Structure and Form in the Design Curriculum

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Abstract

This paper identifies subject matter included in an introductory lecture course concerned with structure and form, delivered to design students at the University of Leeds. By way of background, recognition is made of the important contribution made by scientific investigators at Leeds to the understanding of a range of related issues. Fundamental structural elements are identified and brief synopses are given of the nature of tilings, patterns and polyhedra. Various inter-related concepts, associated with the Fibonacci series and the golden section, scale symmetry and modularity are introduced. A range of important literature is identified and a series of assignment briefs is included. The intention is to identify curriculum material that has proved to be of value in the qualitative improvement of the practice-based activity of design students at Leeds. The paper is intended primarily as a guide to design teachers whose current professional duties involve the development of theoretical components to underpin a largely practice-based design curriculum.

Background

For much of the twentieth century, the University of Leeds played a pivotal role in the analysis and interpretation of patterns – the three-dimensional patterns which are the basis of crystal structures and the two-dimensional patterns which are the basis of fabric design, tessellations and tilings. This role may be said to have begun with the Nobel Prize-winning work of W. H. Bragg, Cavendish Professor of Physics, and his son W. L. Bragg. Working as a team, using x-ray diffraction techniques, they solved the first crystal structures in 1913. In the 1930s H.J. Woods of the Department of Textile Industries presented a comprehensive appraisal of symmetries in patterns [1 - 4]. Drawing on concepts which have their origin in the study of crystal structures, Woods was the first to present the complete and explicit enumeration of the two-color one- and two-dimensional patterns, visionary work which was several years ahead conceptually of the theoretical developments emanating from crystallographers worldwide. Today it is acknowledged widely that Woods helped to lay the foundation for our current thinking on the geometry of regular repeating patterns and tilings and, in particular, our knowledge of color (counterchange) symmetry. Between the 1930s and 1940s W.T. Astbury, building on work initiated by J. B. Speakman, also of the Department of Textile Industries at Leeds, pioneered the use of x-ray diffraction techniques to the elucidation of wool fiber structure, work which (it could be argued) led directly to the discovery of the structure of DNA. The underlying threads throughout all the diverse work charted above are structure and form, the ultimate determinants of aesthetic effect as well as physical performance.

The Leeds tradition continues and ideas more commonly associated with the nano structures of polymers continue to stimulate research thinking in the areas of patterns and structures. Contributions have been made at Leeds to furthering the understanding of patterns within different cultural contexts and historical periods [5 – 7], to identifying concepts of importance to pattern classification [8], and to the analysis and synthesis of counter-change patterns [9, 10]. Concepts have been developed in the field of layer symmetry, to aid advances in our understanding of the geometry of woven textiles [11, 12]. Tilings, tessellations and polyhedra are a research focus currently at Leeds. One project is concerned with the geometric patterning (using varieties of Platonic and Archimedean tilings) by impacting the surface of dyed woven fabrics with fine, high-pressure water jets [13]. Another is concerned with regular polyhedra and in exploring the application of periodic patterns or tilings to such structures [14].

The intention in this paper is to identify a range of concepts, which underpin the design curriculum at Leeds. The syllabus has been informed by the past and current research interests of the authors, as well as by the valuable insights provided by Washburn and Crowe [15, 16], Grünbaum and Shephard [17], Critchlow [18, 19], Elam [20], Kappraff [21, 22], Schattschneider [23, 24], Lawlor [25], Hargittai [26, 27] and Pearce [28]. Lidwell et al [29] presented a well-produced and well-illustrated introductory text that dealt with a wide range of geometric and other concepts of importance to designers; this should prove of value to teachers in the early stages of developing curriculum material.

The following topics are covered: theories of proportion, tiling the plane, regular and semi-regular tilings and tessellations, motifs, patterns and symmetry, pattern construction, modularity, the Fibonacci series and the golden section, polyhedra, scale similarity and fractals. The paper should be of particular value to design teachers whose current professional duties involve the development of theoretical components to underpin a largely practice-based design curriculum. To others, the paper may act as the basis of a simplistic refresher course, or as a source from which to adapt supplementary teaching material. The paper is by no means all encompassing. It simply identifies some of the more important components of the curriculum at Leeds, and identifies appropriate texts that may assist teachers in their quest to develop their own curriculum by including theoretical aspects of structure and form. It is important to stress that practice-based activities should be developed hand-in-hand with the theoretical delivery. By way of example, a few assignment briefs are included in the Appendix. It is worth noting that the relevant lecture course at Leeds is well received by design students. Meanwhile, tutors have remarked that qualitative improvements in the responses to studio/practice-based projects seem to have resulted, at least in part, from the inclusion of the curriculum material presented in summary form in this paper.

Point, Line, Form and Structure

It is of importance that designers develop a visual vocabulary and that they become aware of the inter-related structural elements of importance in art and design. These elements include: point, line, form, pattern and structure. An explanation of these should form the starting point in any lecture course concerned with structure and form. Point is the basic graphic element from which all visual expression springs. A collection of connected points (or a moving point) constitutes a line. Lines have psychological impact, influenced by their direction or orientation, weight and emphasis, and variations in these. Lines may be human-made or may be created by nature. A line may be implied (as an outline between two colors or two textures, for example) and may be orientated horizontally, vertically, diagonally or in any other direction in the plane. Lines may be straight or curved. Combinations of lines constitute forms, and create areas or masses, which define objects in space. In the context of this paper, the term *form* is used to express length and width (in two-dimensions) or length, width and depth (in three dimensions). Meanwhile the term *structure* is used to denote the underlying geometrical skeleton or framework of two- or three-dimensional forms. For definition and discussion of the nature of point, line and form, as well as other elements of a "visual grammar", it is worth referring to Leborg [30]. A series of practice-based assignments could also be developed from this source.

Polygons, Circles and Other Constructions

Polygons are enclosed figures with sides (represented, for example, by lines on paper). Regular polygons have equal sides and equal angles. The names attributed to regular polygons have their origin in Greek: a pentagon with five sides, a hexagon with six sides, a heptagon with seven sides, an octagon with eight sides, a nonagon with nine sides and a decagon with ten sides. Students should become familiar with the construction of both a regular pentagon and a regular hexagon. A circle may be considered as an infinite-sided polygon, without beginning or end. It is the easiest geometric figure to construct with accuracy and, over the years, has had a multitude of uses in the visual arts. Amongst much else, it is associated with rainbows, halos, the prayer wheel, the marriage ring, rose windows in European mediaeval cathedrals and pre-historic stone circles. It is a vital component in geometric construction and the discipline of geometry would have a limited range without it. Lawlor provided a useful review of various geometrical constructions [25].

Regular and semi-regular tilings

The term "tilings" is used when referring to polygons that cover (or tessellate) the plane, edge-to-edge, without gap or overlap. A regular tiling (or tessellation) is comprised of copies of a single polygon of the same size and shape. Only three regular polygons tessellate the two-dimensional plane: equilateral triangles, squares and hexagons. Tessellation is only possible where angles at the vertex (i.e. where the angles meet) add up to precisely 360 degrees. The three possibilities, each using one type of regular polygon, are known as the Platonic or regular tilings. Several forms of notation are in use. For example a tiling may be notated by choosing a vertex and counting the sides of the polygon that touches it as well as the number of polygons involved at the vertex. In the case of the hexagon tiling, three hexagons meet at a vertex and each polygon has six sides; the appropriate notation is 6.6.6. Using this system, the other regular tilings can be notated by 4.4.4.4 and 3.3.3.3.3.3. Tessellations of the plane, using two or more regular polygons, are also possible. With the further restriction that all vertices must be alike, there are only eight possibilities, these are known as the Archimedean or semi-regular tilings. Regular and semi-regular tilings are illustrated in numerous text books. Particularly informative texts have been provided by Grünbaum and Shephard [17] and Critchlow [19].

Motifs, patterns and symmetry

Motifs are the building blocks of patterns. The principal characteristic of a regular repeating pattern is the repetition of a motif by a given distance across the plane. Patterns are considered to have symmetry characteristics. In this case, the meaning of the term "symmetry" extends beyond its common every-day usage to cover geometric actions beyond bi-lateral symmetry. Patterns may exhibit one or more of four symmetry operations or symmetries. These are: translation, by which a motif undergoes repetition vertically, horizontally, or diagonally at regular intervals while retaining the same orientation; rotation, by which a motif undergoes repetition round an imaginary fixed point; reflection, by which a motif undergoes repetition across an imaginary line, known as a reflection axis; glide-reflection, by which a figure is repeated in one action through a combination of translation and reflection, in association with a glide-reflection axis. Patterns may be classified with respect to their symmetry characteristics. Combinations of the four symmetry operations yield seventeen possibilities (or classes). An explanation of fundamental concepts was given by Stevens [31], Hann and Thomson [32], Hann [10], and Washburn and Crowe [15]. A useful text from which to develop an understanding of symmetry in tilings was provided by Schattschneider [24]. It is worth developing a series of exercises that require students to identify the symmetry characteristics of motifs and patterns. Flow diagrams to aid identification of a pattern's symmetry class were provided in Washburn and Crowe [15].

Aperiodic Tilings

The regular and semi-regular tilings, mentioned above, translate (or repeat) in two distinct directions across the plane without gap or overlap. These may also be referred to as periodic tilings or tessellations. There is also a class of tilings that do not exhibit regular translation or repetition, but nevertheless cover the plane without gap or overlap. These are termed non-periodic or aperiodic. During the latter part of the twentieth century, the British mathematician Roger Penrose developed an aperiodic tiling, which exhibited five-fold rotational symmetry, at various points within its non-repeating structure. The tiling is comprised of two rhombi (known as kites and darts), one with angles of 36 and 144 degrees and one with angles of 72 and 108 degrees. In constructing the tiling it is necessary to adhere to a series of rules specified by Penrose [33].

Polyhedra

A polyhedron is a solid consisting of polygonal faces. These faces meet at edges, and the edges meet at vertices (singular vertex). There are five regular polyhedra (known as the "Platonic solids") each comprised of combinations of one specific type of regular polygon. With each Platonic solid, the faces are thus identical in size and shape, and the same number of faces meets at each vertex. The five Platonic solids are as follows: the tetrahedron (four faces), the cube or hexahedron (six faces), the octahedron (eight faces), the dodecahedron (twelve faces) and the icosahedron (twenty faces). A further set of polyhedra (thirteen in total) can be obtained from the Platonic solids. These are known as the "Archimedean solids", and each is formed from combinations of two or more types of regular polygonal faces. They are considered "semi-regular" and, in each case, the vertices are identical. Seven of the Archimedean solids can be obtained from the Platonic solids by slicing off either vertices or edges and producing "truncated polyhedra". An additional four solids can be obtained by "expansion" of a Platonic solid and one of the previous Archimedean polyhedra. The remaining two solids are obtained through manipulation of the cube and the dodecahedron. A good explanatory text was provided by Cromwell [34].

Fractals and Self-similarity (or Scale Symmetry)

A fractal is a geometrical shape made up of identical parts each of which is (at least approximately) a reduced/size copy of the whole. Fractal, from the Latin fractus meaning fractured or broken, refers to a unique type of geometric shape. Fractals have two distinct properties: they tend to exhibit infinite detail and they conform to the same shape at different scales, a property known as self-similarity. Fractals can be based on mathematical models, but are also common in real life. Examples of nature's fractals are clouds, coastlines, lightning, various vegetables (e.g. cauliflower and broccoli) and mountains. Although the roots of fractal geometry can be traced to the late 19th century, it was the work of Benoit Mandelbrot, in the 1960s and 1970s that popularized the concepts, and made fractal geometry accessible to a wider audience [35]. An interesting account of fractal geometry in architecture and design was given by Bovill [36].

Modularity (Minimum Inventory and Maximum Diversity)

Modularity embraces the concept of “minimum inventory and maximum diversity”. In other words, from a few basic modules (such as two or three tile shapes), a large collection of different structures (or solutions) is possible. The concept is of relevance to science, art and design, and can be detected throughout the natural world. It offers potential for innovation in the decorative arts and design, and is common in two-dimensional repeating patterns as well as architecture. Comprehensive treatises are provided by Pearce [28] and Jablan [37].

In Conclusion

A close inter-relationship between structure, form and performance can be found throughout nature, science, art, design, engineering and architecture. The search for the secrets of the inter-relationship has preoccupied thinkers for millennia. Plato, Aristotle and Pythagoras took on the challenge, but probably relied on systems of knowledge conveyed from the ancient cultures of ancient Egypt, Mesopotamia and India. Structure, form and performance continue to be of paramount importance in the modern world. The necessity to characterize and mathematically define these concepts is of relevance at all levels (nano, micro and macro). An understanding of what nature builds at the nano level can inspire humans to create grand macro-level structures such as geodesic domes. Most importantly it should be realized that certain geometric concepts and ideas, although sourced in ancient times, transcend the boundaries between art, science, mathematics and design. With insight and vision they still offer immense potential as problem-solving design tools in the twenty-first century. On gaining a basic understanding of the concepts outlined in the sections above, students should be able to conduct structural analyses of naturally occurring phenomena, human-made objects, images, paintings, sculpture, patterns, tilings and other forms of two- and three-dimensional designs. For a good, concise treatise dealing with the structural aspects of design (especially product design) it is worth referring to Elam [20]. A series of example assignments, which reflect some of the subject matter dealt with above, is presented in the Appendix to this paper. Where possible these or similar assignments should be integrated with conventional studio-based activity. Most importantly, design teachers must select and adapt syllabus material to address the requirements of their own students.

Appendix: Example Assignments

Assignment 1: Polygons, Tilings and Patterns. (a) Using a pair of compasses, a ruler and a pencil present precise drawings of a hexagon and a pentagon. (b) Explain why there are only three Platonic (or regular tilings) and provide a clear, precisely-drawn illustration of each together with an appropriate notation. (c) Present a precisely-drawn illustration for each of the eight Archimedean (or semi-regular) tilings. (d) Select twenty patterns from any published or observed sources. List the symmetry characteristics of each.

Assignment 2: Symmetry and Proportion. Select a frontal photograph of a human face that you consider to be very attractive. You are required to conduct a geometrical analysis of this image, with the objective of establishing the degree of bilateral symmetry and the presence of proportions/ratios, which conform to the Fibonacci series and the associated golden section. You may wish to conduct the following measures: top of head to tip of chin; centre of mouth to tip of chin; centre of mouth to tip of nose; tip of nose to bridge of nose; bridge of nose to ear; bridge of nose to pupil of eye; pupil to pupil; width of nose at outer part of nostrils; pupil to eyelash; eyelash to eyebrow; eyebrow to eyebrow; any other measure you believe to be appropriate. You may then wish to establish if there are any apparent relationships between these measures. You may wish also to draw a mid-way line, down the centre of the image, and to measure features to the right and left of this line. Once measurements and calculations have been completed, you are required to present you data in tabulated form, and to briefly discuss the significance of your findings.

Assignment 3: Symmetry in motifs. Making reference to the schematic illustrations of class cn and dn motifs, presented in Figure A1, classify the motifs presented in Figure A2.

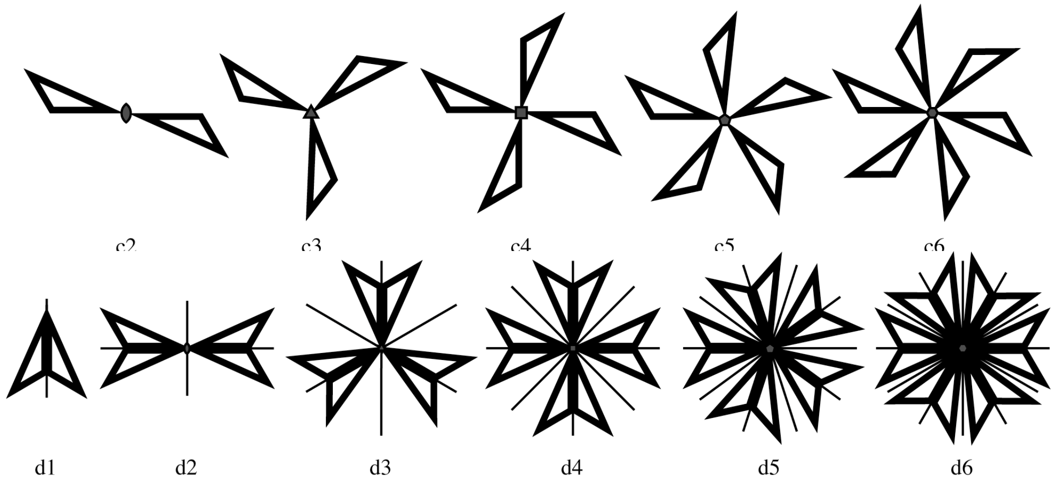

Figure A1 *Schematic illustrations of cn and dn motifs.*

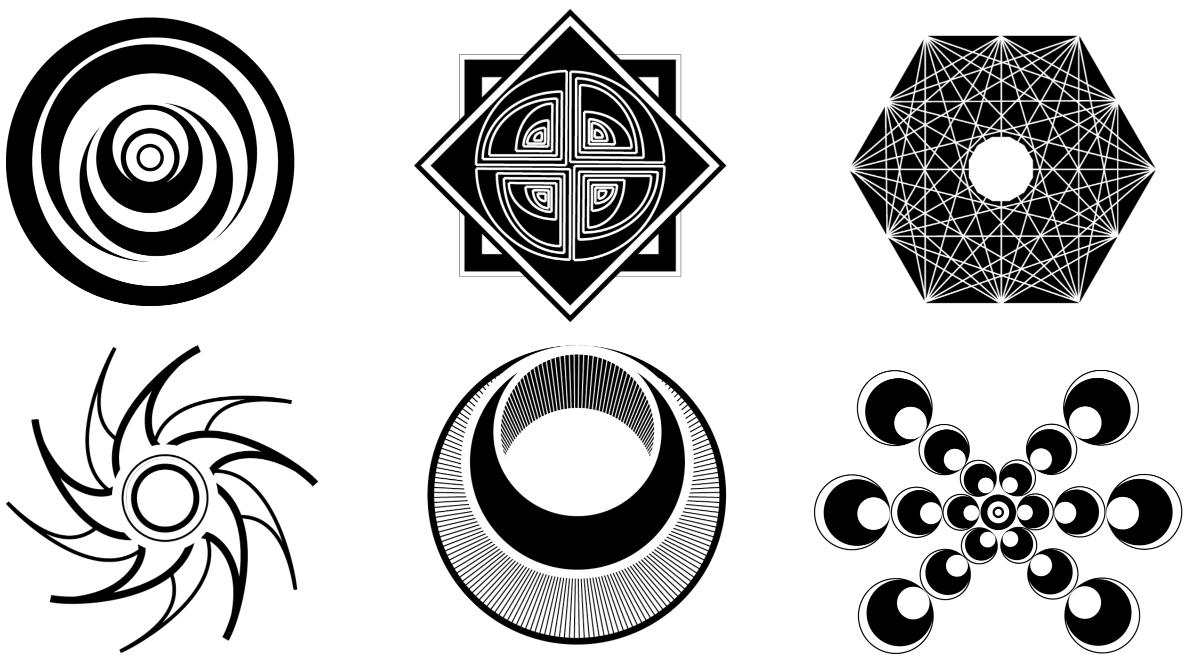

Figure A2 *Examples of cn and dn motifs.*

Assignment 4: Modularity and Pattern Construction. You are required to produce a collection of repeating designs, each created from tiling elements cut or drawn from a regular polygon (six designs from elements of a square, six designs from elements of an equilateral triangle and six designs from elements of a hexagon). To begin, draw a square to dimensions of your choice. Cut into two or more unequal parts. You have thus produced two or more tiles of different dimensions. Color each tile with a color of your choice. Make multiple copies (by scanning or photocopying each colored tile). Use these two or more different shaped tiles (in any numerical proportion you wish) to create a collection of six periodic tilings, which cover the plane without gap or overlap. A minimum of four repeats of each design must be shown. Each design must be original, precisely drawn, and distinctly different, and must not relay solely on a change of scale as a means of differentiation. Feel free to use computing software of your choice. Repeat the process using a regular hexagon and an equilateral triangle. Examples of twelve modular designs created from an equilateral triangle and a square are shown in Figure A3.

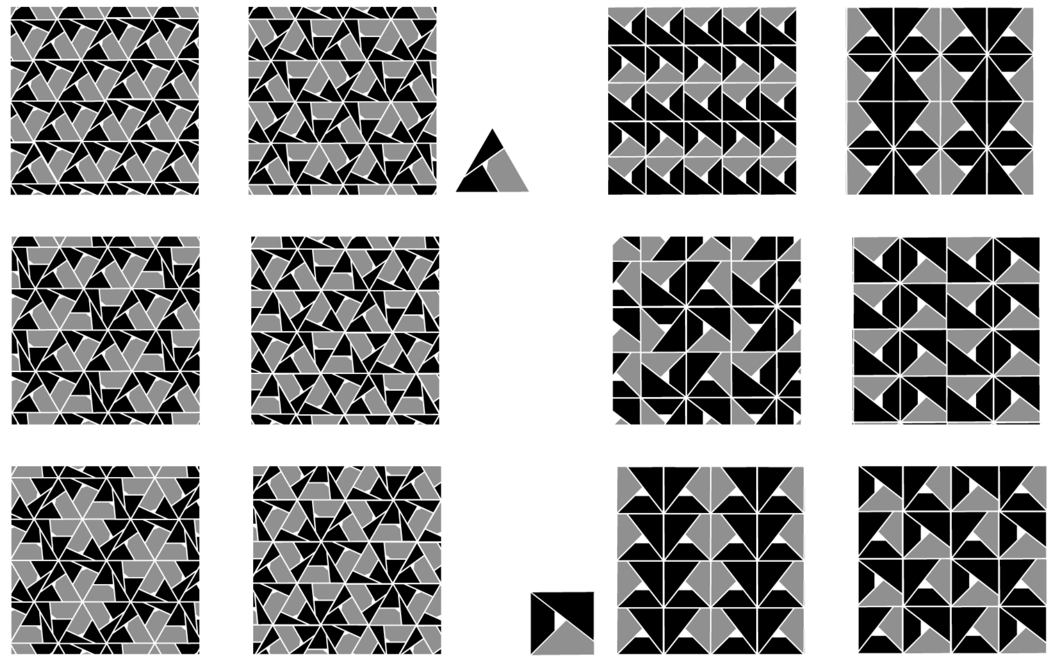

Figure A3 *Modular designs created by tessellating units cut from an equilateral triangle and a square.*

Assignment 5: Coloring Polyhedra. What is the minimum number of colors required to systematically color each of the five Platonic solids? Your answer must be subject to the rule that two faces with a common edge are not allowed to have the same color. Provide clearly-labeled, precisely-drawn illustrations.

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