Mathematical Book Forms for Teachers

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The sequential properties of basic mathematics facilitate the creation of math art book forms. This workshop presents three artists book forms with mathematical significance for school teachers. Scissors, glue stick, protractor, straight edge and pre-cut paper are the only required equipment to make all three forms.

Book Form 1: Single Sheet Instant Books
Book Form 2: Star Books
Book Form 3: Trihexaflexagons

These basic forms use elementary geometric knowledge, while letting Student explore creative ways to use mathematical information in adding the content to the pages or faces of these forms.

Book Form 1: Single Sheet Instant Books

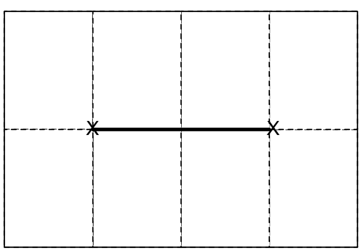

Figure 1

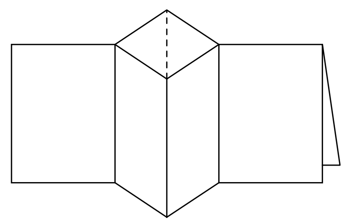

Figure 2

The basis for this book is a single sheet of photocopying paper (8.5" x 11"). The great thing about these books is that you only need to have images on one side of the paper to create a book with 2-sided pages. Start by folding the paper in half, then each half in half again, like you were making a fan. Then unfold and fold in half the other way, lengthwise. See figure 1. Unfold again and make a slit in the last fold between the two X's in figure 1. To assemble the book, fold lengthwise again, open up the slit and pinch closed, perpendicular to the original slit. See figure 2. Now, fold all the pages together, and you will end up with an 8-page book. This form works well for books about counting, fractions and other basic concepts. Since it is one-sided, it is perfect for making large editions using photocopied images.

Book form 2: Star Books

Each book requires at least 4 sheets of square paper. Take one sheet and fold it in half. Unfold, and then fold in half in the other direction. Make sure both folds are going the same way (both "hills"). See figure 3. Unfold again. Now fold diagonally, creating a valley fold. Unfold your square. It should now look like figure 4.

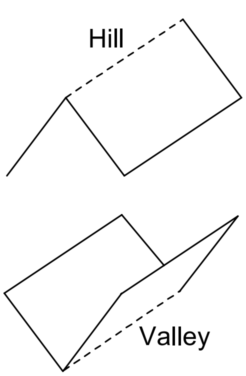

Figure 3

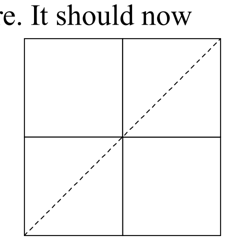

Figure 4

Pinch in at the diagonal valley fold and flatten to form a small square. This is the basic component of the book. Repeat this process for all other sheets of paper. When all of the square components are complete, they are glued together, folded point to folded point, one on top of the other to make a stack. When the glue is dry, the outside corners can be opened to reveal a star. This type of pop-up book has great visual impact, because there are multiple elements each student can make one square that can be assembled into a group book as a class project. With a slight variation in gluing and more sheets of paper, this form can also be made into a snake book that can be displayed open along a wall.

Book Form 3: Trihexaflexagons

Martin Gardner defined Flexagons in Scientific American in 1956: 'Flexagons are paper polygons, folded from straight or crooked strips of paper, which have the fascinating property of changing their faces when they are "flexed".' [1] Garner attributes the discovery of Flexagons to Arthur Stone, an English graduate student at Princeton in 1939. Trihexaflexagons are shaped like hexagons and have three distinct faces. The construction of Trihexaflexagons requires more accurate folding and can be frustrating for someone too young. I start with a strip of paper in a 1 to 7 ratio, and then draw a line with 30 degree angle from the corner. See figure 5. Cut off the little right triangle. Now you are ready to fold: bring the corner point up to the top edge and create the first fold to make an equilateral triangle. See figure 6.

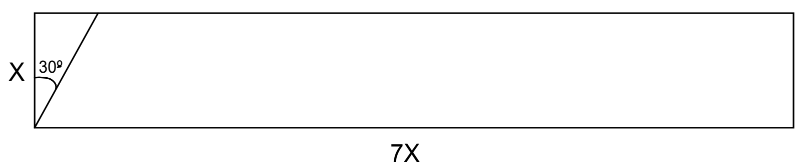

Figure 5

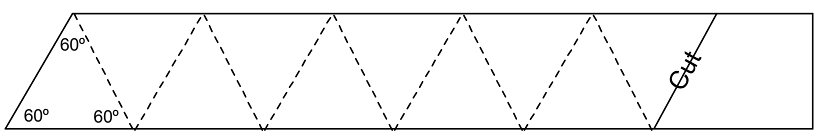

Figure 6

Continue folding, under and over, like a fan. Bring the corner to the edge, folding carefully, so that each time you make a new equilateral triangle. When you have 10 equilateral triangles, stop folding. You should have some paper left over, cut that off. Right now you should have a stack of folded triangles. Take the last triangle, wrap it around, and glue it to the first triangle. Once the glue dries you can flatten out your Trihexaflexagon. You can change the face on your Trihexaflexagon by pinching in towards the center and then re-opening on another fold. Decorating the three faces of the Trihexaflexagon is the fun part. Seeing how the faces change can be an interesting way to explore equilateral triangles and to discuss symmetry.

Assembling these forms corresponds with Van Hiele's levels of understanding geometry [2]. The 2nd level, analysis, relates to identifying the properties of rectangles within rectangles in Form 1. The 3rd level, abstraction, relates to the relationship between the square and the triangles in Form 2 and the interaction between the equilateral triangles in Form 3. Folding flat sheets of paper into 3D shapes helps visualize the relationship between 2D and 3D geometry. Several topics could be addressed: the study of scale and surface area (Form 1) and the study of angles (Forms 2 and 3).

References

[1] Martin Garner, *Hexaflexagons and Other Mathematical Diversions*, University of Chicago Press, Chicago, 1959

[2] P.M. van Hiele, *Structure and Insight. A Theory of Mathematics Education*, Academic Press, Orlando, 1986