Moving Beyond Geometric Shapes: Other Connections Between Mathematics and the Arts for Elementary-grade Teachers

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Abstract

When classroom teachers are asked to identify connections between mathematics and art, they typically refer to geometric concepts. In an attempt to broaden their understanding of potential connections, this paper presents activities that involve common vocabulary, probability, and imagery.

Introduction

In 1989, the National Council of Teachers of Mathematics published its *Curriculum and Evaluation Standards for School Mathematics* [1]. The Council proposed ten standards. Five focused on content and five presented processes. Since the publication of this document as well as its revised edition in 2000, many states have rewritten their own state documents to reflect the vision of the Council. Some states have chosen to include all five process standards while others have chosen to embed processes within the content strands.

The process that is often embedded within the content listings is "connections." This standard encourages teachers to develop curriculum that enables students to make connections among mathematical representations (e.g., concrete, pictorial, and abstract levels of knowledge), between mathematical domains (e.g., algebra and geometry), and between mathematics and other content domains.

Since the *Standards* were originally published over 15 years ago, one could hope that students entering teacher education programs today would be aware of numerous connections within mathematics and between mathematics and areas outside of mathematics. The hope might be even higher when considering inservice teachers who have experienced teacher professional development either through university-level methods courses or conferences sponsored by mathematics-related professional organizations. Unfortunately, this hope is easily dashed when one interviews both preservice and inservice teachers.

Two groups of teachers (one inservice and one preservice) were asked to identify how mathematics could be connected to five areas common to an elementary or middle-school curriculum: science, social studies, art, language arts, and physical education. Many of both the inservice and preservice teachers indicated they'd experienced more difficulty thinking of connections between mathematics and art than with any of the other content areas. One of the preservice teachers, who has a bachelor's degree in music, stated he'd had a "tough time thinking about a connection between math and art" as his degree was in music. He pointed out that the data collection sheet had indicated "art" and not "the arts," so he'd thought about "paintings, not music or dance."

A quick perusal of their responses showed that the variety of connections between mathematics and art was smaller for the inservice teachers than the preservice teachers. Both groups tended to focus on geometric connections but one or two within each group mentioned patterns, golden ratio, and music. Only one student (a preservice teacher) mentioned anything other than content (e.g., angles, proportions, shapes, color, timelines). She stated that both mathematicians and artists use spacial [sic] reasoning.

In an effort to broaden teachers' perspectives of how mathematics may be connected to "the arts," several classroom-based activities will be discussed in this paper. While some will result in products with geometric bases, the focus will be on how other mathematical content is used to generate the end product.

Activities

It's All in the Name. Investigating patterns generated by children's names can be an introduction to algebraic thinking. While this activity does not necessarily produce a "work of art," it provides a visual image for concepts that aggravate many middle-grade teachers and students: common multiples and least common multiples.

Give students a sheet of centimetre grid paper. Have students outline a 5x5 section. Then, beginning in the upper-left corner of the section, have students lightly print their names, one letter per cell, wrapping to the next row if their name is longer than 5 letters. Continue printing until all 25 cells are filled. Using two colors of markers, students color vowels in one color and consonants in another. When complete, students look for and describe patterns in their design. If they cannot see a pattern in the grid, have them add rows and continue their name and coloring. Then students look for other students who have the same design. Samples for the names of Ginny and Marilyn are shown below with consonants shaded and vowels left white.

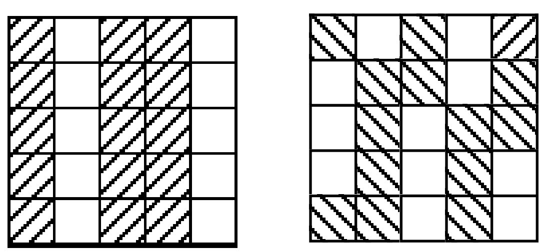

Figure 1: *5x5 grid for the names Ginny and Marilyn.*

Once students have discussed patterns generated on a 5x5 grid, students can be asked a variety of questions such as:

a) What might you pattern look like if the grid were 6x6 rather than 5x5?
b) How is the pattern of a name with an even number of letters the same as (or different from) the pattern of a name with an odd number of letters?
c) What would the pattern of a 9-letter name look like on a 5x5 grid?

Variation of It's All in the Name. Again provide students with cm grid paper. Have students outline a 10x10 section. After printing their names as in the previous activity, have them use a black (or other dark color) to shade in the cells that have the last letter of their name. Have students find a partner whose design is different from their own. Then, numbering the cells from 1 to 100 from the upper-left corner and moving across the grid, have them find the numbers of the darkened cells. For example, for the name Ginny, the darkened cells would be 5, 10, 15...100 while Marilyn's would be 7, 14, 21...98. Then have students find all the darkened cells they have in common (i.e., common multiples) as well as the first darkened cell in common (least common multiple).

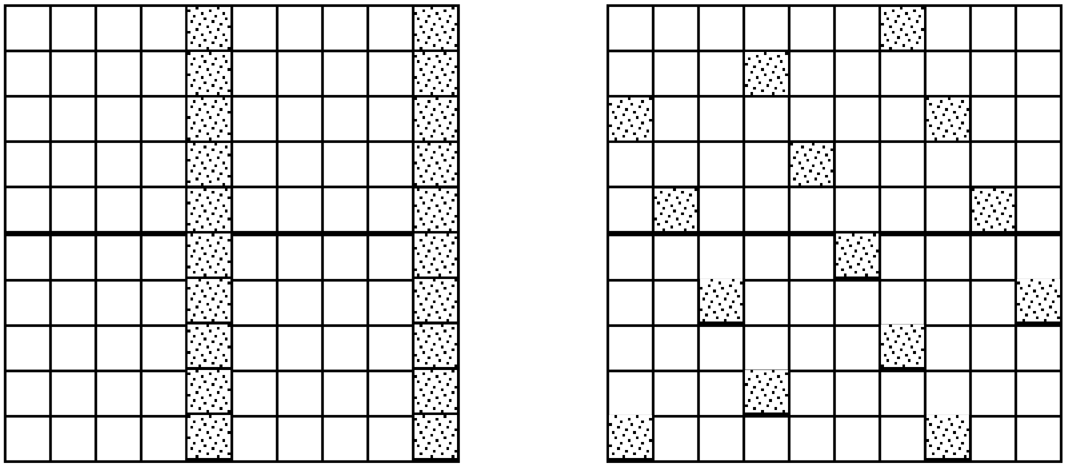

Figure 2: *10x10 grids for last letters of Ginny and Marilyn.*

Clapping Names. A similar activity for finding common multiples has a musical foundation and may be more appropriate for auditory learners. This activity is taken from the work of Schaffer, Stern, and Kim [2].

Students determine which letters in their names are consonants and which are vowels. While quietly spelling their names, they slap their thighs for consonants and clap their hands for vowels. For example, *Ginny* would be Slap, Clap, Slap, Slap, Clap. Have students make the final sound of their name louder than the rest of the word. Also, have students practice their pattern several times, being sure not to make a break at the end of the name. So, if *Ginny* was done four time, the sequence would be Slap, Clap, Slap, Slap, **Clap**, Slap, Clap, Slap, Slap, **Clap**, Slap, Clap, Slap, Slap, **Clap**, Slap, Clap, Slap, Slap, **Clap**. Then, working in partners, have students do their slap-clap patterns multiple times, using the same rhythm. Have them determine when their final sounds meet. Whether the final sound is a vowel or consonant is immaterial as the common multiples appear as a result of the louder sound at the end of the names. For *Ginny* and *Marilyn*, the first loud sound together would be the 35th sound. This activity may need a third person to count the number of sounds as children (and adults) often get lost in the slap-clap sequence.

S C S S **C** S C S S **C** S C S S **C** S C S S **C** S C S S **C** S C S S **C** S C S S **C**
S C S C S C **S** S C S C S C **S** S C S C S C **S** S C S C S C **S** S C S C S C **S**

Figure 3: *Ginny and Marilyn as slaps and claps.*

Conga Rhythms. A variation of the Clapping Names activity provides a connection between mathematics and rhythms and symbols in music. In *Conga Drumming: A Beginner's Guide to Playing with Time*, Dworsky and Sansby introduce a form of representation that uses both numbers and symbols. Again, multiples and common multiples are found when sounds converge. [3]

Chancey Art. While many teachers quickly identify geometric connections between mathematics and art, few would recognize the potential of using probability to generate artistic products. However, that is the basis for many of the wall paintings of Sol LeWitt, a contemporary minimalist. Some activities result in an almost Picasso-looking picture.

Collect pictures from magazines (one for each student) and trim them into squares. Have students fold a picture into 16 squares (fold twice horizontally and twice vertically) and cut along the fold lines. Then, onto a backing paper, have them piece the 16 cells back into the original picture (do not glue yet). Using a regular die, students roll the die, perform the associated movement on the first cell (upper-left corner), and glue the piece in place. Students repeat the roll, move, and glue sequence on each of the other 15 squares. The following moves might be used:

1 one-quarter turn clockwise
2 one-quarter turn counter-clockwise
3 half turn
4 flip so backside is facing upwards
5 trade piece with one in some other cell
6 do not change the piece's position

An extension of this activity will require two copies of each original picture. After students have folded, cut, and moved the cells of the original picture, have the extra copy of each picture available for display. Students need to match the uncut picture with the transformed one. Having pictures with similar colors makes the activity slightly harder.

Another extension focuses on the mathematical foundation of the activity. Provide each student with a 1x4 strip similar to the one below.

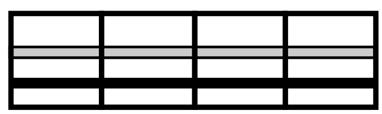

Figure 4: *1x4 strip used in Chancey Art follow-up lesson*

Have students separate the strip into 4 square cells and put them back into a 1x4 strip that looks like the original (i.e., lines match). Then each student rolls a die and performs the move on the first cell (left-most cell). Have students determine how many other students have a 1x4 strip that looks exactly like theirs. What is each student's chance of finding a matching strip? One might think that, since there are 6 potential actions, the chance of finding a potential match is 1 in 6. However, this is only true if each move results in a unique "look." If the strip shown above had only one line across the middle, both quarter turns would end up with the line in the same position. Therefore, it is critical that each move result in a different look.

Then have students roll the die and move the next piece. Again, have them find matching strips and discuss why there might be fewer matches. Continue with the third and fourth cells.

Since this activity quickly results in few matching strips, it may be easier for students to find patterns and apprehend the influence of combinations if a modified set of moves is used. For example, rather than having each number on the die produce a different movement, changes in the position of the cell could be simply based on whether the result of the die is odd or even. Then, there would be 2 outcomes for the first cell, 2 for the second, 2 for the third, and 2 for the fourth, or 16 combinations. The length of the strip can also be shortened for younger children.

Circle Art. Roll a regular die. On a 5x5 grid, use any point as the center of a circle that will have a radius of 1. If the number on the die is 1 or 4, draw a quarter circle. If the number is 2 or 5, draw a half circle. If the number is 3 or 6, draw three-quarters of a circle.

Roll again. Draw another arc (“unfinished” circle) using the second end-point of the previous arc as the beginning point of the new arc. Repeat 8 more times. Each curve must stay within the limits of the 5x5 grid. If using the last end-point will force you out of the grid, you may choose another point. You may retrace previously drawn curves and reuse end-points.

When you have rolled the die 10 times, transfer your design to the upper-left quadrant on a 9x9 grid. Complete the 9x9 grid by flipping the design over the vertical and horizontal lines. Shade your final product in a symmetrical fashion.

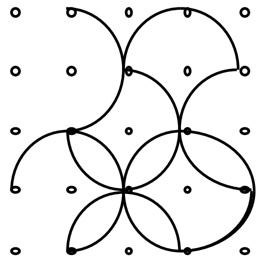

Figure 5: *Result of rolling 2, 3, 1, 1, 5, 6, 2, 1, 4, 3*

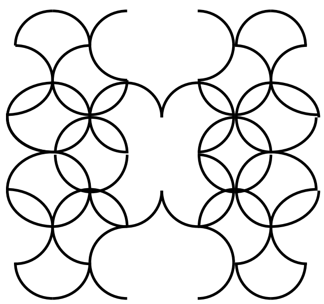

Figure 6: *Design flipped over horizontal and vertical axes, grid points removed.*

Thistle Quilts. Quilts are often used to demonstrate the use of transformational geometry. However, they can also be used to teach about sequencing. Provide students with square pieces of paper 8 or 10 cm on a side. Have them find the midpoints of the sides. If they use a folding method, the edges of the paper should just be pinched.

Beginning at either a vertex or a midpoint, have them draw a line segment to a non-adjacent vertex or midpoint. They should continue drawing segments beginning at a vertex or a midpoint, keeping the following rules in mind:

1. They may not cross a segment already drawn. That is, they must stop when a new line segment reaches a previously-drawn segment, and
2. At least one of the regions created by the new line segment must be a triangle.

They may use the same beginning point more than once. But the only time an ending point may be used more than once is if it is on the perimeter of the square. Draw segments until 8 regions have been created. Quilt squares are then traded with a partner. Students determine the order in which the line segments were drawn. Encourage students to determine whether the quilt square could have resulted from more than one sequence.

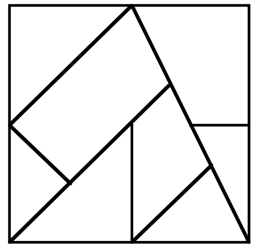

Figure 8: *Sample of a thistle quilt block.*

Taxi-cab Art. Students can investigate geometric concepts through taxi-cab geometry. The co-ordinate plane is likened to city streets that run parallel or perpendicular to each other. To travel from one point to another, a taxi-cab must stay on the streets (grid lines) and may not move into a cell as that would be viewed as driving into or through buildings.

First, have students choose a number between 4 and 8. Later in the activity, this number will be used to determine when their travels are finished. Students roll a regular die. On a 5x5 grid, they begin at any point and draw a line that shows a taxi-cab's trip. The trip covers as many "blocks" as the number on the die. During the trip, the cab can turn corners but cannot make U-turns at or between intersections.

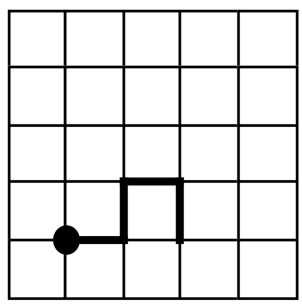

Figure 9: *Sample of a four-block trip beginning at the circle.*

Students roll again and draw another cab ride. The new trip starts where the last one ended. During this and subsequent trips, students may go "around blocks," thus creating a closed figure. They may also create a closed figure using parts of previous trips. Each trip must stay within the limits of the 5x5 grid. They continue rolling and drawing cab rides until they have generated as many closed figures as the number they choose at the start of this activity.

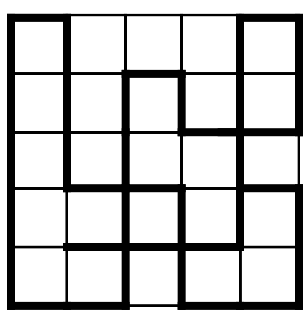

Figure 10: *Taxi-cab trips resulting in 5 closed figures.*

When they have made the appropriate number of closed figures, they transfer their design to the upper-left quadrant on a 9x9 grid and complete the 9x9 grid by flipping the design over the vertical and horizontal axes. The final step of this activity is to shade the end product in a symmetrical fashion.

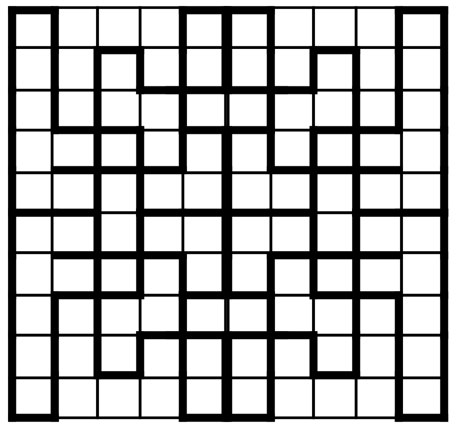

Figure 11: *Taxi-cab trips resulting in 5 closed figures, reflected across horizontal and vertical axes.*

Imaginings. While many activities produce a tangible artistic product, some classroom activities develop mental skills, such as visualization. Below are several exercises that are purported to develop one's ability to visualize. More activities can be found at <http://math.ucsd.edu/~doyle/docs/mpls/handouts/handouts.html>.

1. Picture your first name. Read off the letters backwards. Try other words.
2. Imagine a square. Picture the midpoints of each side. Cut off each corner of the square, cutting from midpoint to midpoint. Sketch the result of the leftover shape. Rearrange the cut-off points to form a new square.
3. Visualize 2 squares. Place the second square centered over the first square but at a forty-five degree angle to the first. Sketch the intersection of the 2 squares.
4. Mark the sides of a square into thirds. Cut off each of its corners back to the marks. Sketch the result.
5. How many colors are required to color the faces of a cube if no two adjacent faces have the same color?

This activity is rich with ambiquity. Results of the imaginings often depend upon the original images created in the student's mind. A debriefing session held at the end of each exercise allows students to describe their results and provides opportunities to enrich vocabulary and develop precise meanings and definitions. An alternative closure to each activity is to have students instructor a partner to sketch the image they describe and discuss words and phrases which were helpful or un-helpful.

Some "imaginings" have more than one possible answer. Have students choose one and reword it so there is only one possible answer. Also have students reword an imagining so there is more than one possible answer.

Common Terminology. In addition to sharing certain processes such as visualization, mathematics and the arts share terminology. However, their definitions and uses are not always consistent. Provide students with a list of words similar to that given below. Have them discuss how the term is used in both mathematics and the arts and decide whether the meanings are similar or not.

Additive/Subtractive
Complement/Complementary
Line
Positive/Negative
Proportion

Figure 7. *Sample words used in both mathematics and the arts.*

While "additive/subtractive" and "proportional" may have relatively similar meanings to both a mathematician and an artist, the other words in the list may not. To an artist, "complementary" refers to the hue directly opposite on the color wheel; to a mathematician it could mean two angles whose sum is 90 degrees. To a mathematician, "positive" and "negative" refer to a number's relationship to zero; to an artist, they relate to the foreground and background. Possibly the biggest difference occurs with the word "line." To a mathematician, a line has no beginning or end, is straight, and has no width; to an artist a line is a mark left by a dot or point moving continuously through space or over a surface. It starts somewhere, ends somewhere, and may have texture. It may be thick or thin; straight, curvy, or zigzagged; horizontal, vertical, or diagonal. When artistically-talented students express frustration with mathematics, it may be rooted in their confusion about terminology.

Each of the described activities focuses on connections between mathematics and art as a visual, and typically 2-dimensional product. Other connections could involve dance or music and symmetry or fractions. In addition to the numerous connections involving concepts, teachers should also investigate how processes such as problem solving and communication are common to both mathematics and the arts.

References

[1] National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA, 1989.

[2] K. Schaffer, E. Stern, & S. Kim. *Math Dance*. URL: http://www.mathdance.org

[3] A. Dworsky & B. Sansby. *Conga Drumming: A Beginner's Guide to Playing with Time*. Dancing Hands Music: Minneapolis, MN, 1994.