on the lower left in Figure 1 (which is red on a white background), they will immediately identify it as the English flag, and will probably start talking about football – flags with a red cross on a white background can be seen hanging out of car doors and windows of houses whenever the England football team is in action. English children may well cheer; Welsh, Scottish or Irish will probably not! However, no matter what their emotional reaction, they will all have a similar story in mind, in which the England football team is either wining gloriously or losing spectacularly. The red cross on a white background acts as a prompt for, and a symbol of, this story.

The number of words required to write the last paragraph indicates to me how powerful symbols are, and how, in comparison, words are a far less concise and frequently much clumsier form of expression. The power of a symbol lies partly in its simplicity and partly in the wealth of meaning ascribed to it. Symbols do not exist in the real world in the way that, say, a table does or a flag, but they are objects we create to enable us to communicate and to share meaning. Indeed, shared symbols are part of what forms a community.

Symbols have always been fundamental to mathematics also – numbers, operations, algebraic variables, geometric diagrams, and so on, are all symbols in that they encapsulate meaning beyond the marks on a piece of paper of which they are actually composed. Figure 2 contains a translation of a verse by Niccolo Tartaglia (c1500-1557) which he included in a letter written in 1539 to Geronimo Cardano (1501-1576). In this letter, Tartaglia explains: "I want you to know, that, to enable me to remember the method [for solving certain cubic equations] in any unforeseen circumstances, I have arranged it as a verse in rhyme, because if I had not taken this precaution, I would frequently have forgotten it …" [1]. The equivalent expressions in modern symbols are shown on the right of Figure 2. The development of such symbols was hugely important in enabling mathematicians to express and solve problems.

When the cube and the things together
Are equal to some discrete number,
Find two other numbers differing in this one.
Then you will keep this as a habit
That their product should always be equal
Exactly to the cube of a third of the things.
The remainder then as a general rule
Of their cube roots subtracted
Will be equal to your principal thing.

Given $x^3 + cx = d$, find u, v, such that $u - v = d$ and $uv = \left(\frac{c}{3}\right)^3$ then $x = \sqrt[3]{u} - \sqrt[3]{v}$

Figure 2: *Part of Tartaglia's method of solving cubic equations [1]*

Symbols are also very significant in places of worship. The shapes used in architecture and decoration are not random and are rarely simply utilitarian, but are a means of expressing important truths and concepts. Many of the shapes used are geometrical, and their mathematical properties have contributed to the development of their symbolic meaning. Once we give significance to our buildings and the symbols associated with them, they become part of us, or as Sir Winston Churchill put it [2]:

> On the night of May 10, 1941, with one of the last bombs of the last serious raid, our House of Commons was destroyed by the violence of the enemy, and

we have now to consider whether we should build it up again, and how, and when. *We shape our buildings, and afterwards our buildings shape us.* [My italics]

Patterns and Circles

The circle is probably the most common and universal of religious symbols. From earliest times, people depended on the light of the sun and the moon. Looking into the sky, they saw the circular shapes of the sun and the moon and organised their lives around their appearance and disappearance. Their significance led to both the sun and the moon being viewed as gods, or symbolising gods, by civilisations of many different times and places.

A circle is an ideal shape around which to develop a rich symbolism. It is one of the simplest shapes to describe, and is easily drawn with some kind of marker, a piece of stick and a length of string. Geometrically, it has no beginning and no end. It has an infinite number of lines of symmetry and an infinite degree of rotational symmetry. It can be used by religions which prohibit the representation of God, humans or animals. Indeed, because of prohibitions on such representation in many traditions within the Abrahamic religions, particularly Islam, it has come to symbolise completeness and so, by extension, God and/or eternity.

Circles are frequently found as part of other patterns also. Like the circle, pattern has long been part of religious symbolism. The concept of a god who brings order from chaos is common to many religious traditions, and pattern is a way of visually symbolising this process. Symmetric patterns also appeal to our senses, and so, for both religious and aesthetic reasons, are often used in places of worship. Their initial creation is then a form of worship, and the result becomes part of worship for those who wish it to be so.

A simple geometric pattern, which can be extended indefinitely, carries with it a sense of the infinite. Islam in particular has made significant use of geometric patterns in decoration, and many mosques are richly provided with geometric patterning. One reason may have been because they wished to 'ennoble matter' [3]; an alternative explanation is that geometric pattern expresses the unchanging nature of God's laws, thus providing a 'visual analogy to religious rules of behavior' [3].

Circles and Circular Arcs in Places of Worship

Investigating circular windows. Figure 3 shows a set of circular windows with decorative circular arcs of a type often seen in places of worship, particularly buildings from the nineteenth century in England. The circular arcs generally show a symmetrical arrangement of 3, 4, 6, 8 or 12 arcs. Interestingly, 5 lobes are also seen sometimes, as in the (less simple) example from the Greek Orthodox Cathedral in London.

Figure 3: *Circular windows (right, Orthodox Cathedral, London)*

The first task for students could be to draw such windows accurately. This requires them to understand the connection between the radius of the outer circle and that of the inner circles (2:1 for those shown here). A further question then is whether this is the only possible ratio, or whether similar windows could be drawn for other ratios. The other information students need to know in order to construct their own diagrams is the angle between the axis of one arc and the next, and how to construct such angles accurately.

If the students know the various accurate construction techniques likely to be required, the task is more interesting and mathematically richer if they are shown the diagrams and asked to work out how to construct them, than if they are simply given instructions which they then follow without further thought.

As an alternative to accurate construction, questions that might be asked are:

- If a stonemason wishes to construct a window of this type accurately using only simple construction methods (such as drawing circular arcs using a marker, a length of string plus a stake to hold the string stationary at one end, and a straight edge), which windows are possible? Which are not?
  [This requires knowing the interior angles in polygons, and which angles can be constructed accurately using only a straight edge and a pair of compasses.]
- If the outer part of the window is not glazed, but is covered up in some way, what is the difference in the amount of light which each window lets through?
  [This requires calculating the glazed area in each case, and could be extended as an algebraic problem by finding the formula for the *n*th case.]

Circles in decorative designs. Many mosques have detailed, intricate, geometrical designs, some of which involve circles or near circles (Figure 4). These are difficult for students to copy accurately, but form a rich source for discussing symmetry. Although completely made up of straight lines, the intricacy of the design shown here creates the effect of a set of circular units with a flower-shape at the centre of them.

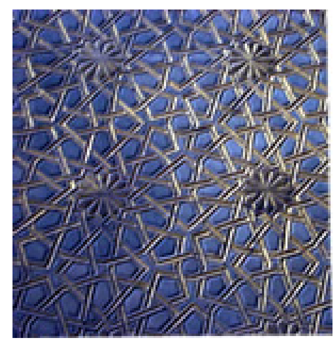

Figure 4: *Geometric Islamic design (mosque, Istanbul)*

For sheer visual beauty, rose windows (Figure 5) should be included in any study of circular designs in buildings of worship. These are often spectacularly beautiful, and are also rich sources for mathematical work. Both the windows in Figure 5 have unusual symmetries: rather than the more usual 12-fold symmetry, the left-hand window has 18-fold symmetry, and the right-hand has 10/20-fold symmetry.

Figure 5: *Two rose windows from medieval RC chapels (Koblenz, Germany, left; London Colney, UK, right)*

Copying complex designs can be difficult and unrewarding for students, but they can be used to motivate creative mathematical design of their own (as in the example in Figure 6). Solving the problems of how to construct the image they want can be a rich mathematical task and the results can be used for a stunning classroom wall display.

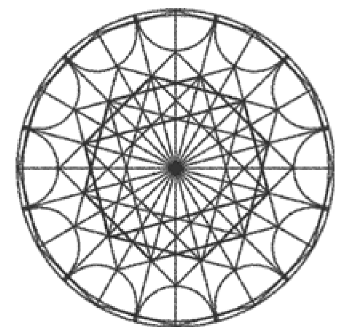

Figure 6: *Design for a rose window*

Creating arches. Many arches look as if they are formed from complex shapes, but in fact are often made up of sections of circular arcs. This is perhaps not surprising, since a circular arc is such an easy shape to produce accurately.

Figure 7 shows a simple arch, which is constructed by putting an arc of a circle on each of two sides of an equilateral triangle. Supporting sides for the arch can then be added if desired. Yet although the arch is so simple to construct, it can still motivate useful mathematical discussion. Questions which might be asked include:

- What fraction of a circle is each of the arcs? How do you know?
- How could you calculate the length of each arc?

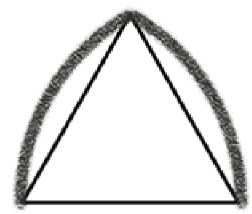

Figure 7: *Basic arch*

The ogee or ogive arch originated in India, and was imported into Western Europe via the Arab world in the fourteenth century. Examples can be seen in many religious buildings of different faiths from this period onwards. One way of constructing such an arch is demonstrated in Figure 8, but there are other methods also.

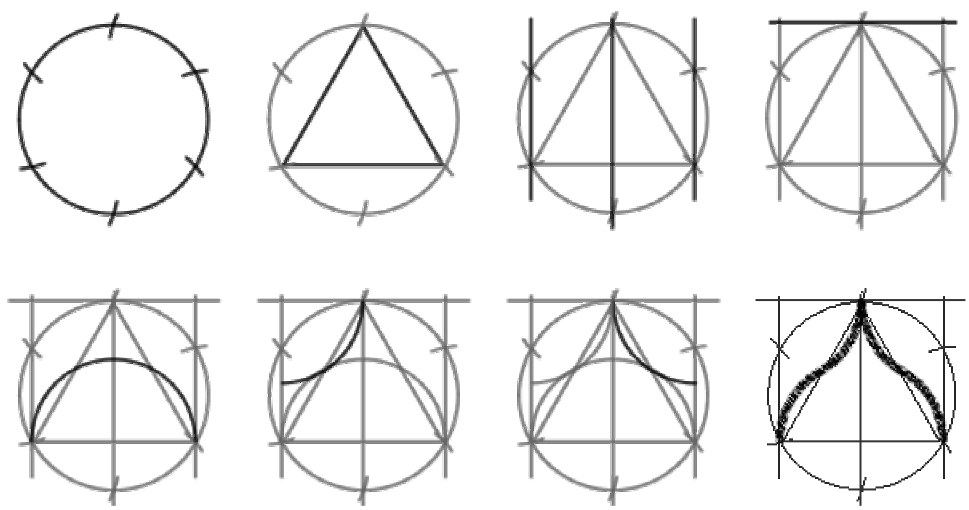

Figure 8: *Construction of an ogee arch*

Students could be given this set of drawings and asked to draw their own arch. This will require them to decide on the construction methods for each stage, and then to carry them out accurately, if they are to obtain a finished arch where the convex and concave arcs meet each other at the correct point with the two halves symmetric about the central axis. They could also be challenged to find other ways of constructing the arch accurately.

Symmetry

Another mathematical concept which is very important in places of worship is that of symmetry. Symmetry can be found everywhere – in stonework, windows, floor tilings, decorative panels, … Examples are given here of symmetry to be found in floor tilings and decorative panels.

Tiling patterns and decorative panels and friezes (examples shown in Figure 9) can be used to motivate a discussion of both reflective and rotational symmetry. A single pattern can be looked at in more than one way, depending on which motif is made the focus, as the bottom two examples of floor tilings in Figure 9 show.

Figure 9: *Floor tilings and decorative friezes (two top left floor tilings, Ely Cathedral, UK; three friezes on the right and two bottom floor tilings, Westminster RC Cathedral, London, UK)*

Students can make similar patterns using tiles cut out of coloured card, or draw the patterns on squared or isometric paper. This will allow them to explore symmetrical patterns for themselves. Choosing the appropriate type of paper, or selecting shapes which will tessellate, are both mathematical tasks in themselves, as is constructing a symmetrical design.

Figure 10: *Jewish decorative pattern (synagogue in West London)*

Synagogues often have designs which involve the Star of David set in circular designs, such as that in Figure 10. It is often difficult to take photos which show true angles and lengths, because of the angle at which the photo is taken, so the first task students might be set is to draw the design accurately, so that they can see the true proportions and relationships in the design. Further questions that could then be asked include:

- The design has two Stars of David. How are they related to each other?
  [This question could be about the ratio of the side lengths, or ratios of area, or axes of symmetry.]
- There is a further shape in this design, apart from the two Stars of David, and the outer circle. What shape is this, and how does it relate to the other shapes? How many axes of symmetry does it have, and where are they?

The Workshop

The workshop associated with this paper will provide participants with the opportunity to explore several of the mathematical tasks described here, and also to discuss the ideas about symbolism with which I start this workshop when I do it with children.

References

[1] Fauvel, John, and Jeremy Gray (eds) (1987) *The History of Mathematics: A Reader,* Milton Keynes: Open University Press: 255f.

[2] Speech by Sir Winston Church in the House of Lords (UK), on 24 October 1943, which can be found at http://www.winstonchurchill.org/i4a/pages/index.cfm?pageid=388. (Accessed 29 January 2008).

[3] Article on *Islamic Patterns and Geometry*, http://www.salaam.co.uk/themeofthemonth/march02_index.php?l=3. (Accessed 30 January 2008).