Spiral Developable Sculptures of Ilhan Koman

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Abstract

In this paper, we present spiral developable sculptural forms invented by Ilhan Koman approximately 25 years ago. We identify the mathematical procedure behind Koman's spiral developable forms and show that using this procedure, a variety of spiral developable forms can be constructed.

1 Introduction and Motivation

This paper presents the spiral developable forms invented by sculptor Ilhan Koman during the 1970's. Figure 1 shows Ilhan Koman in front of one of his spiral developable sculptures. For detailed photographs see Figures 2 and 4C.

Figure 1: *A Composite of Ilhan Koman in front of one of his Spiral developable sculptures. (The photograph of Ilhan Koman by Tayfun Tunçelli, 1980's).*

Ilhan Koman has not recorded his method to construct these spiral sculptures. By examining his sculptures, we were able to reconstruct his method. In this paper, we demonstrate Ilhan Koman's method that is reconstructed by us, hereinafter we simple refer as *our method*, but, it is, in fact, Koman's method.

To demonstrate that our method is viable, we show that using our method it is possible to construct circular developable sculptures from one piece of rectangular paper. Note that using the same method we can construct a spiral shape by changing the curvature. As shown in Figure 3B, a circular developable sculpture can be constructed using our method from one piece of rectangular paper shown in Figure 3B.

Figure 2: *A detailed photograph of one of the spiral developable sculptures of Ilhan Koman.*

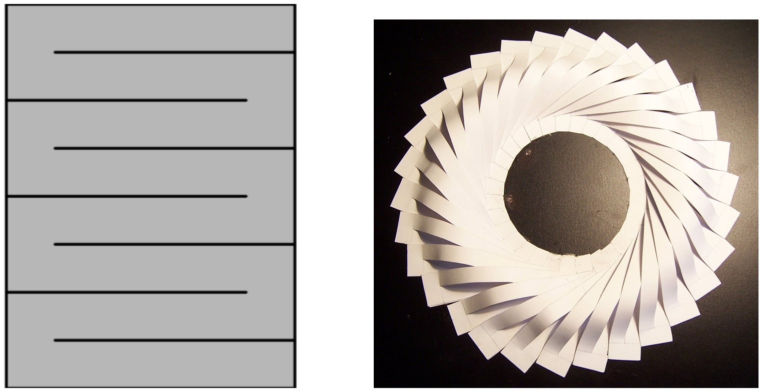
Our method can also provide a spiral by using a quadrilateral piece of paper, shown in Figure 4. Figure 4C shows one of Ilhan Koman's spiral sculptures. Our method allows us to create a comparable form as seen in this comparison. However, Koman's metal sculpture looks much more elegant than ours as seen in this comparison.

2 Overview

Ilhan Koman is one of the innovative sculptors of the 20th century who frequently used mathematical concepts in creating his sculptures and discovered a wide variety of sculptural forms that can be of interest for the art+math community. Koman was born in 1921, Edirne, Turkey, studied at the Art Academy in Istanbul, opened his first workshop and exhibition in Paris, 1948, moved to Sweden in 1958, where he taught at the Konstfack School of Applied Art in Stockholm until his death in 1986. His works cover a wide spectrum of styles and materials, including 12 public monuments in Sweden and 4 in Turkey. He is represented in several museums including Moderna Museet, Stockholm, Museum of Modern Art, New York, Musée d'Art Moderne de la Ville de Paris. During the last twenty years of his life, he worked on inventing diverging geometrical forms which he developed as prototypes to be realized in large-scale projects. Among the geometrical shapes he developed, tetraflex was a flexible polyhedron which he registered at the Swedish Patent Office in 1971. At the time he had created and proposed these structures as modules for architecture, constructions in space and aviation fuel tanks and published his method in Leonardo [8] For a detailed Ilhan Koman biography see [8, 9, 1].

Ilhan Koman also invented a variety of developable forms, but he did not published his methods. Some of his developable surfaces were the subject of our earlier paper [1]. Developable surfaces are defined as the surfaces on which the Gaussian curvature is 0 everywhere [18]. The developable surfaces are useful since they can be made out of sheets metal or paper by rolling a flat sheet of material without stretching it [14]. Most large-scale objects such as airplanes or ships are constructed using un-stretched sheet metals, since sheet metals are easy to model and they have good stability and vibration properties.

Sheet metal is not only excellent for stability, fluid dynamics and vibration, but also one can construct aesthetic buildings and sculptures using sheet metal or paper. Developable surfaces are frequently used by



(A) (B)

Figure 3: *Constructing a circular developable sculpture from one piece of rectangular paper shown in (A) using our method.*

contemporary architects, allowing them to design new forms. However, the design and construction of large-scale shapes with developable surfaces requires extensive architectural and civil engineering expertise. Only a few architectural firms such as Gehry Associates can take advantage of the current graphics and modeling technology to construct such revolutionary new forms [11].

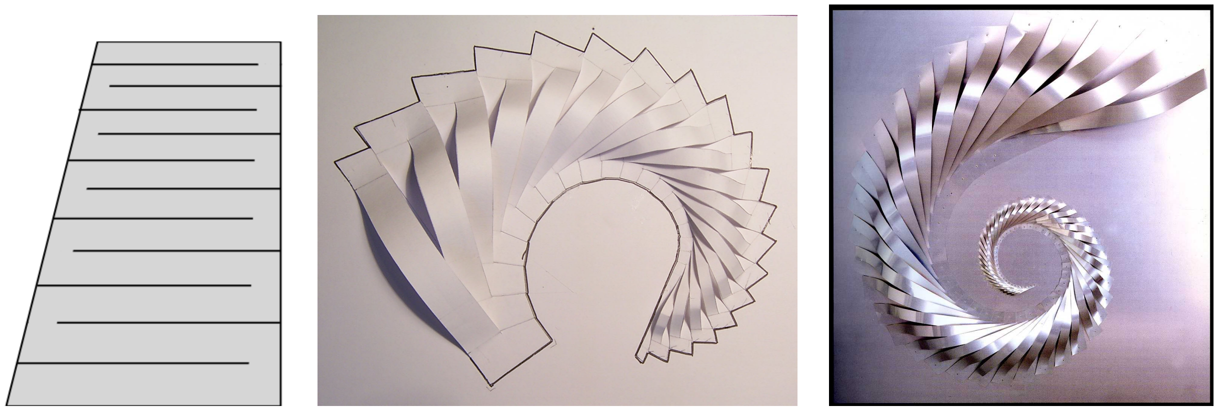
Developable surfaces are particularly interesting for sculptural design. It is possible to find new forms by physically constructing developable surfaces. Recently, very interesting developable sculptures, called D-forms, were invented by the London designer Tony Wills and introduced by Sharp, Pottman and Wallner [15, 12]. D-forms are created by joining the edges of a pair of sheet metal or paper with the same perimeter [15, 12]. Pottman and Wallner introduced two open questions involving D-forms [12, 6]. Sharp introduced anti-D-forms that are created by joining holes [16]. Ron Evans invented another related developable form called Plexagons [5]. Paul Bourke has recently constructed computer generated D-forms and plexons [3, 5] using Evolver developed by Ken Brakke [2].

Ilhan Koman's spiral developable sculptures are the result of a procedure that transforms a single flat developable surface to spiral shapes in 3D. Spirals are one of the most common shapes in nature, mathematics, and art [23]. Behind the beauty of many natural objects such as snail shells, seashells and rams' horns lies their spiral shapes [22]. The spiral forms exist in almost all cultures as artistic and mystical symbols. This widespread usage of the spiral form may imply that humans innately find it aesthetically pleasing and interesting.

Spirals are also popular in mathematical art. Spirals frequently appears Fractal art [21]. Spiral forms can be seen Charles Perry's mathematically inspired sculptures [13]. Daniel Erdely's Spidrons also shows spiral structures in 3D [19, 20]. Spirals are among the most studied curves since ancient Greek times. Although spirals are usually represented by parametric equations, there are a wide variety of methods that can be used to construct and represent spirals. There exist a wide variety of spirals such as the Archimedean spiral, the Fermat's spiral, the Logarithmic spiral and the Fibonacci spiral [23].

3 Our Method to Construct Ilhan Koman's Developable Spiral Forms

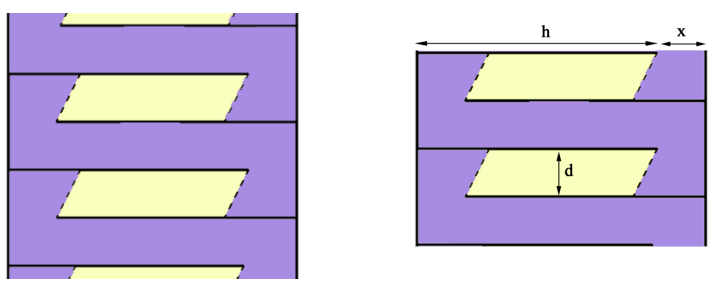
In this paper, we present our reconstruction of Ilhan Koman's method to build developable spiral sculptures. We have developed first a method to build circular sculptures. Spiral sculptures can be obtained by changing the curvature of circular sculptures. The way we change the curvature can directly affect the shape of the spirals.



(A) (B) (C)

Figure 4: *Constructing a spiral developable sculpture (B) from one piece of a quadrilateral paper shown in (A) using our method. We traced the boundary with pen to show the the structure better. (C) shows a photograph of a spiral sculpture of Ilhan Koman.*

Figure 5 shows the the basic procedure to create a circular sculpture. Figure 5A differentiates two different parts of the rectangular paper strip that is used to construct circular sculpture. Dark-blue pieces always stay planar and light-yellow pieces eventually become curved and create the 3D structure of the sculpture.



(A) (B)

Figure 5: *Square paper that is used to create circular sculpture. We color coded paper to differentiate the two types of pieces. Blue pieces stays planar and yellow pieces becomes curved. The values of d, h and l can be chosen independently.*

Our method starts by gluing or stapling the dark-blue piece in the boundary to a planar surface. Once it is flattened, the next neighboring dark-blue piece is flattened by rotating with an angle a and translating such that the second dark-blue piece enters under the now-curved light-yellow piece that is between two dark-blue pieces as shown in Figure 6A. Note that this procedure just makes the yellow piece shorter and forces it to be curved. If we continue this procedure using the same angle a, we start to see a circular arc as shown in Figure 6C. Note that if $a = 2\pi/n$ where n is an integer, the circular arc is closed as an n sided regular polygon and n becomes the number of blue pieces.

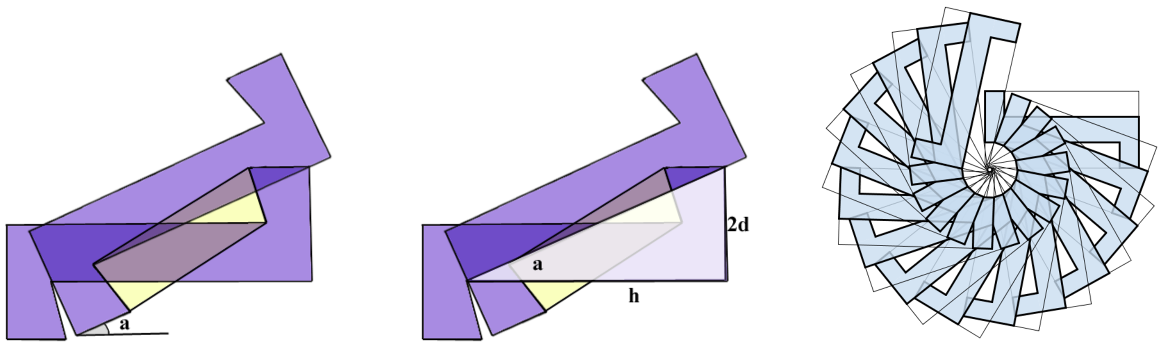
The value of a can be uniquely determined by the values of h and d as

$$a = \arctan(2d/h)$$

(See Figure 6B). In practice, $2d$ is chosen much smaller than h as in the example shown in Figure 3B. If $2d$ is much smaller than h, then the value of a becomes very small. For very small values of a, the regular polygon closely approximates a circle with a radius r where

$$r \approx \frac{nd}{2\pi} = \frac{d}{a} = \frac{d}{2d/h} = \frac{h}{2}$$

since $\tan a \approx a = 2d/h$. Note that $r \approx h/2$ is also visible in Figure 3B. For a circle, curvature is constant and can be computed $1/r$. Therefore, we can change the curvature by changing either d or h or both.



(A) (B) (C)

Figure 6: *Basic procedure. The triangle drawn in (B) shows the relationship between angle a, d and h. The circular structure in (C) is drawn by using exaggerated values:* $d = 4$ *and* $h = 21$. *Therefore, the angle a is not small, which is approximately* 20^0.

One possible approach is to change only the d value and keep a constant. The value of a can be kept constant by choosing h/d constant. We have created our spiral sculptures this way by changing only the d value and keeping a constant as seen in Figure 4B. In this example, since the d value is changed linearly, our spiral is not a logarithmic spiral, which is more common in nature and considered more aesthetic [22]. We think that Koman's sculpture is more like a logarithmic spiral and therefore it looks more aesthetic.

Although, it is possible to create logarithmic spirals by changing only the d value, for logarithmic spirals d values quickly becomes smaller and it is very hard to cut such a thin slits by hand with scissor. We have not developed a software for this purpose, but, it is possible to automatically create such a drawing and cut the paper or metal using a laser cutter. On the other hand, since laser cutters were not available in 1980, this was not an option for Ilhan Koman. Observing his sculptures it is clear that he did not change d value drastically, instead he made h exponentially smaller. We are sure that he used a simple procedure to change the values of h and d. However, we do not know how he changed the values and what kind of procedure he used to change them.

4 Conclusion and Future Work

In this paper, we presented Ilhan Koman's spiral developable sculptures. We have identified the mathematical ideas behind these spiral forms. We have also introduced a method that can allow the construction of variety of a spiral forms by changing only two parameters.

Using the same type of cuts, Ilhan Koman also created more complicated developable surfaces. Our next goal is to understand how he created those more complicated sculptures and disseminate our findings.

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