Using Art To Teach Maths * Using Maths To Create Art

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Abstract

In recent years, there has been a move away from approaching topics such as number and algebra, shape and space, probability, and so on, as distinct units of work. Alternative approaches focus on the connections between different areas of maths, using a variety of techniques to explore these. The activities covered in this workshop can be used to give students experience across all these mathematical areas while also linking them to art, history, and other curriculum areas.

Delegates will have hands-on experience of up to six different activities, which they will be able to explore at their own pace during the workshop. There will also be opportunities to discuss implications for classroom practice, and ways in which the ideas could be used and extended for students of different abilities and with varying interests.

Introduction

This workshop (see [5] and [9] for further ideas) has been designed to promote the teaching of mathematics in ways that are visually stimulating for students of a wide range of abilities from the ages of around 10 to 18. Not all students who are 'good at maths' like maths, and one aim of this approach is designed to stimulate able students who might otherwise find maths uninteresting. We also hope that it will encourage students who find maths difficult to discover that it has something to offer them also. Further objectives are to:

- increase students' self-esteem in mathematics lessons
- challenge preconceptions of what maths is
- explore the beauty of maths
- make connections between different areas of maths in a meaningful way
- make cross-curricular links
- develop students' independent learning skills
- increase student motivation and enjoyment

Random Art

What makes an image art? Can an image which is created randomly be called art? Can the creation of an image ever be entirely random? The activities considered here give students an insight into the concept of randomness, and also prompt them to consider the question, ‘But is it art?’ Students can then be asked to develop these ideas further independently: what other ways can they develop for generating ‘art’ randomly?

Grids or coordinate axes may be set up in a number of different ways using squared or isometric paper. Dice or spinners are then used to determine position and the colour to use.

If desired, an isometric grid could first be constructed from scratch by hand using a pair of compasses and a straight edge; alternatively, isometric paper could be provided for students. They then take a die numbered from 1 to 6, choosing each number to correspond to one of 6 colours. (How is it possible to decide on these colours randomly?) Add colour to the triangles systematically, working from left to right across each line, from top left to bottom right, deciding the colour that each triangle is to take by rolling the die (Figure 1, left-hand image).

Figure 1: *random art on an isometric or coordinate grid*

Next take a Cartesian grid whose *x*- and *y*-axes are numbered from 1 to 6. Roll two dice, twice, to get a start and a finish coordinate for a straight line. Then roll a single dice to determine the colour of the line. Draw the line as decided by the roll of the dice and repeat the procedure a predetermined number of times. The drawing above (Figure 1, right-hand image) was produced in this way, then scanned and the image enhanced using computer software. It could of course be argued that such manipulation means that the process is no longer random...

Digit-Sum Spirals

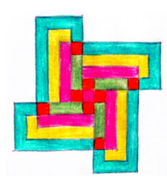

Figure 2: *digit-sum spiral*

This activity is an exploration of the images that result when digit-sums are found for the multiples of a given number, and then a “spiral” is drawn on a squared grid with the lengths of the lines determined by the numbers obtained as the digit-sums (Figure 2).

Firstly, what is a digit-sum? Using 149 as an example: 1 + 4 + 9 = 14 and 1 + 4 = 5, so the digit sum of 149 is 5.

To create a digit-sum spiral for multiples of 7:

- Find the digit-sums for the first 20 or so multiples of 7.
- Notice that there is a pattern in the sequence of digit sums.
- Start to draw the spiral (0.25 cm squared paper is recommended).
- Starting point is at the left hand side of the paper, half way up.
- Draw the first line: UP 7 squares.
- Turn clockwise 90 degrees, then RIGHT 5 squares …
- Turn clockwise 90 degrees, then DOWN 3 squares ...
- Turn clockwise 90 degrees, then LEFT 1 square ...
- Continue turning and using the digit sums until the starting point is reached: the 'spiral' is complete (Figure 3).

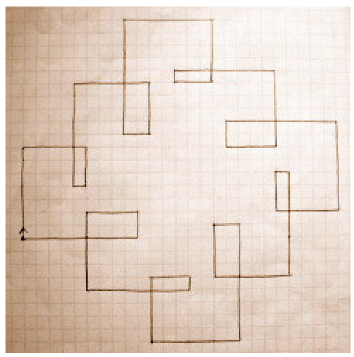

Figure 3: *digit-sum spiral*

Students can now explore this activity by considering multiples of other numbers. There is a meaningful context for collaborative work here: if each member of a group of students takes a different starting number then the group will quickly have, between them, a set of spirals about which they can ask questions such as:

- Do any numbers result in similar digit-sum spirals? Which ones?
- What about multiples of 10? 11? 12? Is it possible to make any generalised statements?
- Are there any different types of spiral that could be drawn?
- What happens if you use the digit-sums of square numbers? Cube numbers? Prime numbers?

A simple task such as this can raise sophisticated questions such as 'How do you know the digit-sums for 7 will always follow that same pattern?' leading students naturally into the ideas of mathematical proof.

Islamic Art

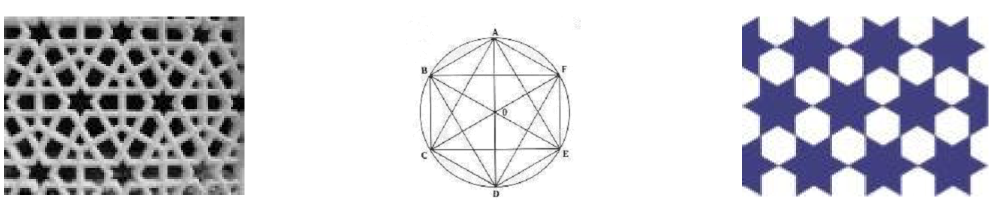

Figure 4: *examples of Islamic art*

Many examples of Islamic art [1], [2] are based on the arrangement of a repeat unit to produce an overall design (Figure 4), the repeat unit being determined by constructions based on a circle. The circle is a fundamental starting point in Islamic art symbolising wholeness and unity. The circle also symbolises eternity: it has no start point and no end. A particular characteristic of Islamic geometrical patterns is the prominence of star and rosette shapes, achieved by dividing the circle into equal parts - 5, 6, 8, 10 or 12 points being the most common divisions.

The only equipment that will be used during the workshop is a straightedge, a pair of compasses and a sharp pencil. There is no need for a ruler as a measuring device – the pair of compasses measure equal distances, and that is the only measurement that is needed.

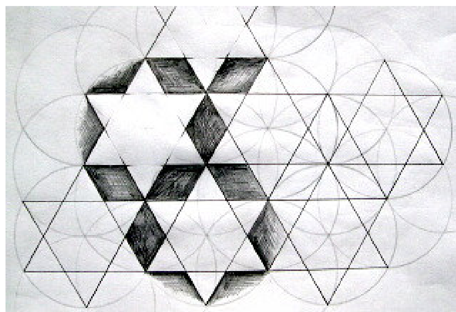

Figure 5: *construction of a repeating Islamic pattern*

By first using compasses to construct a regular hexagon, it is straightforward to create an infinitely repeating pattern of interlocking circles upon which to build the repeating pattern of 6-point stars and rhombuses (Figure 5).

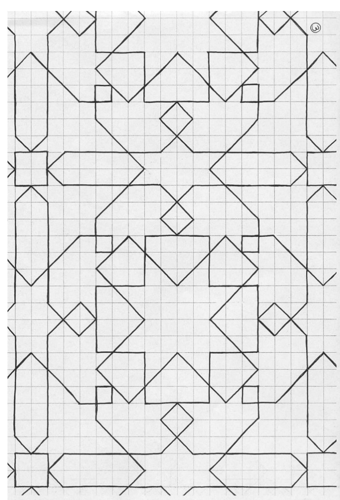

Figure 6: *using a grid to create a repeating pattern*

A starting point for an investigation of area and perimeter could be to take a grid like Figure 6 and ask students to identify, by outlining, a number of different shapes on the grid. Then ask them to find the area and perimeter of each of their chosen shapes. They can then consider which shapes have area greater than perimeter and vice versa. Do any shapes have the same area as perimeter?

When do regular polygons have the same area and perimeter? The ease with which students will be able to answer this will depend on the polygon they choose to consider.

Fractals

Fractal figures are a rich source of mathematical investigation, [7], such as the well-known Koch (snowflake) curve, the Sierpinski gasket, and so on. However students can be encouraged to investigate their own fractals which can be produced by applying simple rules repeatedly. They can also be introduced to the widespread occurrence of fractals in nature, and think about why nature is fractal.

Suppose we start with a square, then use this algorithm:

1. Mark the central third of each straight line edge.
2. Add (or remove) a square of side equal to that third.

This is the algorithm for the Koch curve adapted to make a square-based fractal, rather than one based on the equilateral triangle. The first stages are shown below for squares added (Figure 7) and squares removed (Figure 8).

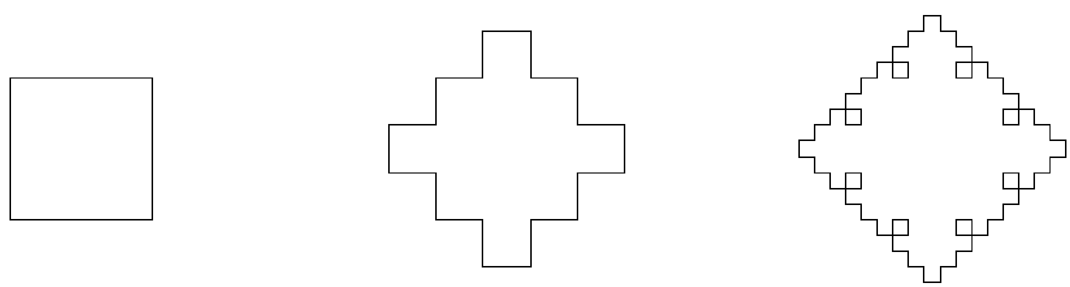

Figure 7: *fractal created by adding squares*

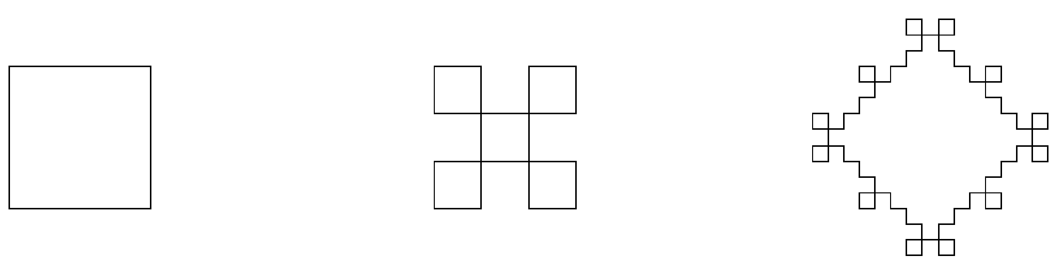

Figure 8: *fractal created by removing squares*

Students should start by drawing by hand the first four or five stages on squared paper (three stages are shown in Figure 7 and Figure 8). Once they can do this, they could either answer questions like those below, or be challenged to make a Logo program to create this or other fractals. Students should be encouraged to make up their own rules and design and explore their own fractals, using questions at a suitable level to motivate their exploration.

- Fractals are defined as possessing self-similarity – what does this mean?
- What is the scale factor for length between each stage?
- What symmetry does each stage show?
- What symmetry would the nth stage show?
- How does the perimeter (including loops) increase from one stage to the next?
- If area in a loop is counted as inside the figure for the first set (square added) and outside for the second set (square removed), what is the area for each stage?
- What is the area for the *n*th stage?
- How many loops are there for each stage?
- How many loops will the *n*th stage have?
- At each stage, and at the *n*th stage, what area is contained in loops?
- What is the fractal dimension?

A different way of creating a fractal is through tiling half or a quarter of a polyomino, [4]. For instance, we can start with a cross-shaped pentomino, cut it in half through its centre and use this half-pentomino to tile around the original shape, and then repeat this in successive layers (Figure 9). At each stage, the size of the tile is reduced so that the diagonal of the tile fits along one edge unit of the shape.

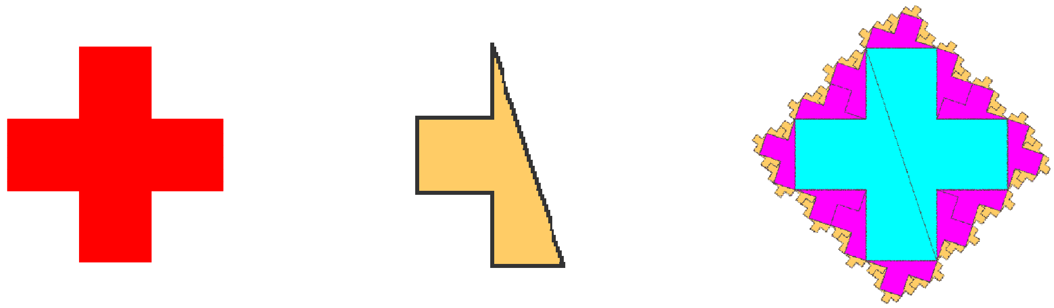

Figure 9: *fractal tiling of half pentominos around a pentomino*

Again a number of questions can be posed to students:

- How is self-similarity shown here?
- What is the scale factor for areas and for lengths at each stage of the tiling?
- What is the scale factor for areas and lengths for the *n*th stage?
- How many tiles are there at each stage, and at the *n*th stage?
- What is the perimeter and area of the tiled shape at each stage, and at the *n*th stage?
- Can this be done to all polyominos – if not, which can be tiled in this way, and which cannot?

Labyrinths

Labyrinths have a long history from very early times – for instance, the Greek legend of Theseus and the Minotaur, and examples of labyrinths in cave paintings in many places in the ancient world – to the labyrinths built in Christian churches and cathedrals in mediaeval and more recent times. Studying labyrinths is an opportunity to link maths with ancient myths, history and religious experience, [8].

The classic labyrinth, supposedly the one that housed the Minotaur on the island of Crete, is built up from a seed. In Figure 10, the initial seed (left, top row) and the following two stages are shown, then the 6th stage (right, top row) and finally the final stage (below). Students can practise drawing labyrinths from this and other seeds, then consider:

- Does it make a difference to the finished labyrinth where you start on the seed, or in which direction you draw the lines?
- What is the length of the path of a labyrinth compared to its area? (It helps to draw the labyrinth on squared paper for this question).
- Can you design your own labyrinth using your own seed?
- Do they all have just one entrance, one centre and one path through them? (More than one path means they are not labyrinths, but mazes).

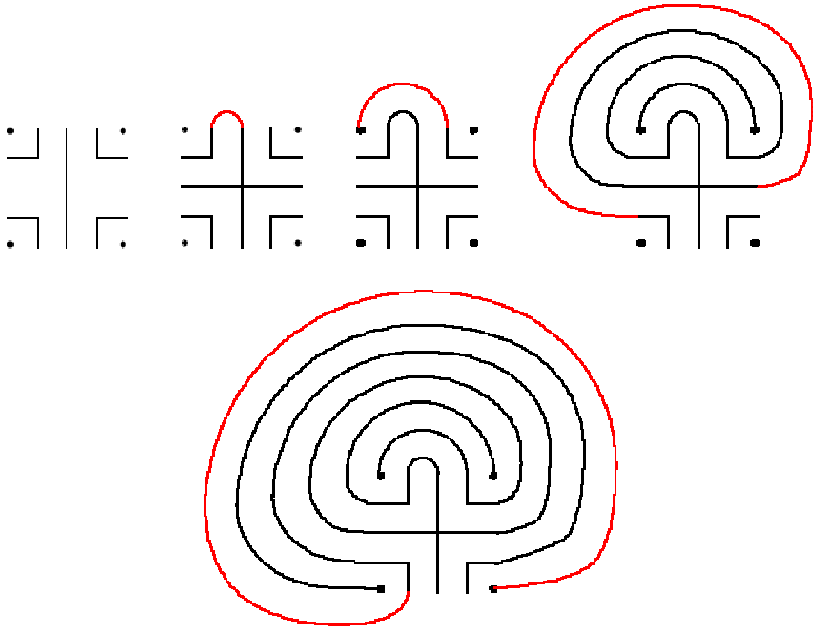

Figure 10: *Using a seed to create a labyrinth*

Celtic Knots

Celtic Knot work is another rich source of mathematical investigation, [6], which can also be linked with other curriculum areas, since Celtic Knots were used extensively by the Celtic peoples to decorate stone, wood, metal and literature [3]. Like Islamic art, Celtic artists were inspired by the supreme perfection of God.

Figure 11 shows a straightforward way to draw a Celtic Knot using a square grid, placing 'crosses' first on all horizontal lines, and then adding them to all vertical lines, and finally connecting these to make the finished knot. Once students are familiar with drawing the knots, they can investigate them further:

- How does the number of separate pieces of thread required for different size knots relate to the dimensions of the starting grid?
- Can you predict this for any size grid?
- How do these dimensions relate to the number of crossing points? Can you make predictions?
- What happens if you use a circular grid rather than a square one?

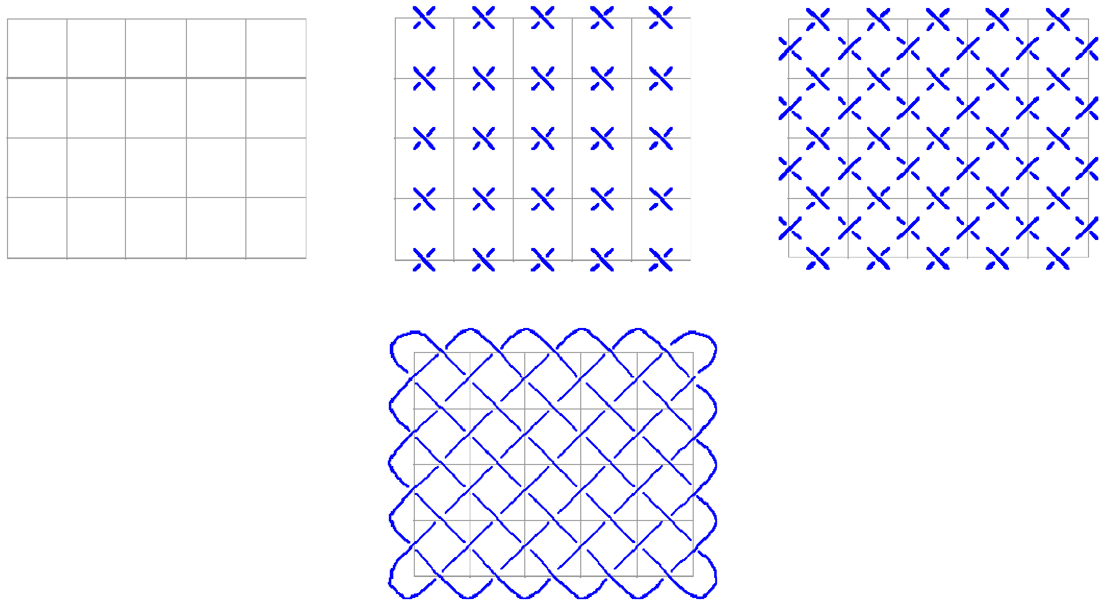

Figure 11: *Drawing a Celtic Knot on a grid*

References

[1] S.J. Abas, 'Islamic geometrical patterns for the teaching of mathematics of symmetry', *Symmetry in Ethnomathematics*, 12(1-2), 53-65. Budapest, Hungary: International Symmetry Foundation. 2001. www.ethnomath.org/resources/abas2001.pdf

[2] K. Critchlow, *Islamic Patterns – An Analytical and Cosmological Approach,* Thames & Hudson, London. 1976.

[3] Courtney Davis, *The Celtic Art Source Book*, Blandford, London. 1988.

[4] Robert W. Fathauer, 'Fractal Tilings Based on Dissections of Polyominoes', *Bridges London*. 293-300. 2006.

[5] http://www.maths2art.co.uk/ (Julie Dobson's website Maths2Art)

[6] http://motivate.maths.org/conferences/conference.php?conf_id=99 (project work on Celtic Knots)

[7] http://motivate.maths.org/conferences/conference.php?conf_id=107 (project work on Fractals)

[8] http://motivate.maths.org/conferences/conference.php?conf_id=108 (project work on Labyrinths)

[9] http://motivate.maths.org/conferences/conference.php?conf_id=98 (project work on Maths and Art)