Entwined Circular Rings

Rinus Roelofs
Sculptor
Lansinkweg 28
7553AL Hengelo
the Netherlands
E-mail: rinus@rinusroelofs.nl
www.rinusroelofs.nl

Abstract

Entwined circular rings ("In elkaar gevlochten cirkelvormige ringen") was the title that M.C. Escher gave to a little research project he started around 1945. Although the start of this project was very well set up he didn't come further than the first page on which he shows us how you can make patterns of entwined circular rings based upon a square grid. Because I often make use of these kinds of patterns in my own work I decided to pick up this research project and try to find out how Escher would have continued this project.

1. Introduction

1.1. Escher's drawings. In 1979 I had the opportunity to go through Escher's notebooks in the Haags Gemeentemuseum in the Hague, Holland. In one of the notebooks we can find several drawings of patterns of entwined circular rings. And one of the pages is set up as a start of research project, probably with the goal to find out the different possibilities of these kinds of patterns. The first page represents a list of four such patterns based on a square grid. The title on the page with these drawings (see Figure 1) says: "A: four-fold axes at each center point.". On other pages in this notebook we can find some more drawings of patterns with entwined circular rings based upon a triangular grid (Figure 2). All these drawings are published in Doris Schattschneider's book M.C. Escher: Visions of Symmetry (ref. [1]).

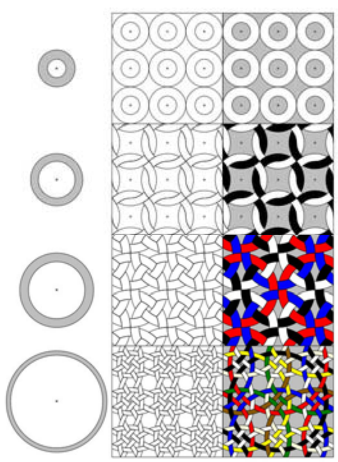

Figure 1: *M.C. Escher - Entwined circular rings.*

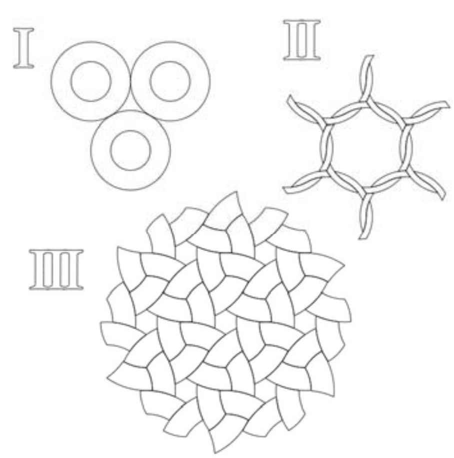

Figure 2: *Entwined circular rings.*

1.2. Escher's intention. Based on the fact that Escher wrote "A: .." in the title we may assume that he intended to go on with another display to be called "B: ..". The next one would certainly be based upon a triangular grid because the first sketches for the display, numbered 1, 2 and 3 (Figure 2) are in the same notebook.
But besides that, Escher might have had a special reason for studying this group of patterns. For between the pages of the notebook there were a few sheets with sketches which were made as a preparation for a stamp that Escher designed for the Wereldpostvereniging, Holland (1949). On the stamp we can see a sphere decorated with entwined circular rings. So I think that besides the possibilities of using patterns with entwined circular rings in the plane, Escher was also interested in the more complex problem: which patterns can be made on three dimensional objects like the sphere?

Figure 3: *The decorated sphere on the stamp.*
M.C. Escher's "10 cents stamp" - copyright 2007 The M.C. Escher Company B.V. - Baarn - Holland.

2. From the plane to the sphere

2.1. Cube. Taking a closer look at the decoration of the sphere upon the stamp we can conclude that Escher might have used the cube as a step in between. Starting with a flat pattern of entwined circular rings (Figure 4) we can cut out six squares as a plan of a cube and fold them together to the cube, like in Figure 5. While looking at Escher's sketches we can assume that this is the way he did it.

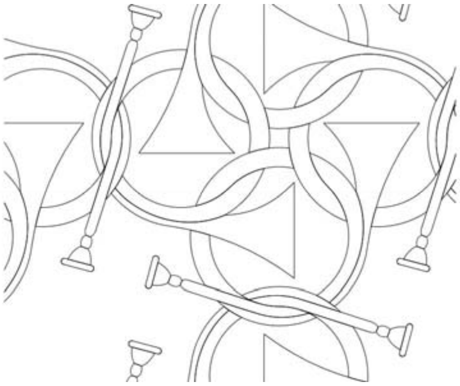

Figure 4: *Escher's sketch*

. **Figure 5**: *A cube decorated with Escher's pattern*

So if going from 2D to 3D was an underlying reason, it is even more obvious that the next step in his 'Entwined Circular Ring'-project was to make a display of entwined circular ring patterns based on a triangular grid. Because from a triangular grid we can not only go to tetrahedron, but also to an octahedron, icosahedron or in general to deltahedra, including the infinite structures.

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2.2. Cube and octahedron. In space there is an easy way to go from squares to triangles: the dual of the cube is the octahedron. Escher was very familiar with the relationship between the cube and the octahedron. He made several drawings of this combination and one of them is on the page in the notebook with the three circular patterns based on the triangular grid. And in the sketches for the stamp we can find some studies of using the pattern based on a triangular grid in order to be able to use the octahedron as the step in between to come to the decorated sphere (Figure 7).

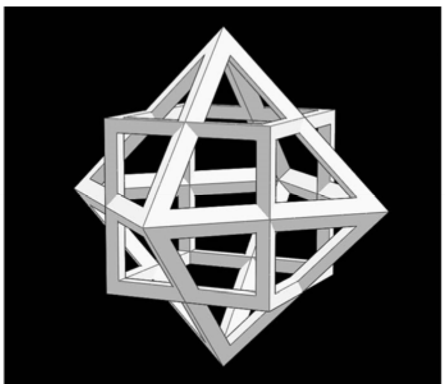

Figure 6: *Cube and Octahedron.*

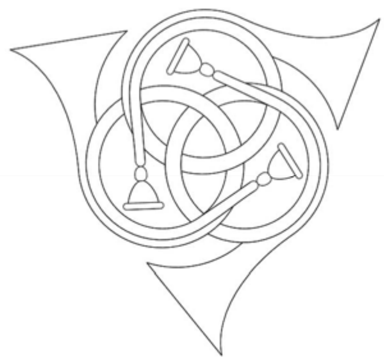

Figure 7: *Escher's triangular pattern.*

3. Folding Deltahedra (intermezzo)

3.1. Strip Plan of the Octahedron. To make an octahedron we can make a plan of eight connected triangles. We can see this as a part of triangular pattern. One way of doing this is the strip of triangles shown in Figure 8.

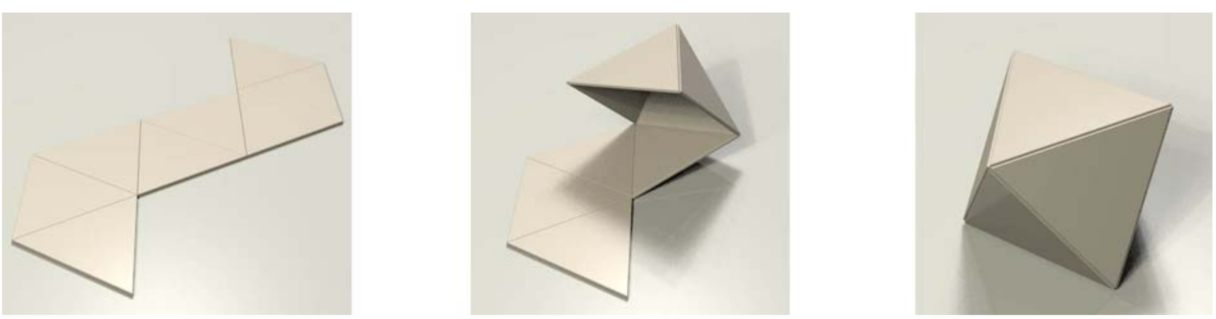

Figure 8: *Plan of an octahedron.*

Now we can extend the strip by adding pairs of triangles. The next member of this family of shapes we can create in this way is the pentagonal dipyramid. And with long strips we can create cylindrical shapes like the tetrahelix. In general most of the 3d constructions which are built with triangular faces can be "unrolled" as a strip of triangles. In Figure 9 you can see the "strip plan" of the icosahedron. There is close relationship between this "strip plan" and the Hamilton path on the dodecahedron, the dual of the icosahedron. Another nice example is the strip plan of the Kepler Star shown in Figure 10. So the group of shapes that can be formed with equilateral triangles is not only big but there are also interesting connections between the 3d shapes and the 2d plans. Reason enough for Escher to continue his research with the patterns based on the triangular grid.

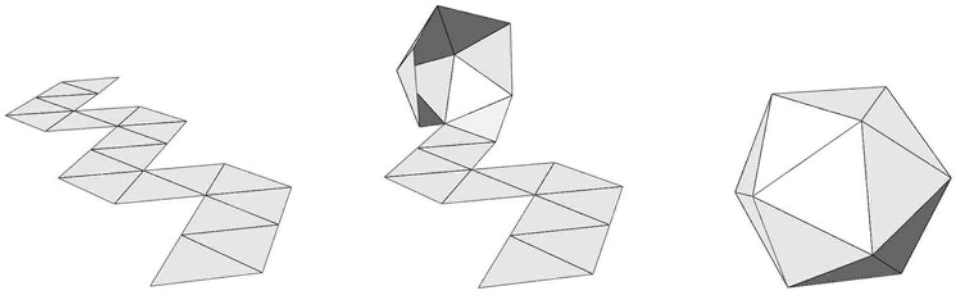

Figure 9: *Icosahedron.*

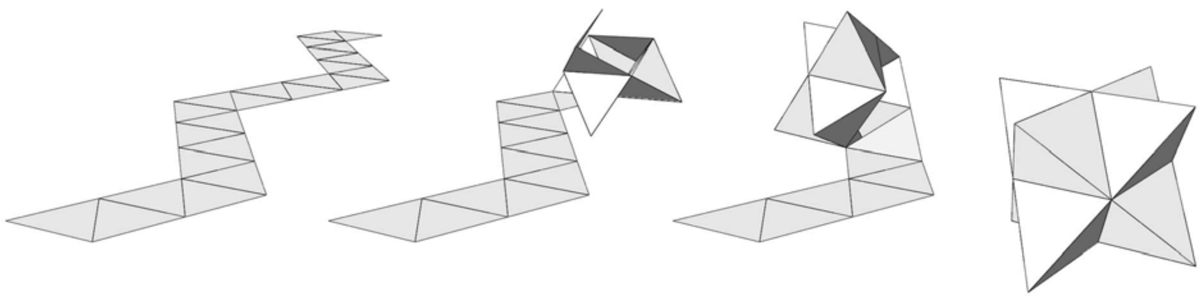

Figure 10: *Kepler Star.*

3.2. Display B. Setting up display B in the way Escher made display A will result in the picture shown in Figure 11. Steps 1, 2 and 3 can be found in Escher's notebook. In 1950 Escher made the drawing shown in Figure 12, which can be seen as the possible fifth step for display B. It was used in one of his designs for banknotes.

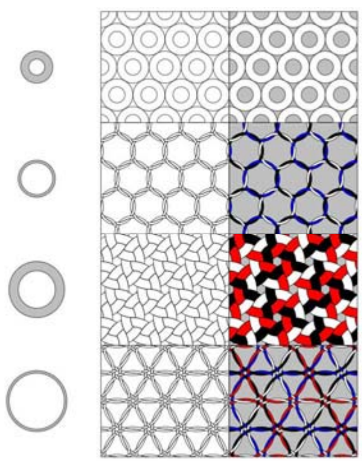

Figure 11: *Display B*

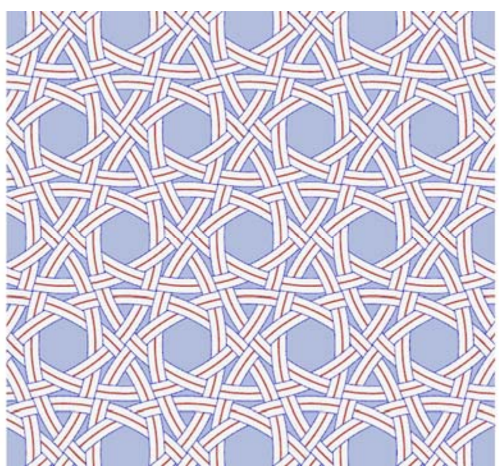

Figure 12: *Fifth pattern for display B*

4. The pattern on the stamp

4.1. Cubic. Now let's take a closer look at the pattern on the sphere on the stamp. When we try to "unfold" the sphere in a cubic way to come back to the plane pattern we see that it is not really one of the patterns of display A that is used for this design. And when we study one of Escher's preliminary sketches we can see that he has used a pattern with squares and octagons, the Archimedean tiling (4.8.8), instead of just the square tiling.

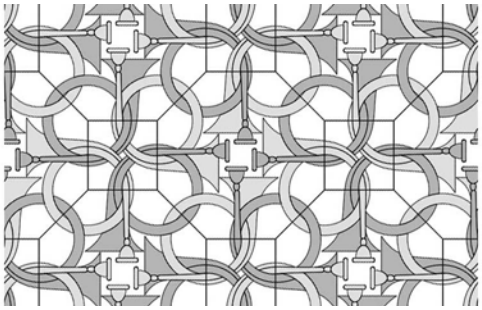

Figure 13: *Escher's horn pattern.*

So we may conclude that display B was not intended to be the last page of his research. In fact all the other Archimedean tilings can be used as basic grids for entwined circular patterns. In Figures 14 to 16 you can see display C (based on the Archimedean tiling (4.8.8)), display D (based on the Archimedean tiling (3.6.3.6)) and display E (based on the Archimedean tiling (3.3.4.3.4)). The pattern on the stamp can now be recognized as step 2 in display C.

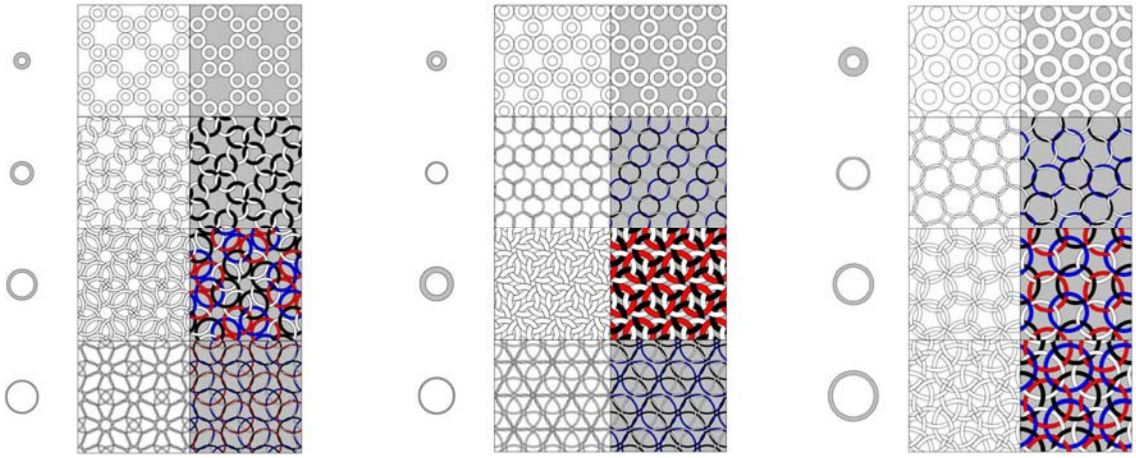

Figure 14, 15, 16: *Displays C, D, E.*

5. Interwoven layers (intermezzo 2)

5.1. Layers with circular holes. On a drawing entitled Japansche Vlakversiering made by H.J. de Vries in 1891 (ref. [2]) we can recognize one of the entwined circular patterns of display B. The basic set up of the entwined circular rings can be seen in the upper right corner (Figure 17). However in the other part of the drawing this basic pattern is filled in, in such a way that the final drawing can be seen as three interwoven layers with circular holes.

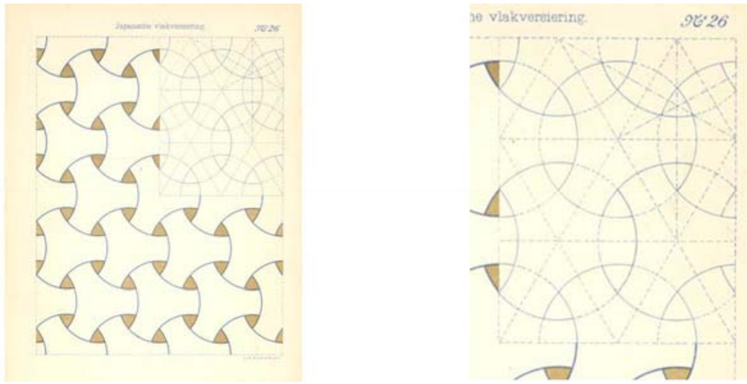

Figure 17: *Japanese ornamental design.*

5.2. Other examples. We can apply this idea also on a few other circular ring patterns. For instance the second pattern of display A can be transformed into two interwoven layers (Figure 18). And the third drawing of display A can be transformed into four interwoven layers (Figure 19). The number of layers corresponds to the minimum number of colors mentioned by Escher.

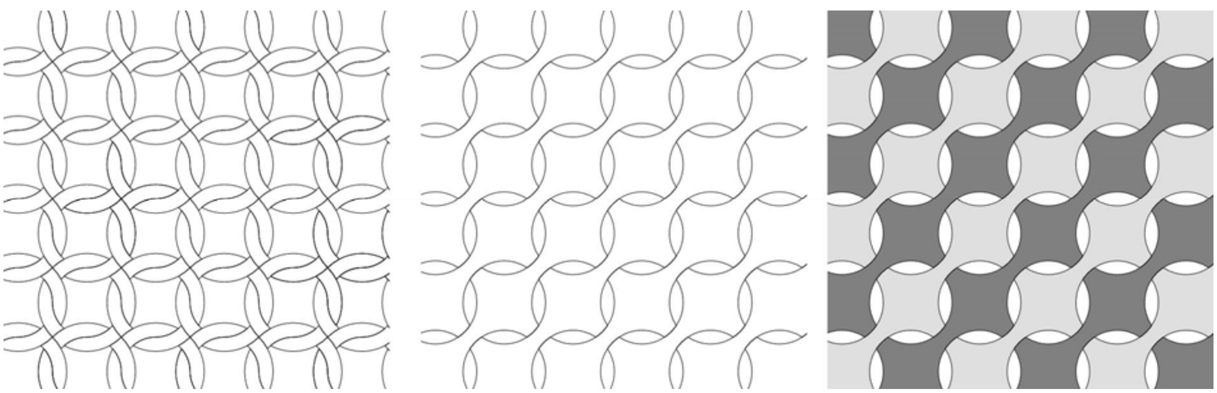

Figure 18: *Two interwoven layers*

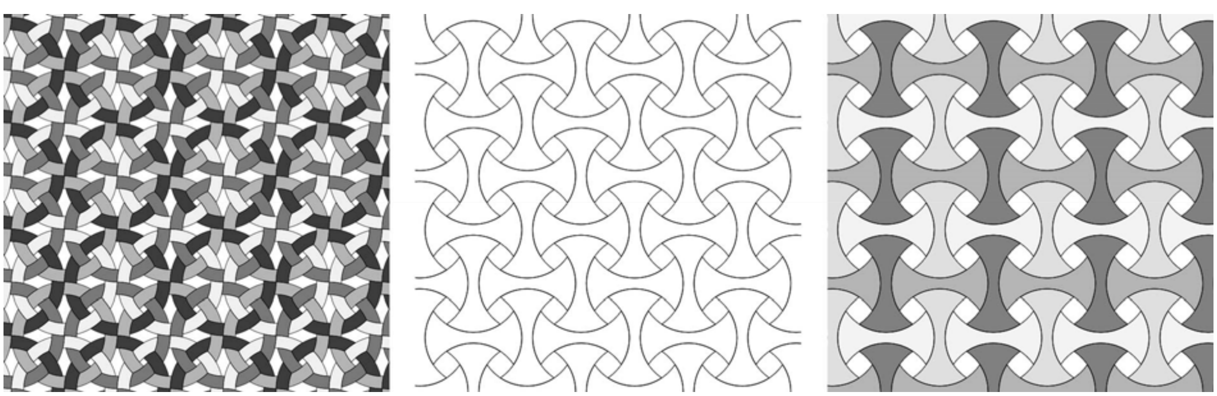

Figure 19: *Four interwoven layers .*

5.3. Holes, tiles and rings. The pattern of the four interwoven layers can now be transformed into a well-known tiling. Making the holes in the layers as small as possible brings us to the pattern of Figure 20c. And now we can interpret the convex arcs of each tile as the upper part of a circular ring (Figure 21). So the tiling can be seen as a drawing of entwined circular rings, not laying flat in a plane as in Escher's display but as a three-dimensional structure.

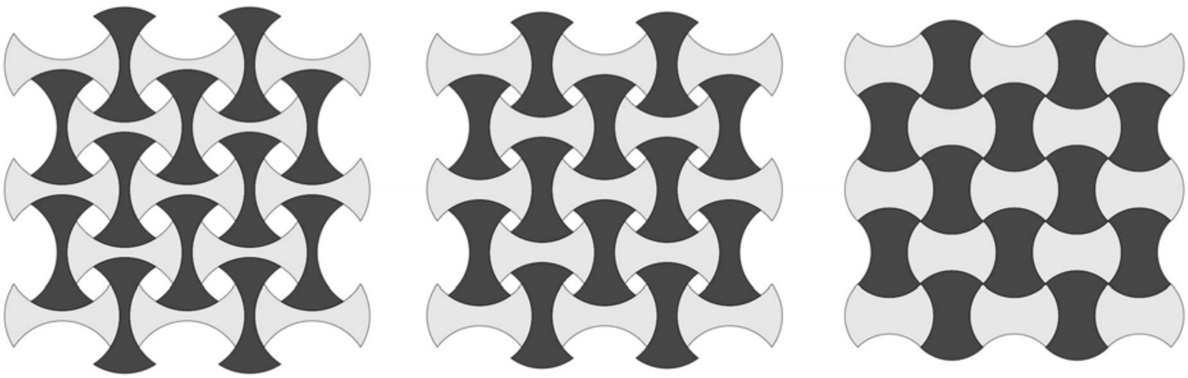

Figure 20 a,b,c: *Holes, tiles.*

Figure 21: *Three Dimensional Ring Structure*

6. From Ring Patterns to Ring Structures

6.1. Entwined Ring Structures. Another area of entwined circular rings, worth investigating, has arisen. The steps taken in Figure 20, 21 can be applied on other similar tiling patterns, as can be seen in Figures 22 and 23.

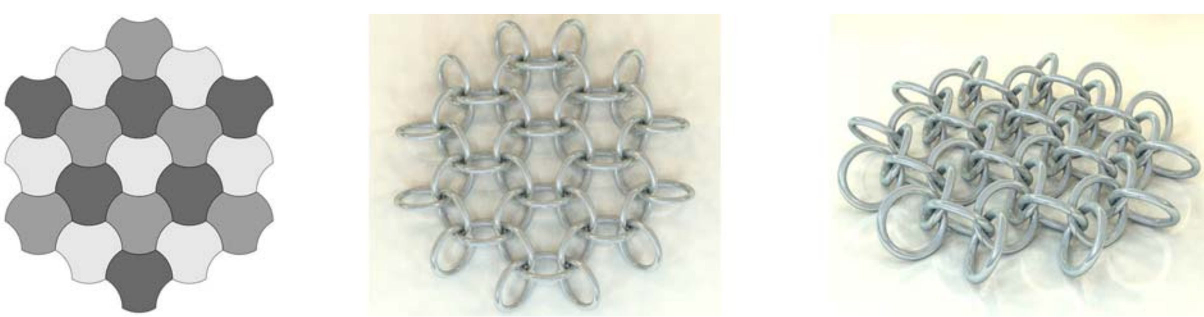

Figure 22: *Threefold symmetry.*

Figure 23: *Sixfold symmetry.*

6.2. Cylindrical sets. The entwined circular ring structure can be seen as a concatenation of cylindrical sets of six rings (Figure 24). In a way these sets are the basic elements of the total structure. This brings us to another way of composing new entwined circular ring structures. The entwined circular ring structure of Figure 25 is made out of cylindrical sets of 8 rings.

Figure 24: *Cylindrical sets.*

Figure 25: *Entwined circular ring structure.*

6.3. Coloring. In these ring structures we can also use colors to distinguish subsets. Escher mainly used colors in a way that entwined rings would never have the same color. In this particular group of entwined ring structures I will follow another rule. In these structures strings of rings can be emphasized by coloring. And when we do this we see that the structure in a matter of fact is an entwined chain structure .

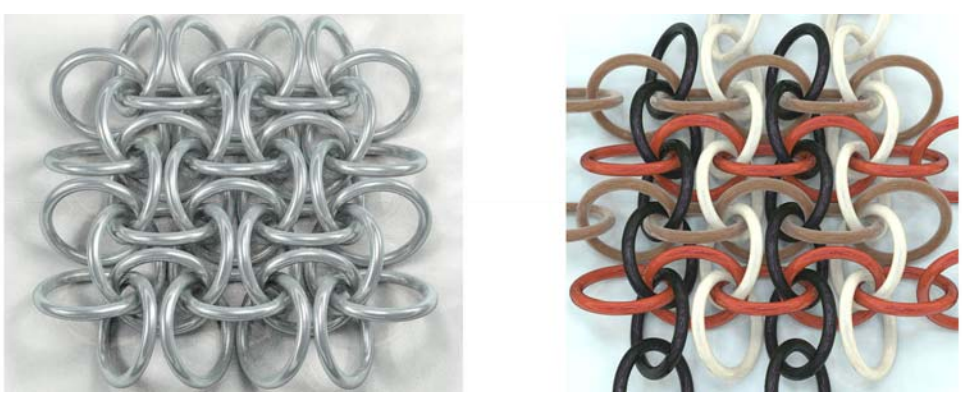

Figure 26: *Chains - 4 colors.*

Figure 27: *Chains - 3 colors.*

7. Infinite Polyhedra

7.1. Cubic ring structure. There is another way of creating three dimensional entwined circular ring structures. And in some sense this way is more connected to Escher's search. We start with a set of rings in a cubical space frame structure as in Figure 28 in which you can recognize Escher's first drawing of display A. Now we add a second set of rings to connect all the rings. This second set has the same structure as the first one but can also be seen as the dual set.

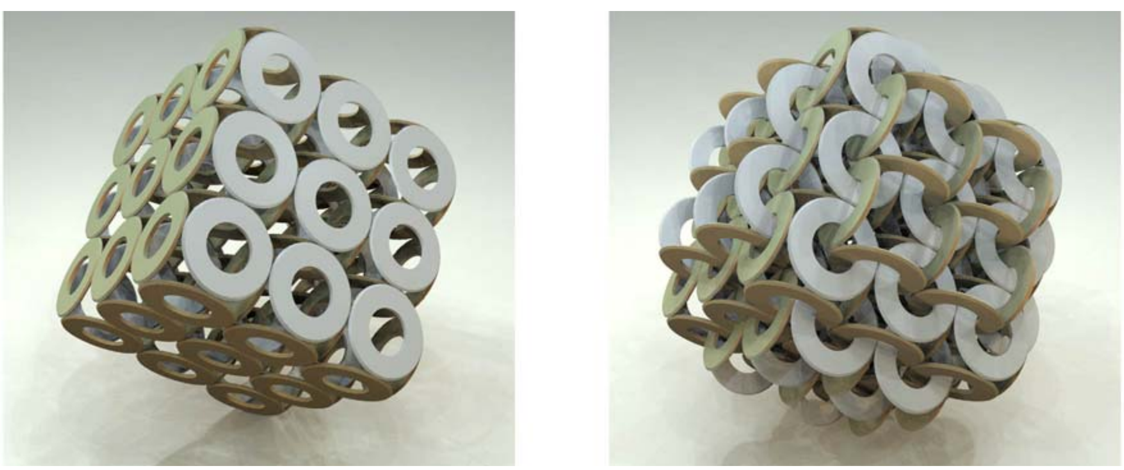

Figure 28: *Cubical ring structure.*

7.2. Tetrahedral ring structure. Starting with the first entwined ring pattern of display B (Figure 11) we get the tetrahedral entwined ring structure of Figure 29.

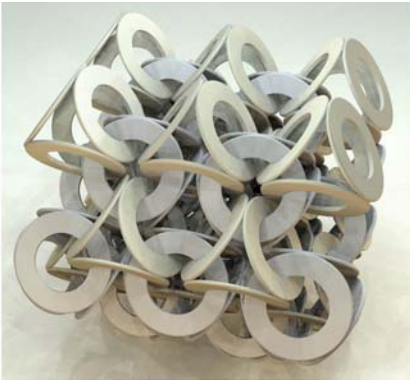

Figure 29: *Tetrahedral.*

7.3. Triangles and squares. In the structure of Figure 29 we can distinguish subsets of 4 rings each. The rings of such a subset are lying on the triangular faces of a tetrahedron. It might be a surprise that, seen from a certain angle, the structure looks exactly the same as the pattern of Figure 21, which is based on a square grid (Figure 31). And something similar occurs when we take the cubic ring structure of Figure 28: seen under the right angle we will see threefold symmetry.

Figure 30: *Tetrahedral - threefold symmetry.*

Figure 31: *Tetrahedral - twofold symmetry.*

8. Conclusion

8.1. Escher's research. Somehow it's a pity that we do not know how Escher would have continued his research. But the first display of entwined circular rings already appears to be very inspiring and opens up many directions for further research.

References

[1] Doris Schattschneider, *M.C. Escher: Visions of Symmetry,* Abrams 2004
[2] Herman J. de Vries, *Meetkunstig Vlakornament,* De gebroeders van Cleef 1891
[3] M.C. Escher, *Notebooks (unpublished),* Haags Gemeentemuseum